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LMI-based sliding-mode observer design method for reconstruction of actuator and sensor faults

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Abstract. *This paper presents a method to design Sliding Mode Observers (SMO) for a class of linear system using Linear Matrix Inequalities (LMIs). The switching surface is set to be the difference between the observer and system output. In terms of LMIs, a necessary and sufficient condition is derived for the existence of a sliding-mode observer guaranteeing a stable sliding motion on the switching surface. The gain matrices of the sliding-mode observer are characterised using the solution of the LMI existence condition. Next, we discuss how these SMO can be used for detecting and reconstructing actuator and sensor faults by analysing the equivalent output error injection signal required to maintain the observer in a sliding motion. Finally, the validity and the applicability of this approach are illustrated by a lateral axis model of an L-1011in cruise flight conditions.*

Keywords: *Observer; Sliding mode; LMI; Equivalent output error injection; Fault reconstruction*.

1. Introduction

The fundamental purpose of Fault Detection and Isolation (FDI) scheme is to generate an alarm when a fault occurs and also to identify the nature and location of the fault. Many FDI methods are observer based: the basic idea is to reconstruct the outputs of the system of interest from the measurements, and use the estimation error as the residual. The observer feedback gain enters the calculation of the residual generator and the gain design problem provides freedom for achieving the required performance. However, the model of the system about which the observer is designed will possess uncertainties. These uncertainties could cause the FDI

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scheme to trigger a false alarm when there are no faults, or even worse, mask the effect of a fault. Hence, there is a need for robust FDI schemes using sliding mode observers [7, 8 and 9].

The concept of sliding mode emerged from the Soviet Union in the late sixties where the effects of introducing discontinuous control action into dynamical systems were explored. By the use of a judicious switched control law, it was found that the system states could be forced to reach and subsequently remain on a pre-defined surface in the state space. Whilst constrained to this surface, the resulting reduced-order motion – referred to as the sliding motion – was shown to be insensitive to any uncertainty or external disturbance signals which were implicit in the input of the system. This inherent robustness property has resulted in world wide interest and research in the area of sliding mode control. These ideas have subsequently been employed in other situations including the problem of state estimation via an observer.

Several authors have proposed sliding mode observer design methods [1-5]. Utkin [1] presents a discontinuous observer strategy whereby the error between the estimated and measured outputs is forced to exhibit a sliding mode and measurement noise effects are reduced. Dorling and Zinober [2] explore the practical application of this observer to an uncertain system and report difficulties in the selection of an appropriate switched gain that will cause the system to exhibit a sliding mode without excessive chattering. It should be noted that this study did not consider the application of a continuous approximation to the discontinuous observer action as traditionally used in the implementation of variable structure control systems. Walcott and Zak [3] use a Lyapunov-based approach to formulate an observer design which, under appropriate assumptions, exhibits asymptotic state error decay in the presence of bounded non-linearities/uncertainties. In particular, this method seeks to render the observer error system totally insensitive to matched uncertainty. This framework, although intuitively appealing, necessitates the use of algebraic manipulation tools to solve the associated constrained Lyapunov problem effectively for systems of reasonable order. Edwards and Spurgeon [4] propose an observer strategy, similar in style to that of Walcott and Zak, which circumvents the use of a symbolic manipulation and offers an explicit design algorithm. This paper builds on the work of Tan and Edwards [5] by considering an LMI-based sliding-mode observer design method. This approach does require not only of finding state transformation matrices but also changing coordinates to obtain the canonical form. Using LMIs a necessary and sufficient condition is derived for the existence of a sliding-mode observer guaranteeing a stable sliding motion on the switching surface that is set to be the difference between the observer and system output. In terms of the solution of the LMI existence condition, explicit formulas of the gain matrices of the sliding-mode observer are derived. Because the approach is based on LMIs, it offers degrees of freedom which can be used to improve the design [6].

Despite SMO's excellent ability to generate unbiased estimates of the system states under modeling errors, relatively few researchers have investigated the area of fault diagnosis using SMO: Sreedhar, Fernandez and Masada [7] consider a model-based sliding mode observer approach although in their design procedure it is assumed that the states of the system are available; a different approach is adopted by Hermans and Zarrop [8], who attempt to design an observer in such a way that in the presence of a fault the sliding motion is destroyed. In this paper, the observer is designed to maintain a sliding motion even in the presence of faults which are detected by analysing the so-called equivalent output injection. The manipulation of the equivalent output injection signal can be used for explicitly reconstructing fault signals. This may be allied to the equivalent control signal which appears in the analysis of sliding mode based feedback control systems [14].

