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Stability and Stabilization for uncertain switched systems, a polyquadratic Lyapunov approach

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Abstract. *This paper discusses the problem of stability and stabilization of discrete time switched systems, focusing on the design of a robust state feedback control, and a robust static output feedback control. The results are derived using the direct Lyapunov approach and the poly-quadratic function concept. The stabilization conditions are written through linear matrix inequalities relations. The polyquadratic Lyapunov approach provides a constructive way to tackle uncertainty in the switched framework. The feasibility is illustrated on two examples of discrete time uncertain switched systems.*

Keywords. *Polyquadratic stability and stabilization, switched discrete time system, polytopic uncertainty, state feedback control and static output feedback control.*

1. Introduction

The literature has shown a growing interest on switched systems since switched control systems exist widely in engineering technology and social systems [8]. Switched systems are an important class of hybrid systems defined by a set whose elements are dynamic continuous (or discrete) time models and a commutation law which governs, in time, the jumps between the elements, defining a non stationary dynamic system. Many important progress and remarkable achievements have been made on issues such as controllability, reachability and stabilizability [9],[10], control and switching law design [11], [12], [13], [14], optimal control [15], [16]...

Among others, stability analysis and stabilization control are two important topics. The basic problems considered include stability analysis for systems with specific switching laws [19], stability analysis for systems with arbitrary switching laws [13] and design of stabilizing switching laws [14]. Many contributions to analyze stability

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of arbitrary switched systems use conservative arguments, the most pessimistic ones assuming the existence of a common lyapunov function [17],[18]. Even if these conditions are easily tractable, they can be used in a very few applications. More recently, less conservative conditions, using multiple lyapunov function, have been proposed. [19],[20],[5]. In J. Daafouz, G.Millerioux and C.Iung, 2002 [2] a sufficient (but relatively non restrictive compared to the quadratic approach) stability condition for discrete switched systems is provided using the poly-quadratic approach proposed by J.Daafouz and J.Bernussou 2001[1] for stability analysis and stabilization control of Linear Time Varying systems.

This paper proposes an extension of this works in the case when the switches are made among uncertain LTI systems. The control investigated is of state and static output feedback control.

2. Problem formulation

Consider the autonomous switched system given by:

$$x(k+1) = A_i x(k) \quad (1)$$

Where: $\{A_i : i \in e\}$ is a family of matrices parametrized by an index set $e = \{1,2,...,N\}$ i is a switching signal taken value in *e* depending on k or x (k), or both, or deriving from another hybrid scheme.It is assumed that the switching pattern is not known a priori but the switching index is measured in real time. This class of systems is one of the most commonly treated in literature. One among the basic problems reported in Liberzon and Morse (1999) concerns finding conditions which guarantee that the switched system (1) is asymptotically stable for any switching signal.

The stability of this kind of systems can not be only deduced from the stability of each subsystem $x(k+1) = A_i x(k)$. Example 1 exhibits a case where all the sub-systems are stable and the overall switched system is unstable for given switching law.

3. Stability analysis

The solution to solve the problem presented in Jamal Daafouz, Gilles Millerioux and Claude Iung [2] using the polyquadratic stability concept is the following one: the system (1) can be viewed as a polytopic system considered in [1] with the particularity that the allowable values for the dynamical matrix are those corresponding to the vertices of the polytope:

$$\xi_k^l = \begin{cases} 1 \text{ when the switched system is described by the matrix } A_1. \\ 0 \text{ otherwise} \end{cases}$$

$$A(\xi_k) = \sum_{l=1}^{N} \xi_k^l A_l \; , \xi_k^l \geq 0, \sum_{l=1}^{N} \xi_k^l = 1 \cdot \tag{2}$$

Then the autonomous switched system (1) can also be written as:

$$x(k+1) = \sum_{l=1}^{N} \xi_k^l A_l x(k)$$

Using this fact, the stability condition given in [1] is adapted to the case of switched systems in [2] by the following proposition:

Theorem 1

The discrete system (1) is asymptotically stable if and only if there exist N symmetric matrices $S_1 \ldots . S_N$ and N matrices $G_1 \ldots \ldots G_N$ satysfing :

$$\begin{bmatrix} G_i + G_i^T - S_i & G_i^T A_i^T \\ A_i G_i & S_j \end{bmatrix} > 0 \qquad \forall (i, j) \in (e \times e) \tag{3}$$

The proof is made using the lyapunov function:

$V(x(k), \xi(k)) = x^T(k) \sum_{l=1}^{N} P_l(\xi_k^l) x(k)$, $P_l = S_l^{-1}$, value of which is decreasing along the discrete trajectory of (1).

