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Robust Fuzzy Sliding Mode Control for Antilock Braking System

M. Oudghiri, M. Chadli, A. El Hajjaji

Centre de Robotique, d'Electrotechnique et d'Automatique, EA 3299
7, Rue du Moulin Neuf - 80000, Amiens – France.
Phone: (33) 3 22 82 76 80
{mohammed.oudghiri, mohammed.chadli, ahmed.hajjaji}@u-picardie.fr

Abstract. *This paper focuses on the development of a robust fuzzy sliding-mode scheme for controlling a vehicle motion system by continuously adjusting the brake torque, Fuzzy logic known for its properties of universal approximator and sliding mode control for its robustness in the presence of parameter variations and the disturbances are employed to control the wheel slip rate in emergency braking maneuver. Simulations under various road conditions are performed to demonstrate the effectiveness of the proposed robust fuzzy sliding mode control.*

keywords. *Antilock Braking System (ABS), Fuzzy Control, Sliding Mode Control, Lyapunov*

1. INTRODUCTION

One of the main areas of research being undertaken in the automotive industry is that of vehicle chassis control in terms of handling performance, ride comfort and traction/ braking performance [Abe 1999]. The principle aim of this research is to satisfy increasing safety, efficiency and comfort requirements. Significant progress has been achieved by the introduction and development of active safety systems like anti-lock braking systems (ABS), traction control (TC), electronic stability program (ESP) which have become an integral part of modern passenger vehicles. One of the most popular active safety systems is the ABS which has dramatically improved vehicle handling in braking maneuvers, it has been developed initially to prevent wheel lock-up when the brakes are activated by automatically modulating the brake pressure during the emergency stop. An ABS controls the slip of each wheel to prevent it from locking such that a high friction is achieved and steerability is maintained. ABS controllers are characterized by robust adaptive behavior with respect to highly uncertain tyre characteristics and fast changing road surface

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properties [SAE 1992]. Some recent research proposes to use the advanced control to design a new generation of the ABS [Tan1990; Unsal 1999; Buckhort 2002; Layne 1993].

Research in the field can be classified according to the control methodology used. Below, we present recent studies, which are to be compared with the solution given in this study. The model-based approach in [Drakunov, 1995] applies a search for the optimum brake torque via sliding modes. This approach requires the tyre force, hence, a sliding observer is used to estimate it. The approach is tested in hardware in the loop simulator [Kawabe 1997] and also in a vehicle. A derivative part depending on the rotational acceleration is introduced in order to reduce the chattering of the sliding controller. In [Freeman 1995] another theoretical approach is presented where authors design an adaptive Lyapunov-based nonlinear wheel slip controller. This controller has been extended in [Yu 1997] by introducing speed dependence of the Lyapunov function and also including a model of the hydraulic circuit dynamics. A feedback linearization to design a slip controller is suggested by [Liu and Sun 1995] where gain scheduling is used to handle the variation of the tyre friction curve with respect to speed. In [Taheri an Law 1991] a simple PD wheel slip controller by the Ziegler-Nichols rule is designed, focusing on the desired slip value. The desired slip is estimated by evaluating the switching of a conventional ABS. Additionally, a modification of the desired slip according to the steering angle is also proposed. In [Solyom 2003], the authors propose a gain-scheduling control structure based on tire-slip and maximum friction coefficients are not directly measurable but can be estimated. A method based on static-state feedback of longitudinal slip is proposed in [Baslamisli 2007], it does not involve controller scheduling with changing vehicle speed or road adhesion coefficient estimation

The present study propose a robust control method for ABS which combines fuzzy logic and sliding mode for wheel slip control in emergency braking case. The structure of this paper is as follows. In section 2, ABS formulation is presented, Section 3 provides an introduction to sliding mode control and to fuzzy logic, and describes the structure and design method of the control algorithm. The effectiveness of the proposed method verified by simulation under various road conditions is presented in Section 4. In Section 5, the main conclusions of the work are drawn.