The structure of the paper is as follows: Firstly the preliminaries and background work for this paper will be presented. Next, the sliding mode observer and its design method will be introduced. Following that, a method to reconstruct actuator and sensor faults will be described. Finally, the efficacy of this method will be demonstrated with a lateral axis model of an L-1011 in cruise flight conditions taken from the literature.

2. System description

Consider the nominal linear system subject to certain faults described by

$$\dot{x}(t) = Ax(t) + Bu(t) + F\,f_a(t) \tag{1}$$

$$y(t) = Cx(t) + f_s(t) \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are the state variable, the input and the output respectively, the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $F \in \mathbb{R}^{n\times q}$ are assumed to be time invariant, and $q \le p < n$. Assume that the matrices *C* and *F* are full rank. The functions $f_a(t)$ and $f_s(t)$ are deemed to represent actuator and sensor faults respectively, and the signal $f_a : \mathbb{R}_+ \times \mathbb{R}_m \to \mathbb{R}^q$ is assumed to be unknown but bounded so that

$$\| f_a(t) \| \le \beta \tag{3}$$

where the positive scalar β is known. The objective is to design an observer to generate a state estimate $\hat{x}(t)$ and output estimate $\hat{y}(t)$ such that a sliding mode is attained in which the output error

$$e_y(t) = \hat{y}(t) - y(t)\ . \tag{4}$$

is forced to zero in finite time. It will be shown that, provided a sliding motion can be attained, estimates of $f_a(t)$ and $f_s(t)$ can be computed from approximating the equivalent output error injection signal required to maintain a sliding motion.

Consider initially the case when $f_s(t) = 0$.

Edwards and Spurgeon [4] have shown a sliding mode observer exists if and only if

(A1) the matrix CF is full rank,

(A2) the invariant zeros (if any) of (A, F,C) lie in the left half plane

Furthermore if these two conditions hold then, there exists a change of co-ordinates in which the system triple *(A, F,C)* from *(1)-(2)* has the following structure :

$$\overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix}, \ \overline{F} = \begin{bmatrix} 0 \\ \overline{F}_2^0 \end{bmatrix}, \ \overline{C} = \begin{bmatrix} 0 & T \end{bmatrix} . \tag{5}$$

where the sub-matrices $\overline{A}_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$; $\overline{F}_2^0 \in \mathbb{R}^{q\times q}$ is non-singular and $T \in \mathbb{R}^{p\times p}$ is orthogonal. Define $\overline{A}_{211}$ as the top *(p-q)* rows of $\overline{A}_{21}$. By construction, the pair $(\overline{A}_{11},\overline{A}_{211})$ is detectable and the unobservable modes of $(\overline{A}_{11},\overline{A}_{211})$ are the invariant zeros of *(A, F, C)* [10]. Also for convenience, define $\overline{F}_2 \in \mathbb{R}^{p\times q}$ as the bottom *p* rows of *F* (which therefore includes the matrix $\overline{F}_2^0$).

3. A sliding mode observer

3.1. A description of the sliding mode observer

Consider the following observer with the same form of Tan and Edwards [5]

$$\begin{aligned} \dot{\hat{x}}(t) &= A\,\hat{x}(t) + B\,u(t) - G_l\, e_y(t) + G_n\, \upsilon \\ \hat{y}(t) &= C\,\hat{x}(t) \end{aligned} \tag{6}$$

Will be considered where $G_l \in \mathbb{R}^{n\times p}$ is a traditional Luenberger observer gain used to make $(A - G_l C)$ stable and $G_n \in \mathbb{R}^{n\times p}$.

The discontinuous output error injection vector υ is defined by

$$\upsilon = \begin{cases} -\rho(t,y,u)\left\| P_0 \mathcal{F}_2 \right\| \dfrac{e_y}{\left\| e_y \right\|} & if\, e_y \neq 0 \\ 0 & otherwise \end{cases} \tag{7}$$

where $P_0 \in \mathbb{R}^{p\times p}$ is a symmetric positive definite (s.p.d) matrix. The matrices P_0 and $\mathcal{F}_2$ will be defined formally later. The scalar gain function $\rho : \mathbb{R}_+ \times \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}_+$ must be an upper bound on the magnitude of the fault signal.