Remark 1:

It is possible to consider some constrained switching laws for instance the ones in which, from sub-system *i* only some sub-systems are admissible for the next jump; the $\forall (i, j) \in (e \times e)$ condition is then replaced by $\forall (i, j) \in (e \times J)$ where $J \subset e$ depends on *i.*

Remark 2 :

As shown in [5] the condition given in the previous theorem is equivalent to the following one:

$$\begin{bmatrix} S_i & S_i^T A_i^T \\ A_i S_i & S_j \end{bmatrix} > 0 \qquad \forall (i, j) \in (e \times e)$$

The main difference is the introduction of an additional variable G.

The introduction of an additional variable G has been first used in [3] for Linear Time Invariant discrete time systems. Unlike [3] where this variable cannot be dropped since it plays a key role for stability analysis and control design, in the case of switched systems the additional variable can be omitted. However, we prefer to formulate the results on the basis of condition (3), to allow a direct extension of the result to the polytopic uncertain case and to prevent conservatism.

4. Uncertain switched system stability analysis

We consider an uncertain discrete time switched system where the so-called subsystems are uncertain with a polytopic uncertainty, as roughly illustrated in (fig.1) where the uncertainty domains are polytopic shaped with different number of vertices to cope with maximal generality. The model can be stated as follows:

$$x(k+1) = \sum_{l=1}^{M} \xi_k^l \sum_{i=1}^{Nl} \alpha_{i(l)} A_{i(l)} x(k) \tag{4}$$

l is the switching index.

M the number of uncertain systems domain and D_l is the uncertainty domain for subsystem l defined by:

$$D_l = \left\{ A_\alpha : A_\alpha = \sum_{i(l)=1}^{Nl} \alpha_{i(l)} A_{i(l)}, \alpha_{i(l)} \geq 0, \sum_{i(l)=1}^{Nl} \alpha_{i(l)}(k) = 1 \right\}$$

The $\alpha_{i(l)}$ are uncertain but invariant in time .

Nl is the number of the vertices of the polytop D_l.

$$\xi_k^l = \begin{cases} 1 \text{ when the state matrix is defined into } D_l \text{ domain.} \\ 0 \text{ otherwise} \end{cases}$$

$$\xi_k^l \geq 0, \sum_{l=1}^{M} \xi_k^l = 1 . \quad \xi_k = \left[\xi_k^1, \ldots\ldots, \xi_k^N\right]^T$$

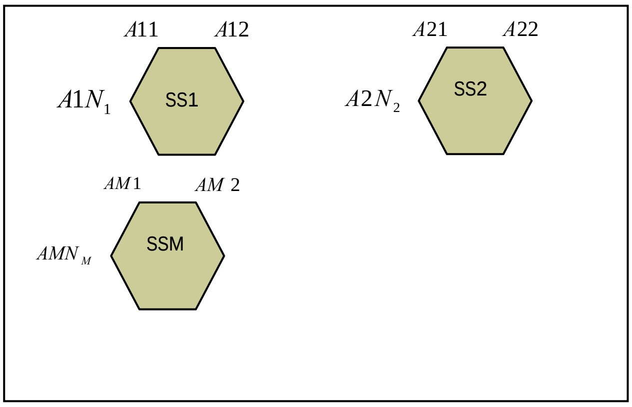


Fig. 1 Autonomous uncertain switched system

4.1. Polyquadratic stability

Theorem 2

The system described by (4) is polyquadratically stable if and only if there exist H symmetric positive definite matrices S_{11} ………S_{MN} and M matrices G_1………G_M of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l)l} & G_l^T A_{i(l)l}^T \\ A_{i(l)l} G_l & S_{i(j)j} \end{bmatrix} > 0 \ \forall (l, i(l), i(j), j) \in (e \times e_l \times e_j \times J) \qquad (5)$$

$H = \sum_{l=1}^{M} N_l$ *,e={1….M},e*e_l*={1……*N_l*},* e_j*={1……*N_j*} J function of l.*

<u>Proof</u>**:** to prove the precedent condition we have to prove that if this condition (5) is verified then condition (3) is verified for any jumps realisation in the uncertainty domains.

We suppose two dynamical matrices A_l and A_j defined respectively in the domain *Dl* and *D j*.