2. ABS FORMULATION

2.1 ABS MODEL

The dynamic equations of ABS are the result of Newton’s law applied to the wheels and the vehicle [Layne 1993].as shown in Figure 1. The vehicle dynamic is determined by summing the total forces applied to the vehicle during a braking operation to obtain

$$\dot{V}_v(t) = \frac{-1}{M_v}\left[4F_t(t) + B_v V_v(t) + F_\theta(\theta)\right] \quad (1)$$

$$\dot{w}_w(t) = \frac{1}{J_w}\left[-T_b(t) - B_w w_w(t) + T_t(t)\right] \quad (2)$$

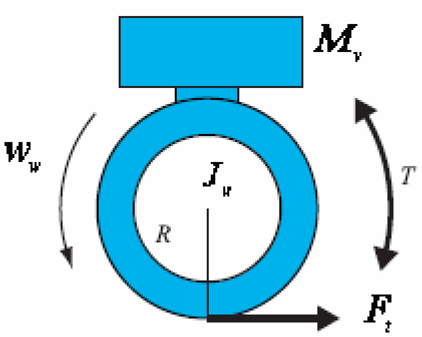

Figure 1Quarter car forces and torques

$V_v(t)$	the velocity of the vehicle
M_v	the mass of the vehicle
B_v	the vehicle viscous friction
$F_t(t)$	the tractive force
$F_\theta(\theta)$	the vertical force applied to the car
B_w	the viscous friction of the wheel
J_w	the rotation inertia of the wheel
$T_b(t)$	the braking torque
$T_t(t)$	the torque generated due to slip between the wheel and the road surface

The expressions of different forces are given as follows

$$F_\theta(\theta) = M_v g \sin(\theta) \quad (3)$$

$$F_t(t) = \mu(\lambda) N_v(\theta) \quad (4)$$

$$N_v(\theta) = \frac{M_v g}{4}\cos(\theta) \quad (5)$$

$$T_t(t) = R_w F_t(t) \quad (6)$$

where θ is the angle of inclination of the road, g is the gravitational acceleration constant, $N_v(\theta)$ is the vertical force applied to the wheel, and $\mu(\lambda)$ id the coefficient of friction.
Note that

$$w_v(t) = \frac{V_v(t)}{R_w} \tag{7}$$

Is the angular velocity of vehicle, where R_w is the radius of the wheel.

The longitudinal slip is defined by

$$\lambda(t) = \frac{w_v(t) - w_w(t)}{w_v(t)} \tag{8}$$

It describes the normalized difference between the angular velocity of vehicle and the angular velocity of wheel. The slip value of $\lambda = 0$ characterizes the free motion of the wheel where no friction force F_t is exerted. If the slip attains the value $\lambda = 1$, then the wheel is locked ($w_w = 0$)

2.2 FRICTION COEFFICIENT

It characterizes the road and has the properties $\mu(\lambda = 0) = 0$ and $\mu(\lambda) > 0$ for $\lambda > 0$. Its typical qualitative dependence on longitudinal slip λ is shown in Figure 1. It shows how the friction coefficient μ increases with slip λ up to a value λ_0, where it attains its maximum value μ_H. For higher slip values, the friction coefficient will decrease to a minimum μ_G where the wheel is locked and only the sliding friction will act on the wheel. The longitudinal force gets smaller as side slip angle is increased. This physical phenomenon is the main motivation for ABS brakes, since avoiding high longitudinal slip values will maintain high steerability and lateral stability of the vehicle during braking. Achieving this by manual control is difficult because the slip dynamics are fast and open loop unstable when operating at wheel slip values to the right of any peak of the friction curve.

The dependence of friction on the road condition is shown in Figure 2. For dry and wet roads