Let $\overline{G}_l$ and $\overline{G}_n$ represent the observer gain matrices in the new coordinate system and define $\overline{A}_0 = \overline{A} - \overline{G}_l \overline{C}$. A suitable choice for the matrix $\overline{G}_n$ is

$$\overline{G}_n = \begin{bmatrix} -\overline{L}T^T \\ T^T \end{bmatrix} P_0^{-1} \tag{8}$$

where

$$\overline{L} = \begin{bmatrix} L^0 & 0 \end{bmatrix} \in \mathbb{R}^{(n-p)\times p} \tag{9}$$

with $L^0 \in \mathbb{R}^{(n-p)\times(p-q)}$ and the orthogonal matrix T is part of the output distribution matrix $\overline{C}$ from *(5)*.

If the state estimation error is defined as

$$e(t) = \hat{x}(t) - x(t) \tag{10}$$

then it is straightforward to show from equations *(1)*, *(2)* and *(6)* that

$$\dot{e}(t) = \overline{A}_0 e(t) + \overline{G}_n \upsilon - \overline{F} f_a(t) \tag{11}$$

The matrices $\overline{G}_l$, $\overline{G}_n$ and P_0 represent the design freedom associated with the observer and are to be determined so that a sliding motion takes place on $S = \left\{ e \in \mathbb{R}^n : e_y = 0 \right\}$.

Proposition 1 [5]: If there exists a s.p.d matrix $\overline{P}$, with the structure

$$\overline{P} = \begin{bmatrix} \overline{P}_1 & \overline{P}_1 \overline{L} \\ \overline{L}^T \overline{P}_1 & T^T P_0 T + \overline{L}^T \overline{P}_1 \overline{L} \end{bmatrix} > 0 \tag{12}$$

where $\overline{P}_1 \in \mathbb{R}^{(n-p)\times(n-p)}$, and satisfies

$$\overline{P}\,\overline{A}_0 + \overline{A}_0^T \overline{P} < 0 \tag{13}$$

then the state error system in equation *(11)* is quadratically stable. Furthermore, a sliding motion occurs in finite time on S governed by the system matrix $\overline{A}_{11} + L\overline{A}_{211}$.

Proof: See Proposition 1 and Corollary 1 in Tan & Edwards [5].

The Next section focuses on design methods to synthesise the gain $\overline{G}_l$ and the Lyapunov matrix $\overline{P}$ which has the structure given in *(12)*. The problems will be posed in such a way that Linear Matrix Inequalities (LMIs) [6] can be used to numerically synthesise the required matrices.

3.2. Synthesis of the observers via solving LMIs

This section describes a tractible design method from [5]. It is based on an LQG type approach and is parameterized by two user defined weighting matrices. The gain $\overline{G}_l$ and the Lyapunov $\overline{P}$ will be chosen so that the matrix inequality

$$\overline{A}_0^T\,\overline{P}+\overline{P}\,\overline{A}_0 < -\overline{P}W\overline{P}-\overline{P}\,\overline{G}_l V\overline{G}_l^T\,\overline{P} \tag{14}$$

is satisfied, where the design weighting matrices W and V are assumed to be symmetric positive definite, and $\overline{P}$ has the structure in *(12)*.
Defining $\overline{Y}^T := \overline{P}\,\overline{G}_l$ to transform a problem of BMIs into LMIs and substituting for $\overline{A}_0$, the inequality *(22)* can be written as

$$\overline{A}^T\,\overline{P}+\overline{P}\,\overline{A}-(\overline{Y}\,\overline{C})^T-\overline{Y}\,\overline{C}+\overline{P}W\overline{P}+\overline{Y}V\overline{Y}^T < 0 \tag{15}$$

Using standart matrix manipulations, inequality *(15)* is identical to

$$\overline{P}\,\overline{A}+\overline{A}^T\,\overline{P}+(\overline{Y}^T-V^{-1}\overline{C})^T V(\overline{Y}^T-V^{-1}\overline{C})-\overline{C}V^{-1}\overline{C}+\overline{P}W\overline{P} < 0 \tag{16}$$

While choosing $\overline{Y}^T = V^{-1}\overline{C}$ eliminates the third term in *(16)*, and hence the necessary and sufficient condition for *(15)* to hold is that $\overline{P}$ satisfies

$$\overline{P}\,\overline{A}+\overline{A}^T\,\overline{P}-\overline{C}V^{-1}\overline{C}+\overline{P}W\overline{P} < 0 \tag{17}$$