Let $A_l = \sum_{i(l)=1}^{Nl} \alpha i(l) A_{i(l)}$, $\sum_{i(l)=1}^{Nl} \alpha i \ (l) = 1$ and $A_j = \sum_{i(j)=1}^{Nj} \alpha i(j) A_{i(j)}$, $\sum_{i(j)=1}^{Nj} \alpha i \ (j) = 1$

From (5) one gets: $\sum_{i(l)=1}^{Nl} \alpha i \ (l) \begin{bmatrix} G_l + G_l^T - S_{i(l)l} & G_l^T A_{i(l)l}^T \\ A_{i(l)l} G_l & S_{i(j)j} \end{bmatrix} > 0$

$$\Leftrightarrow \begin{bmatrix} G_l + G_l^T - \sum_{i(l)=1}^{Nl} \alpha i(l) S_{i(l)l} & G_l^T \sum_{i(l)=1}^{Nl} \alpha i(l) A_{i(l)l}^T \\ \sum_{i(l)=1}^{Nl} \alpha i(l) A_{i(l)l} G_l & \sum_{i(l)=1}^{Nl} \alpha i(l) S_{i(j)j} \end{bmatrix} > 0 \Leftrightarrow \begin{bmatrix} G_l + G_l^T - \sum_{i(l)=1}^{Nl} \alpha i(l) S_{i(l)l} & G_l^T A_l^T \\ A_l G_l & S_{i(j)j} \end{bmatrix} > 0$$

Replacing $\sum_{i(l)=1}^{Nl} \alpha i(l) S_{i(l)l}$ by $S_l > 0$ we obtain: $\begin{bmatrix} G_l + G_l^T - S_l & G_l^T A_l^T \\ A_l G_l & S_{i(j)j} \end{bmatrix} > 0$.

Again $\sum_{i(j)=1}^{Nj} \alpha i(j) \begin{bmatrix} G_l + G_l^T - S_l & G_l^T A_l^T \\ A_l G_l & S_{i(j)j} \end{bmatrix} > 0 \Leftrightarrow \begin{bmatrix} G_l + G_l^T - S_l & G_l^T A_l^T \\ A_l G_l & \sum_{i(j)=1}^{Nj} \alpha i(j) S_{i(j)j} \end{bmatrix} > 0$.

Replacing $\sum_{i(j)=1}^{Nj} \alpha i(j) S_{i(j)j} = S_j > 0$ we obtain: $\begin{bmatrix} G_l + G_l^T - S_l & G_l^T A_l^T \\ A_l G_l & S_j \end{bmatrix} > 0$

Which concludes the proof.

4.2. Local polyquadratic stability

The previous condition (Polyquadratic stability) is global; it associates a Lyapunov function to each vertices of all sub-system and may be very heavy computationally speaking in the case of a large H number. Applying the quadratic concept where a single Lyapunov function is used for each of the uncertain sub-systems, a say local poly-quadratic stability criterion (more restrictive than the previous one but easier computationally speaking) can be described by:

Theorem 3

The system (1) is locally polyquadratically stable if and only if it exist M symmetrical positive definite matrices S_l ………S_M and M matrices G_l………G_M of appropriate dimension solutions of the LMIs

$$\begin{bmatrix} G_l + G_l^T - S_l & G_l^T A_{i(l)l}^T \\ A_{i(l)l} G_l & S_j \end{bmatrix} > 0 \quad \forall (l, i(l), j) \in (e \times e_l \times J_l) \tag{6}$$

e= {1….M}, el= {1……N$_l$}, J$_l$ function of l.

5. State feedback control

Consider the uncertain switched system described by:

$$x(k+1) = \sum_{l=1}^{M} \xi_k^l (A_l x(k) + B_l u(k)) \tag{7}$$

Where $[A_l, B_l] \in D_l = \left\{ (A_l, B_l) = \sum_{i(l)=1}^{Nl} \alpha_{i(l)} \left[A_{i(l)}, B_{i(l)} \right], \alpha_{i(l)} \geq 0, \sum_{i(l)=1}^{Nl} \alpha_{i(l)}(k) = 1 \right\}$

The stabilization problem of the switched system through state feedback is consists in determining a control law of the form:

$u(k) = K_l x(k)$ such that $x(k+1) = \sum_{l=1}^{M} \xi_k^l (A_l + B_l K_l) x(k)$ is stable.