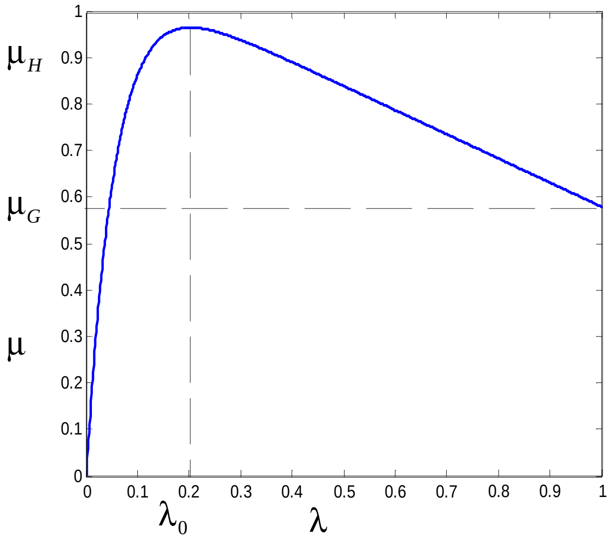

Figure 1 dependence of friction on the road condition

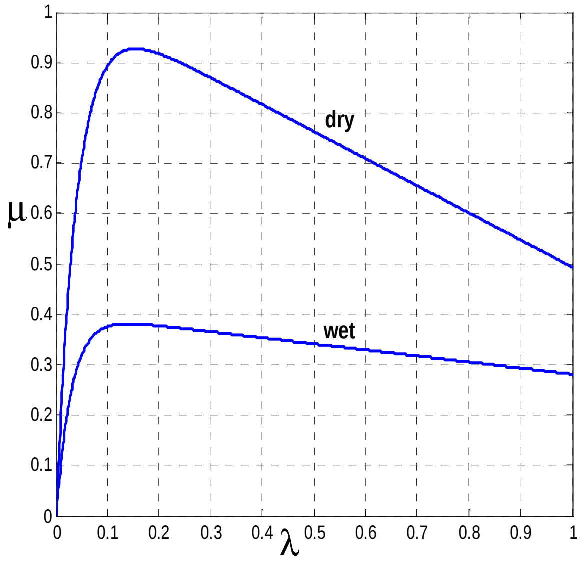

Figure 2 Friction coefficient

Several tire friction models describing the nonlinear behavior are reported in the literature. There are static models as well as dynamic models. The most reputed tyre model is by [Layne 1993] and by [Pacejka 1991], also known as « magic formula » and it is derived heuristically from experimental data. Here we use the expression in [Burckhardt 1993] is derived with similar methodology where μ is expressed as a function of the wheel slip λ, and the vehicle velocity, v

$$\mu(\lambda, v) = \left[C_1\left(1 - e^{-C_2\lambda}\right) - C_3\lambda\right] e^{-C_4\lambda v} \tag{9}$$

Surface conditions	C1	C2	C3
Asphalt, dry	1.029	17.16	0,523
Asphalt, wet	0,857	33,822	0,347
Concrete, dry	1,1973	25,168	0,5373
Cobblestones, dry	1,3713	6,4565	0,6691
Cobblestones, wet	0,4004	33,708	0,1204
Snow	0,1946	94,129	0,0646
Ice	0,05	306,39	0

Table 1Friction parameters

where the parameters are specified for different road surfaces. See Table 1 [Burckhardt 1993]. The parameters in (9) denote the following:

C_1 maximum value of friction curve

C_2 friction curve shape

C_3 friction curve difference between the maximum value and the value at $\lambda = 1$

C_4 wetness characteristic value and is in the range 0.02-0.04s/m

2.3 WHEEL SLIP DYNAMICS

Using (1)-(8), for $v > 0$ and $w_w > 0$, the wheel slip dynamics is obtained by calculating the time derivative of (8) with respect to time

$$\dot{\lambda} = \frac{(1-\lambda)\,\dot{w}_v - \dot{w}_w}{w_v} \tag{10}$$

Substituting (2), (5) and (7) into (10)

$$\dot{\lambda} = F_p(\lambda, t) + G_p u(t) \tag{11}$$

with

$$F_p(\lambda,t)=\left(\frac{4F_t+B_vR_ww_v}{M_vR_ww_v}\right)\lambda+\frac{\dfrac{-(4F_t+B_vR_ww_v+F_\theta)}{M_vR_w}-\dfrac{(-B_ww_w+T_t)}{J_w}}{w_w} \tag{12}$$

$G_p=1/J_w$ is a control gain which is a positive constant, and $u(t)=T_b(t)/w_v$ is a control effort.