The problem considered here is one of minimizing $trace\,(\overline{P}^{-1})$ subject to $\overline{P}$ satisfying *(17)*. By using the Schur complement, the matrix inequality in *(17)* is equivalent to

$$\begin{bmatrix} \overline{P}\,\overline{A}+\overline{A}^T\,\overline{P}-\overline{C}^T V^{-1}\overline{C} & \overline{P} \\ \overline{P} & -W^{-1} \end{bmatrix} < 0 \tag{18}$$

If $\overline{X} \in \mathbb{R}^{n\times n}$ is symmetric positive definite then the LMI

$$\begin{bmatrix} -\overline{P} & I \\ I & -\overline{X} \end{bmatrix} < 0 \tag{19}$$

is equivalent to $\overline{X} > \overline{P}^{-1}$. Thus minimizing $trace\,(\overline{P}^{-1})$ subject to *(17)* is equivalent to minimizing $trace\,(\overline{X})$ subject to the LMIs *(18)* and *(19)*. Writing $\overline{P}$ from *(12)* as

$$\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0 \tag{20}$$

Where $P_{11} \in \mathbb{R}^{(n-p)\times(n-p)}$, $P_{22} \in \mathbb{R}^{p\times p}$ and

$$P_{12} := \begin{bmatrix} P_{121} & 0 \end{bmatrix} \tag{21}$$

with $P_{121} \in \mathbb{R}^{(n-p)\times(p-q)}$, it follows there is one-to-one correspondence between the variables $(P_{11}, P_{121}, P_{22})$ and $(\bar{P}_1, L, \bar{P}_2)$ since

$$\begin{aligned} P_{11} &= \bar{P}_1 \\ L^0 &= P_{11}^{-1} P_{121} \\ P_0 &= T(P_{22} - P_{12}^T P_{11}^{-1} P_{12}) T^T \end{aligned} \tag{22}$$

and the gain $\bar{G}_l$ from *(6)* as

$$\bar{G}_l = \bar{P}^{-1} \bar{C}^T V^{-1} \tag{23}$$

This represents a convex optimization problem. LMI Control Toolbox of PC-Math-Lab such as [11], can be employed to synthesise numerically $\bar{P}$ and $\bar{X}$.

4. Fault reconstruction

If a further linear change of co-ordinates

$$T_L = \begin{bmatrix} I_{n-p} & \bar{L} \\ 0 & T \end{bmatrix} \tag{24}$$

is applied to the triple $(\bar{A}, \bar{F}, \bar{C})$ and its Lyapunov matrix $\bar{P}$, the system matrix, fault distribution matrix and the output distribution matrix will be in the form

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} \end{bmatrix}, \mathcal{F} = \begin{bmatrix} 0 \\ \mathcal{F}_2 \end{bmatrix}, C = \begin{bmatrix} 0 & I_p \end{bmatrix} \tag{25}$$

where $\mathcal{A}_{11} = \bar{A}_{11} + L^0 \bar{A}_{211}$. In the new co-ordinate system, the Lyapunov matrix (12) and the nonlinear gain matrix from (8) will respectively

$$\mathcal{P} = (T_L^{-1})^T \overline{P}(T_L^{-1}) = \begin{bmatrix} \overline{P}_1 & 0 \\ 0 & P_0 \end{bmatrix} \quad (26)$$

and

$$\mathcal{G}_n = T_L \overline{G}_n = \begin{bmatrix} 0 \\ P_0^{-1} \end{bmatrix} \quad (27)$$

As argued in [4], the fact that $\mathcal{P}$ is a block diagonal Lyapunov matrix for $\mathcal{A} - \mathcal{G}_\ell C$ implies that $\mathcal{A}_{11}$ is stable and hence the sliding motion is stable.

The state estimation error in the new co-ordinates system is

$$\dot{e}_L(t) = \mathcal{A}_0\, e(t) + \mathcal{G}_n \nu - \mathcal{F}\, f_a(t) \quad (28)$$

where $\mathcal{A}_0 = \mathcal{A} - \mathcal{G}_\ell C$. Partitioning the state estimation error conformably with *(25)* yields

$$\dot{e}_1(t) = \mathcal{A}_{11} e_1(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1}) e_y(t) \quad (29)$$

$$\dot{e}_y(t) = \mathcal{A}_{21} e_1(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2}) e_y(t) + \upsilon - \mathcal{F}_2 f_a(t) \quad (30)$$

where $\mathcal{G}_{\ell,1}$ and $\mathcal{G}_{\ell,2}$ represent an appropriate partitions of the linear output error injection matrix $\mathcal{G}_\ell = \mathrm{T_L}\overline{\mathrm{G}}_\mathrm{L}$ after the coordinate transformation in (25). Notice this is precisely the canonical form for fault reconstruction proposed in [9].