5.1. Polyquadratic stabilization through state feedback control

Introducing the dynamic matrix $A_l + B_l K_l$ in the condition (5) and after a linearizing change of variables ($R_l = K_l G_l$) the theorem 2 gives rise to the following result in terms of LMI:

Theorem 4:

The system (7) can be stabilized by a state feedback if there exist *H* symmetric positive definite matrices S_{11} ………S_{MN} , *M* matrices G_1………G_M and *M* matrices R_1........R_M of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_{i(l)l} & (A_{i(l)l}G_l + B_{i(l)l}R_l)^T \\ A_{i(l)l}G_l + B_{i(l)l}R_l & S_{i(j)j} \end{bmatrix} > 0 \quad \forall (l, i(l), i(j), j) \in (e \times e_l \times e_j \times J_l) \tag{8}$$

with $H = \sum_{l=1}^{M} N_l$ *,e={1….M},e*

The local state feedback gains are then defined by $K_l = R_l G_l^{-1}$.

5.2. Local polyquadratic stabilization through state feedback control

Extension of the preceding theorem is straightforward and the analogous synthesis result of theorem 3:

Theorem 5:

The system (7) can be stabilized by a state feedback if there exist M symmetric positive definite matrices S_1 ………S_M , M matrices G_1………G_M and M matrices R_1........R_M of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_l & (A_{i(l)l}G_l + B_{i(l)l}R_l)^T \\ A_{i(l)l}G_l + B_{i(l)l}R_l & S_j \end{bmatrix} > 0 \tag{9}$$

$\forall (l, i, j) \in (e \times e_l \times J_l)$

e={1….M},e_l*={1……*N_l*}, J function of l.*

The local state feedback gains are then defined by $K_l = R_l G_l^{-1}$.

6. Static output feedback control

Consider the uncertain switched system described by:

$$x(k+1)=\sum_{l=1}^{M}\xi_k^l A_l x(k)+B_l u(k))$$
$$y(k)=\sum_{l=1}^{M}\xi_k^l C_l x(k) \tag{10}$$

Where

$$\left[A_l,B_l,C_l\right]\in D_l=\left\{(A_l,B_l,C_l)=\sum_{i(l)=1}^{Nl}\alpha_{i(l)}\left[A_{i(l)},B_{i(l)},C_{i(l)}\right],\alpha_{i(l)}\geq 0,\sum_{i(l)=1}^{Nl}\alpha_{i(l)}(k)=1\right\}$$

The stabilization problem of the switched system through static output feedback is consists in determining a control law of the form [2]:

$u(k)=K_l y(k)$ such that $x(k+1)=\sum_{l=1}^{M}\xi_k^l(A_l+B_lK_lC_l)x(k)$ is stable.

6.1. Polyquadratic stabilization through static output feedback

Introducing the dynamic matrix $(A_l+B_lK_lC_l)$ in the condition (5) and after a linearizing change of variables the theorem 2 gives rise to the following result in terms of LMIs and LMEs (linear matrix equalities):

Theorem 6

The system (4) can be stabilized by a static output feedback if there exist H symmetric positive definite matrices S_{11} ………S_{MN}, M matrices G_1………G_M, M matrices U_1........U_M and M matrices V_1........V_M of appropriate dimension solutions of the LMIs, LMEs :

$$\begin{bmatrix} G_l+G_l^T-S_{i(l)l} & (A_{i(l)l}G_l+B_{i(l)l}U_lC_{i(l)l})^T \\ A_{i(l)l}G_l+B_{i(l)l}U_lC_{i(l)l} & S_{i(j)j}\end{bmatrix}>0 \tag{11}$$
$$V_lC_{i(l)l}=C_{i(l)l}G_l$$

$$\forall\,(l,i(l),i(j),\ j)\in(e\times e_l\times e_j\times J_l)$$

$H=\sum_{l=1}^{M}N_l$ *,e={1….M},e_l={1……N_l},ej={1……Nj} J is function of l.*

The output feedback gain is then defined by $K_l=U_lV_l^{-1}$.

6.2. Local polyquadratic Stabilisation through static output feedback

Extension of the preceding theorem is straightforward and the analogous synthesis result of theorem 2:

Theorem 7

The system (1) can be locally polyquadratically stabilized through output feedback if there exist M symmetric positive definite matrices $S_1 \ldots\ldots S_M$, M matrices $G_1 \ldots\ldots G_M$ and M matrices $U_1 \ldots\ldots U_M$ of appropriate dimension solutions of the LMIs:

$$\begin{bmatrix} G_l + G_l^T - S_l & (A_{i(l)l} G_l + B_{i(l)l} U_l C_{i(l)l})^T \\ A_{i(l)l} G_l + B_{i(l)l} U_l C_{i(l)l} & S_j \end{bmatrix} > 0 \qquad (12)$$

$$V_l C_{i(l)l} = C_{i(l)l} G_l$$

$$\forall (l, i(l), j) \in (e \times e_l \times J)$$

e= {1….M}, $e_{l=}$ {1……N_l}, J function of l.