In nominal conditions, the system model is

$$\dot{\lambda}=F_n(\lambda,t)+G_nu(t) \tag{13}$$

where $F_n(\lambda,t)$, G_n represent the nominal values of the system parameters they are measured at $\mu(\lambda)=0.9$. If the uncertainties occur, then the controlled system can modified as

$$\begin{aligned}\dot{\lambda}&=\left[F_n(\lambda,t)+\Delta F_n(\lambda,t)\right]+\left[G_n+\Delta G_n\right]u(t)\\&=F_n(\lambda,t)+G_nu(t)+w\end{aligned} \tag{14}$$

where $\Delta F(\lambda,t)$ and ΔG_n denote the system uncertainties ; w is referred to as the lump uncertainty an is defined as $w=\Delta F_n(\lambda,t)+\Delta G_nu(t)$ with the assumption $|w|\le W$, in which W is a positive constant.

3. DESIGN STRATEGY

3.1 SLIDING MODE CONTROL

Here, we assume that the mathematical model of the ABS system is known. The control objective is to find a control law so that the slip can't rack the desired trajectory λ_d. Define the tracking error as follows:

$$\lambda_e(t)=\lambda_d(t)-\lambda(t) \tag{15}$$

where $\lambda(t)$ is the output and $\lambda_d(t)$ is the reference trajectory. Then, define a sliding surface as

$$s(t)=\lambda_e(t)+k_1\int_0^t \lambda_e(\tau)d\tau \tag{16}$$

where k_1 is a positive constant. The sliding-mode control law is defined as [Unsal 1999]

$$u(t)=u_{eq}(t)+u_{ht}(t) \tag{17}$$

where the equivalent controller $u_{eq}(t)$ is represented as

$$u_{eq}(t)=G_n^{-1}\left[-F_n(\lambda,t)+\dot{\lambda}_d(t)+k_1\lambda_e(t)\right] \tag{18}$$

And the hitting controller $u_{ht}(t)$

$$u_{ht}(t)=G_n^{-1}\left[W\,\mathrm{sgn}\left(s(t)\right)\right] \tag{19}$$

In which $\mathrm{sgn}(\cdot)$ is a sign function. Substituting (17), (18) and (19) into (14), it is revealed that

$$\dot{\lambda}_e(t)+k_1\lambda_e(t)=-w-W\,\mathrm{sgn}\left(s(t)\right)=\dot{s}(t) \tag{20}$$

Then, choose a Lyapunov function as

$$V=\frac{1}{2}s^2(t) \tag{21}$$

Differentiating (21) with respect to time and using (20), it is obtained that

$$\begin{aligned}\dot{V}&=s(t)\,\dot{s}(t)=-s(t)\,w-\left|s(t)\right|W\\&\le\left|s(t)\right|\left|w\right|-\left|s(t)\right|W=-\left|s(t)\right|\left(W-\left|w\right|\right)\le 0\end{aligned} \tag{22}$$

In summary, the SMC (Sliding Mode Control) system presented in (17) can guarantee the stability in the Lyapunov sense under variations.

3.2 FUZZY SLIDING MODE CONTROL

We assume that the mathematical model of the ABS system is not known, to deal with this problem. We propose the following approach

Assume that there are n rules in a fuzzy rule base and each of them has the following form:

Rule i: If s is S_i Then u is $\alpha_i + \beta_i * s$ (23)

where s is the input variable of the fuzzy system; u is the output variable of the fuzzy system ; S are the membership functions and (α_i, β_i) are singleton control actions for . The defuzzification of the FSMC (Fuzzy Sliding Mode Control) output is accomplished by the method of center-of gravity [Lee 1990]

$$u = \sum_{i=1}^{n} w_i \times (\alpha_i + \beta_i s) \Big/ \sum_{i=1}^{n} w_i \tag{24}$$

Where w_i is the firing weight of the i[th] rule. Equation (15) can be rewritten as

$$u = (\alpha^T + \beta^T s)\xi \tag{25}$$

with $\alpha = [\alpha_1, ..., \alpha_n]^T$, $\beta = [\beta_1, ..., \beta_n]^T$ and $\xi = [\xi_1, ..., \xi_n]^T$ is regressive vector with ζ_i defined as

$$\xi_i = w_i \Big/ \sum_{i=1}^{n} w_i \tag{26}$$

3.3 FSMC system with Bound Estimator

Assume that the ideal controller is known and can be obtained as

$$u^*(t) = G_p^{-1}\left[-F_p(\lambda, t) + \dot{\lambda}_d(t) + k_2\lambda_e(t)\right] \tag{27}$$