4.1. Reconstruction of actuator faults

Once a sliding motion has been attained $\mathrm{e_y} = 0$ et $\dot{\mathrm{e}}_\mathrm{y} = 0$ and the discontinuous output error injection term υ can be replaced by $\upsilon_{éq}$, the so called 'equivalent output error injection' required to maintain a sliding motion [4], [9]. From (29)-(30), during sliding mode

$$\dot{e}_1(t) = \mathcal{A}_{11} e_1(t) \quad (31)$$

$$0 = \mathcal{A}_{21}\, e_1(t) - \mathcal{F}_2 f_a(t) + P_0^{-1} \upsilon_{éq} \quad (32)$$

From (32), and using the fact that $\mathcal{A}_{11}$ is stable, it follows that $e_1 \to 0$ and therefore

$$P_0^{-1} \upsilon_{éq} \to \mathcal{F}_2\, f_a(t) \quad (33)$$

Suppose that the discontinuous component in *(7)* is replaced by the continuous approximation

$$\upsilon_\delta = -\rho(t,y,u)\|P_0\mathcal{F}_2\| \frac{e_y}{\|e_y\|+\delta} \tag{34}$$

where δ is a small positive scalar. It can be shown that the equivalent output injection can be approximated to any degree of accuracy by *(34)* for a small enough choice of δ . Since $rank\ (\mathcal{F}_2) = q$ it follows from *(33)* that

$$\hat{f}_a(t) \approx -\rho\ \|P_0\mathcal{F}_2\|(\mathcal{F}_2^T\mathcal{F}_2)^{-1}\mathcal{F}_2^T P_0^{-1} \frac{e_y}{\|e_y\|+\delta} \tag{35}$$

4.2. Reconstruction of sensor faults

Now consider the case when $f_a(t) \equiv 0$ and consider the effect of $f_s(t)$. In this situation equation *(2)* becomes

$$y(t) = C\ x(t) + f_o(t) \tag{36}$$

and therefore $e_y(t) = e_2(t) - f_o(t)$. It follows that

$$\begin{aligned} \dot{e}_1(t) &= \mathcal{A}_{11}e_1(t) + \mathcal{A}_{12}f_s(t) + (\mathcal{A}_{12} - \mathcal{G}_{l,1})e_y \\ \dot{e}_y(t) &= \mathcal{A}_{21}e_1(t) + \mathcal{A}_{22}f_s(t) + (\mathcal{A}_{22} - \mathcal{G}_{l,2})e_y - \dot{f}_s(t) + P_0^{-1}\upsilon \end{aligned} \tag{37}$$

Arguing as before, provided a sliding motion can be attained,

$$0 = \mathcal{A}_{21}e_1(t) - \dot{f}_s(t) + \mathcal{A}_{22}f_s(t) + P_0^{-1}\upsilon\ . \tag{38}$$

If the fault signal $f_s(t)$ is slowly varying compared with the dynamics of the sliding motion in *(37)*, $e_1 \approx -\mathcal{A}_{11}^{-1}\ \mathcal{A}_{12}\ f_s$ and the derivative $\dot{f}_s$ in *(38)* can be ignored. Consequently

$$P_0^{-1}\upsilon_{eq} \approx -(\mathcal{A}_{22} - \mathcal{A}_{21}\ \mathcal{A}_{11}^{-1}\ \mathcal{A}_{12})f_s \tag{39}$$

As in the previous section, the equivalent output injection υ_{eq} can be calculated from *(34)* and consequently if $(\mathcal{A}_{22} - \mathcal{A}_{21}\ \mathcal{A}_{11}^{-1}\ \mathcal{A}_{12})$ is non singular the fault signal can be obtained from equation *(39)*.

A complete FDI scheme using SMO is shown in Figure 1.