The output feedback gain is defined by $K_l = U_l V_l^{-1}$.

Remark 3

Introducing the equality constraints *VC=CG* indeed increases the sufficiency in the relations for the control design. Fortunately in the case when $C_{il} = I \quad \forall (l,i) \in (e \times e_l)$ the conditions defined by (11) and (12) reduces to the classical state feedback stabilising determination.

7. Illustrative examples

7.1. Example 1

To illustrate the poly-quadratic stabilization through state feedback control the discrete switched system presented by Daafouz [2] is considered with some modifications. A similar example has been proposed for the continuous time switched systems in [4].

An uncertainty is put on the *A* matrix, specifically on *A(2,1)* and *A(1,2).*
We consider the system: $x(k+1) = A_{l\alpha} x(k) + B_{l\alpha} u(k)$

Where $\{A_l : l \in e\}$ is a family of polytope, witch represent the subsystems, indexed by a parameter *l* from $e = \{1,2,3,4\}$,

$$A_l : \{ A_\alpha \; : \; A_\alpha = \sum_{i=1}^{Nl} \alpha_i A_i \; , \; \alpha_i \geq 0, \sum_{i=1}^{Nl} \alpha_i(k) = 1 \},$$

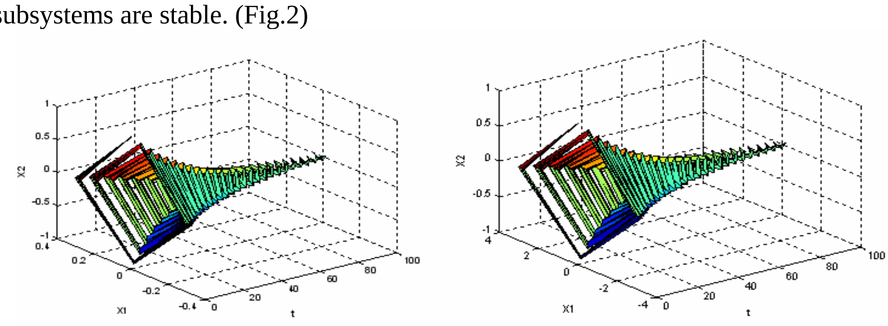
Due to the uncertainty on $A_l(2,1)$ *et* $A_l(1,2)$.

We have:

$$A_{1\alpha} = \begin{bmatrix} 0{,}0094 & 0{,}3010 \pm inc\% \\ -3{,}0098 \pm inc\% & 0{,}0094 \end{bmatrix}, \; A_{2\alpha} = \begin{bmatrix} 0{,}0094 & 0{,}3098 \pm inc\% \\ -0{,}3010 \pm inc\% & 0{,}0094 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

First we consider the autonomous case with $u(k) = 0$ and $x(0) = [0 \; 1]^T$. The two subsystems are stable. (Fig.2)



Subsystem 1 Subsystem 2

Fig. 2. trajectory of x(k) for the two subsystems undependably. (*inc*=5%)

But if the global system is governed by the following switching law:

$$l = \begin{cases} 1 \text{ if } : x1(k) \times x2(k) \geq 0 \\ 2 \text{ else} \end{cases} \quad \text{with} \quad x(k) = \begin{bmatrix} x1(k) \\ x2(k) \end{bmatrix}$$

The system become unstable as shown in trajectory of $x(k)$. (Fig.3)

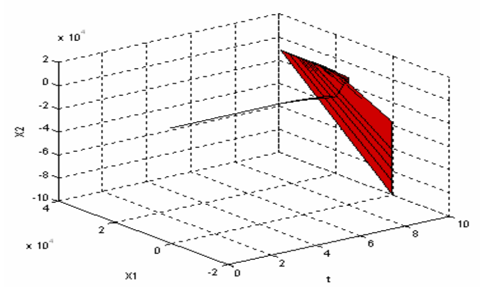


Fig.3. trajectory of x(k) for autonomous switched system

No solution is found applying the stability analysis condition in (5). Neither using the quadratic stabilization, through state feedback, criteria (for *inc%*=0). But it is possible to stabilise the system using the polyquadratic condition (8,9) for different value of *inc*. the result of stabilising gain are listed in table 1.