Substituting (27) into (11) gives

$$\dot{\lambda}_e(t) + k_2\lambda_e(t) = 0 \tag{28}$$

If we well choose the coefficient k_2 it is that $\lim_{t\to\infty}\lambda_e(t)=0$, since the system parameters may be unknown or perturbed, the ideal controller $u^*(t)$ can not be precisely implemented. Therefore, by the universal approximation theorem [Wang 1994], there exists an optimal fuzzy controller $u^*_{fz}\left(s,\alpha^*,\beta^*\right)$ such that

$$u^*(t)=u^*_{fz}\left(s,\alpha^*,\beta^*\right)+\varepsilon=\left[\alpha^{*T}+\beta^{*T}\right]\xi+\varepsilon \tag{30}$$

where ε is the approximation error and is assumed to be bounded by $|\varepsilon|\le E$. Employing a fuzzy controller to approximate $u^*(t)$ as

$$\hat{u}_{fz}\left(s,\hat{\alpha},\hat{\beta}\right)=\left[\hat{\alpha}^T+\hat{\beta}^T s\right]\xi \tag{31}$$

where $\left(\hat{\alpha},\hat{\beta}\right)$ are the estimated values of (α,β). The control law for developed FSMC is assumed to take the following form:

$$u_{sf}(t)=\hat{u}_{fz}\left(s,\hat{\alpha},\hat{\beta}\right)+u_{rb}(s) \tag{32}$$

where the fuzzy controller $\hat{u}_{fz}$ is designed to approximate the ideal controller $u^*(t)$ and the robust controller $u_{rb}(s)$ si designed to compensate for the difference between the ideal controller and the fuzzy controller. By substituting (32) into (11), it revealed that

$$\dot{\lambda}=F_p(\lambda,t)+G_p\left[\hat{u}_{fz}\left(s,\hat{\alpha},\hat{\beta}\right)+u_{rb}(s)\right] \tag{33}$$

Multiplying (27) with G_p, added to (33) and using (15) and (16), the error equation governing the system can be obtained as follows :

$$\dot{\lambda}_e(t)+k_2\lambda_e(t)=G_p\left(u^*-\hat{u}_{fz}-u_{rb}\right)=\dot{s}(t) \tag{34}$$

Define $\tilde{u}=u^*-\hat{u}_{fz}$, $\tilde{\alpha}=\alpha^*-\hat{\alpha}$, $\tilde{\beta}=\beta^*-\hat{\beta}$ and use (30), then

$$\tilde{u}_{fz}=\left[\tilde{\alpha}^T+\tilde{\beta}^T s\right]\xi+\varepsilon \tag{35}$$

Define a Lyapunov function as

$$V_2\left(s(t),\tilde{\alpha},\tilde{\beta},\tilde{E}\right)=\frac{1}{2}s^2(t)+\frac{G_p}{2\eta_1}\tilde{\alpha}^T\tilde{\alpha}+\frac{G_p}{2\eta_3}\tilde{\beta}^T\tilde{\beta}+\frac{G_p}{2\eta_2}\tilde{E}^2 \quad (36)$$

where $\tilde{E}(t)=E-\hat{E}(t)$, $\hat{E}(t)$ is the estimation of the approximation error bound, and η_1, η_2 and η_3 are positive constants

Differentiating (36) with respect to time and using (34) and (35), it is obtained that

$$\begin{aligned}
&\dot{V}_2\left(s(t),\tilde{\alpha},\tilde{\beta},\tilde{E}\right)\\
&\quad=s(t)\dot{s}(t)+\frac{G_p}{\eta_1}\tilde{\alpha}^T\dot{\tilde{\alpha}}+\frac{G_p}{\eta_3}\tilde{\beta}^T\dot{\tilde{\beta}}+\frac{G_p}{\eta_2}\tilde{E}\dot{\tilde{E}}\\
&\quad=s(t)G_p\left(\tilde{\alpha}^T\xi+\tilde{\beta}^Ts(t)\xi+\varepsilon-u_{rb}\right)+\frac{G_p}{\eta_1}\tilde{\alpha}^T\dot{\tilde{\alpha}}+\frac{G_p}{\eta_3}\tilde{\beta}^T\dot{\tilde{\beta}}+\frac{G_p}{\eta_2}\tilde{E}\dot{\tilde{E}}\\
&\quad=G_p\tilde{\alpha}^T\left(s(t)\xi+\frac{\dot{\tilde{\alpha}}}{\eta_1}\right)+G_p\tilde{\beta}^T\left(s^2(t)\xi+\frac{\dot{\tilde{\beta}}}{\eta_3}\right)+s(t)G_p\left(\varepsilon-u_{rb}\right)+\frac{G_p}{\eta_2}\tilde{E}
\end{aligned} \quad (37)$$