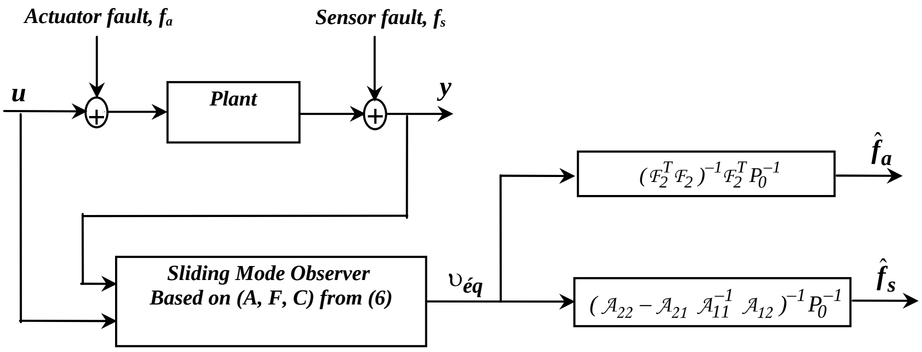


Figure 1. Schematic of the fault detection structure using the sliding mode observer.

5. Illustrative example

The FDI scheme in this paper will now be demonstrated with an example, which is a lateral axis model of an L-1011 in cruise flight conditions taken from reference [4].

The system states, outputs and inputs respectively are

$$x=\begin{bmatrix} \phi : \text{bank angle (rad)} \\ r : \text{yaw rate (rad } s^{-1}) \\ p : \text{roll rate (rad } s^{-1}) \\ \beta : \text{sideslip angle (rad)} \\ x_5 : \text{washed outfilter state} \end{bmatrix}, \quad y=\begin{bmatrix} r_{wo} : \text{washed out yaw rate} \\ p : \text{roll rate (rad } s^{-1}) \\ \beta : \text{sideslip angle (rad)} \\ \phi : \text{bank angle (rad)} \end{bmatrix} \tag{40}$$

$$u=\begin{bmatrix} \delta_r : \text{rudder deflection (rad)} \\ \delta_a : \text{aileron deflection (rad)} \end{bmatrix}$$

The system triple (A,F,C) is given by

$$A=\begin{bmatrix} 0 & 0 & 1.0000 & 0 & 0 \\ 0 & -0.1540 & -0.0042 & 1.5400 & 0 \\ 0 & 0.2490 & -1.0000 & -5.2000 & 0 \\ 0.0386 & -0.9960 & -0.0030 & -0.1170 & 0 \\ 0 & 0.5000 & 0 & 0 & -0.5000 \end{bmatrix} \tag{41}$$

$$B=F=\begin{bmatrix} 0 & 0 \\ -0.7440 & -0.0320 \\ 0.3370 & -1.1200 \\ 0.0200 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C=\begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be verified that the distribution matrix F in (41) results in CF being full rank and the system (A, F, C) does not have any invariant zeros. Hence, the observer design method proposed in § 3.2 was adopted.

5.1. Observer design

The system $(\overline{A},\overline{F},\overline{C})$ in the canonical form of (5) is given by

$$\overline{A}=\begin{bmatrix} -0.0133 & 0.0007 & 0.0172 & -0.2837 & -0.6267 \\ 0 & 0 & -0.0008 & -0.9110 & 0.4125 \\ -0.7071 & 0.0386 & -0.0858 & 0.4192 & 0.9199 \\ -0.1227 & 0.0004 & 4.1018 & -0.8028 & 0.4471 \\ 0.1545 & 0.0009 & -3.5336 & 0.0581 & -0.8692 \end{bmatrix}$$

$$\overline{F}^T=\begin{bmatrix} 0.0000 & 0 & -0.0000 & 0.0000 & 0.8170 \\ -0.0000 & 0 & -0.0000 & 1.0335 & -0.4328 \end{bmatrix} \qquad (42)$$

$$\overline{C}=\begin{bmatrix} 0 & 0 & 0.0265 & -0.4124 & -0.9106 \\ 0 & 0 & -0.0008 & -0.9110 & 0.4125 \\ 0 & 0 & 0.9996 & 0.0103 & 0.0245 \\ 0 & 1.0000 & 0 & 0 & 0 \end{bmatrix}$$