Table 1. gain result from condition (8) and (9)

	Polyquadratic stability		Local polyquadratic stability	
Inc	K1	K2	K1	K2
0%	(-0.0094 -0.3010)	(-0.0094 -3.0098)	(-0.0094 -0.3010)	(-0.0094 -0.3010)
5%	(-0.0189 -0.3010)	(-0.0106 -3.0098)	(-0.0185 -0.2935)	(-0.0103 -3.0098)
10%	(-0.0497 -0.3009)	(-0.0103 -3.0098)	(-0.0374 -0.2861)	(-0.0103 -3.0098)

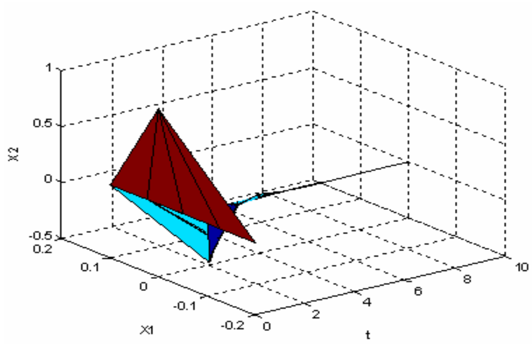


Fig.4. trajectory of x(k) for inc=5%, in local polyquadratic stabilization.

7.2. Example 2 : static output control

To illustrate the poly-quadratic stabilization through static output feedback the discrete switched system presented by Perez P. and Geromel J. 1993 [6] is considered with some modifications:

$$X(k+1) = \begin{bmatrix} x1(k+1) \\ x2(k+1) \\ x3(k+1) \end{bmatrix} = AX(k) + Bu(k)$$

$$y(k) = \begin{bmatrix} x1(k) & x2(k) \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0.2113 & 0.0087 & 0.4524 \\ 0.0824 & 0.8096 & 0.8075 \\ 0.7599 & 0.8474 & 0.4832 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.6135 & 0.6538 \\ 0.2749 & 0.4899 \\ 0.8807 & 0.7741 \end{bmatrix}$$

It is assumed that one of the two actuators can break down so that the B matrix can take 3 different values. An uncertainty is put on the A matrix, specifically on its 3*3 entry. Based on that, the system is an uncertain discrete switched system represented by 3 uncertain sub-systems.

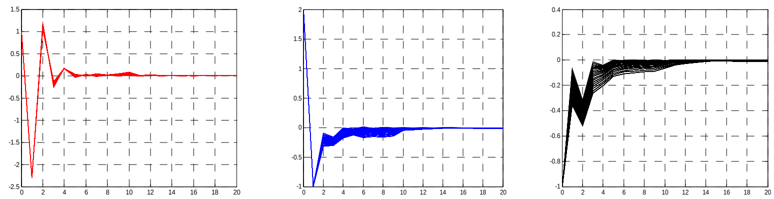
The following results are obtained applying condition (12) with $V_l=CG_lC^{**}$ (C^{**} is the pseudo-inverse of the non square C matrix), for an uncertainly equal to ±30% .

$$K1=\begin{bmatrix}-1.4834 & 1.6831\\ 0.8245 & -2.9073\end{bmatrix}\; K2=\begin{bmatrix}-0.9104 & -0.6476\\ 0 & 0\end{bmatrix}\; K3=\begin{bmatrix}0 & 0\\ -1.0190 & -0.8469\end{bmatrix}$$

The actuator breakdowns are chosen randomly in time.

The figure 5 illustrates the trajectories of the state $X(k)$ for systems taken in all the range of uncertainly with initial condition:

$$x(0)=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$$



x1 x2 x3

Fig. 5. Trajectories of X

8. Conclusion

In this paper a link between polyquadratic stability and stability of uncertain switched systems has been established. The problem of designing a stabilizing output feedback is a complex one for switched systems with uncertain sub-systems. It is known that output control design is difficult when the uncertainties are structured ones, which is the case for polytopic uncertainty. In this paper we propose sufficient conditions using polyquadratic stability to find robust state and output feedback controls for this kind of systems. The conditions proposed are independent from the commutation law. Considering a knowledge on commutation law could allow to give less restrictive conditions: this, and also the problem of the design of dynamic output feedback control, are now among our researches.

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