For achieving $\dot{V}_2\le 0$, we choose

$$\dot{\hat{\alpha}}=-\dot{\tilde{\alpha}}=\eta_1 s(t)\xi \quad (38)$$

$$\dot{\hat{\beta}}=-\dot{\tilde{\beta}}=\eta_1 s^2(t)\xi \quad (39)$$

$$u_{rb}=\hat{E}\,\mathrm{sgn}\left(s(t)\right)\mathrm{sgn}\left(G_p\right)=\hat{E}\,\mathrm{sgn}\left(s(t)\right) \quad (40)$$

$$\dot{\hat{E}}(t)=-\dot{\tilde{E}}(t)=\eta_2\left|s(t)\right|\mathrm{sgn}\left(G_p\right)=\eta_2\left|s(t)\right| \quad (41)$$

Equation (37) can be rewritten as

$$\begin{aligned}
\dot{V}_2\left(s(t),\tilde{\alpha},\tilde{E}\right)&=\varepsilon s(t)G_p-E\left|s(t)\right|\left|G_p\right|\\
&\le-\left|s(t)\right|\left|G_p\right|\left(E-\left|\varepsilon\right|\right)\le 0
\end{aligned} \quad (42)$$

In summary, a Robust Fuzzy Sliding Mode Controller is presented in (32), where $\hat{u}_{fz}$ is given in (31) with the parameters $\left(\hat{\alpha},\hat{\beta}\right)$ adjusted by ((38), (39)) and u_{rb} is given in (40) with the parameter $\hat{E}$ adjusted by (41), by applying this law of control, the system can be guaranteed to be stable.

4 SIMULATION RESULTS

In all simulations, we consider two different roads, a wet road for $t \in [0,3]s$ and an icy road for $t \geq 3s$. In the fuzzy sliding mode control (24), we choose n=5.
The parameters of the ABS used in this study are $M_v = 4\times342$ kg , $B_v = 6$ Ns , $J_w = 1.13$ Nms^2 , $R_w = 0.33$ m , $B_w = 4$ Ns and $g = 9.8$ m/s^2 [Layne 1993]. From figure 1, it is seen that the longitudinal force is near 0.2, so the slip command is chosen as 0.2. Moreover, a reference model is chosen as

$$\dot{\lambda}_d(t) = -10\lambda_d(t) + 10\lambda_c(t) \tag{43}$$

Figures (3.a) and (3.b) show simulations for the ABS, using a sliding mode control (SMC) design method described in section V with $k_1 = 100$ and $W = 25$, we can see that the longitudinal slip λ dose not tend to the slip command λ_c .

The second of simulations is realized by using the proposed fuzzy sliding mode control (FSMC) presented in section VI with the parameters $k_2 = 100$, $\eta_1 = 50$, $\eta_2 = \eta_3 = 1$, we can clearly remark from figures (4.a) and (4.b) that with this method, the slip variable converge to the slip command