Choosing the weighting matrices respectively as $w=0.8I_5$, $V=0.2I_4$ and imposing, the design constraints described earlier the following observer gain matrices (in the original coordinates) were obtained:

$$P_0=\begin{bmatrix} 3.7711 & 0.8538 & 0.1744 & -0.1165 \\ 0.8538 & 3.2996 & 2.8969 & -0.6173 \\ 0.1744 & 2.8969 & 5.6625 & -0.3588 \\ -0.1165 & -0.6173 & -0.3588 & 2.4010 \end{bmatrix} \qquad (43)$$

$$G_l = \begin{bmatrix} -0.0492 & 0.5457 & -0.1383 & 2.1997 \\ 1.2729 & 0.2967 & -0.6963 & 0.0753 \\ -0.6229 & 3.1433 & -1.5543 & 0.5457 \\ 0.2708 & -1.5543 & 1.6611 & -0.1383 \\ -0.1799 & 0.9196 & -0.9671 & 0.1245 \end{bmatrix}$$

$$G_n = \begin{bmatrix} -0.0098 & 0.1091 & -0.0277 & 0.4399 \\ 0.2546 & 0.0593 & -0.1393 & 0.0151 \\ -0.1246 & 0.6287 & -0.3109 & 0.1091 \\ 0.0542 & -0.3109 & 0.3322 & -0.0277 \\ -0.0360 & 0.1839 & -0.1934 & 0.0249 \end{bmatrix} \quad (44)$$

In the canonical form of *(16)* the triple $(\mathcal{A}, \mathcal{F}, C)$ is given by

$$\mathcal{A} = \begin{bmatrix} -0.5778 & -0.1083 & -0.0303 & 0.4130 & 0.0153 \\ -0.1089 & -0.6516 & -0.0043 & 1.6290 & -0.0031 \\ 0.1761 & 0.2452 & -0.9999 & -5.3438 & 0.0049 \\ -0.7043 & -0.9807 & -0.0034 & 0.4583 & 0.0188 \\ 0 & 0 & 1.0000 & 0 & 0 \end{bmatrix}$$

$$\mathcal{F}^T = \begin{bmatrix} 0.0000 & -0.7440 & 0.3370 & 0.0200 & 0 \\ -0.0000 & -0.0320 & -1.1200 & 0.0000 & 0 \end{bmatrix} \quad (45)$$

$$\bar{C} = \begin{bmatrix} 0 & 1.0000 & 0.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

The eigenvalues of $\mathcal{A}_0$ are $\{-2.6942 \pm 2.2925i, -0.6537, -2.0929 \pm 0.1898i\}$. The eigenvalue of $\mathcal{A}_{11}$ is -0.5778 and hence the sliding motion is stable.
From this representation it can be seen that

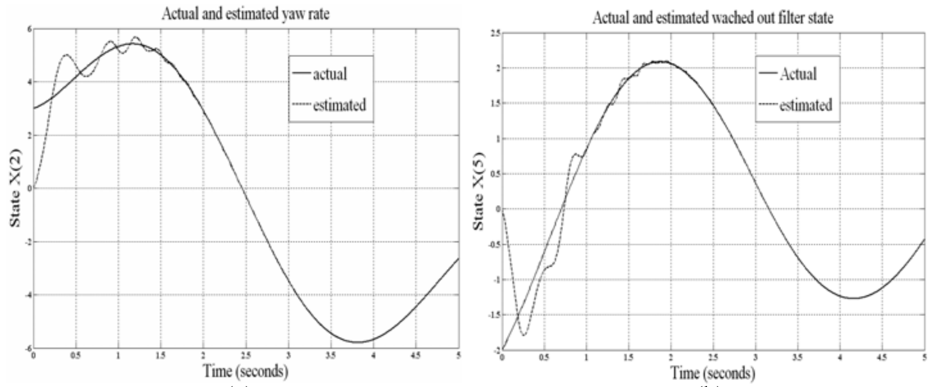
$$\mathcal{F}_2 = \begin{bmatrix} -0.7440 & 0.3370 & 0.0200 & 0 \\ -0.0320 & -1.1200 & 0.0000 & 0 \end{bmatrix}^T \quad (46)$$

5.2. Observer simulations

In the first simulations, the discontinuous observer designed in the previous subsection has been utilized. The scalar design parameter $\rho = 50$, and the discontinuous vector υ has been approximated by

$$\upsilon_{approx} = -\rho \left\| P_0 \mathcal{F}_2 \right\| \frac{e_y}{\left\| e_y \right\| + \delta} \quad with \;\; \delta = 0.025$$

The bank angle, roll rate and sideslip states appear directly as outputs and so theirs estimates need not be showed. However the yaw rate state x_2 appears as linear combination with washed out filter state x_5. Figure 2 shows the states x_2 and x_5 plotted for comparison against the estimated values $\hat{x}_2$ and $\hat{x}_5$. It can be seen that after *1.5 sec* almost perfect tracking in both states is obtained and the sliding motion of the error system is close to zero.