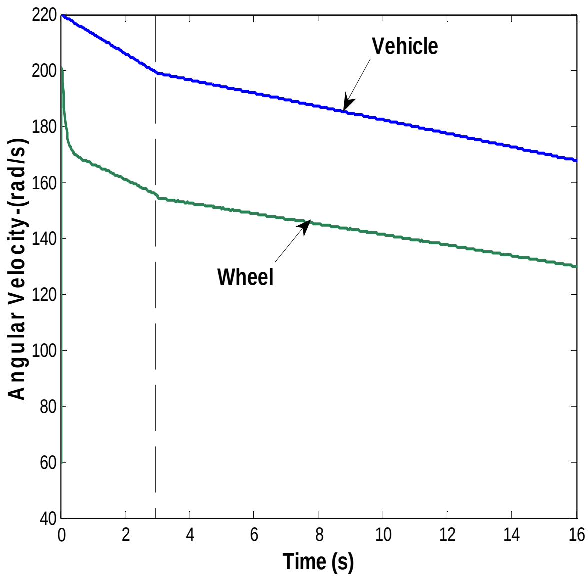

Figure 3.a

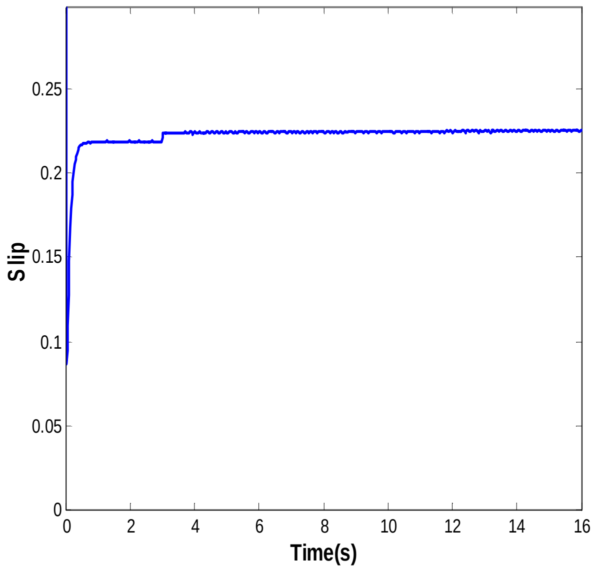

Figure 3.b

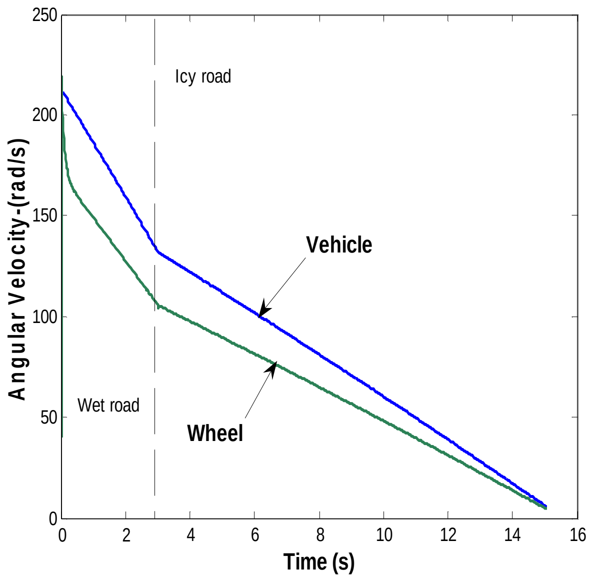

Figure 4.a

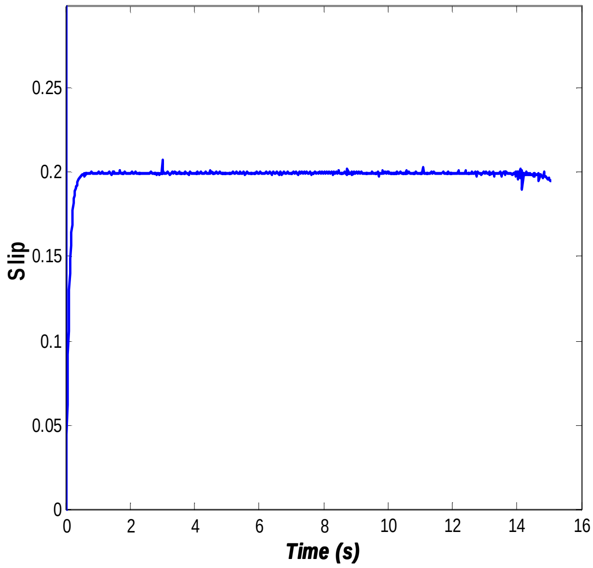

Figure 4.b

5 CONCLUSION

In this study, we have proposed a new control strategy for an antilock braking system (ABS) to maintain the braking force at maximum and achieve robust performance in various road conditions. We have also shown the contribution of the proposed algorithm compared to sliding mod control method. Simulations demonstrate the effectiveness of the algorithm proposed

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