(a) (b)

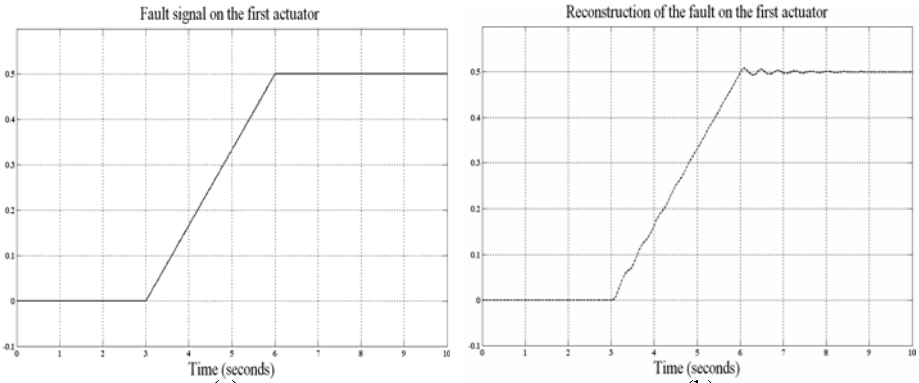
Figure 2. **(a) :** the trajectories of state vector $x_2(t)$ and estimator vector $\hat{x}_2(t)$
(b) : the trajectories of state vector $x_5(t)$ and estimator vector $\hat{x}_5(t)$

5.3. A sliding mode fault detection system

The following simulations show the faults in both actuators and the first sensor, as well as their respective reconstructions, which visually are identical to the faults. This shows that the method presented in this paper is successful.

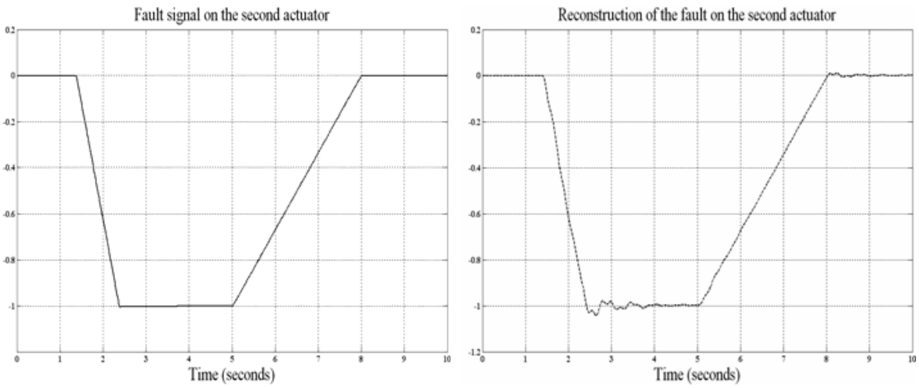
5.3.1 Simulations in the absence of measurement noise

Figures 3 and 4 show the sliding mode observer faithfully reconstructing faults simultaneously occurring in both actuators. Figure 5 shows that the observer reconstructs faithfully the faults acting on the first sensor.



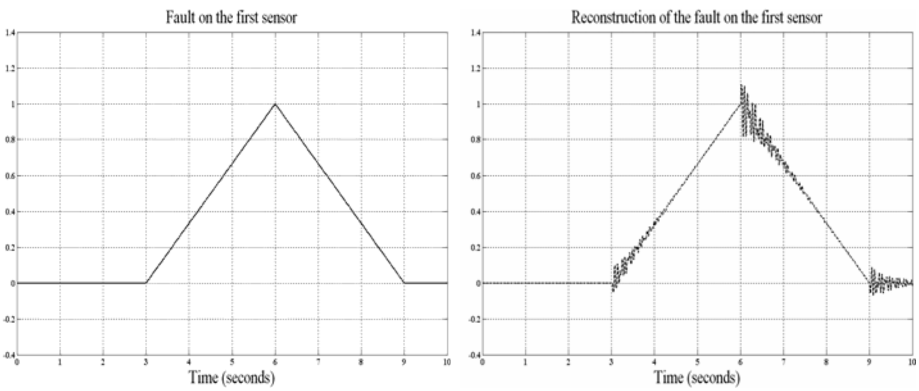
(a) (b)

Figure 3. **(a):** Fault signal on the first actuator (solid line -)
(b): Reconstruction of the fault actuator 1 (dashed line --)



(a) (b)

Figure 4. **(a):** Fault signal on the second actuator (solid line -)
(b): Reconstruction of the fault actuator 2 (dashed line --)



(a) (b)

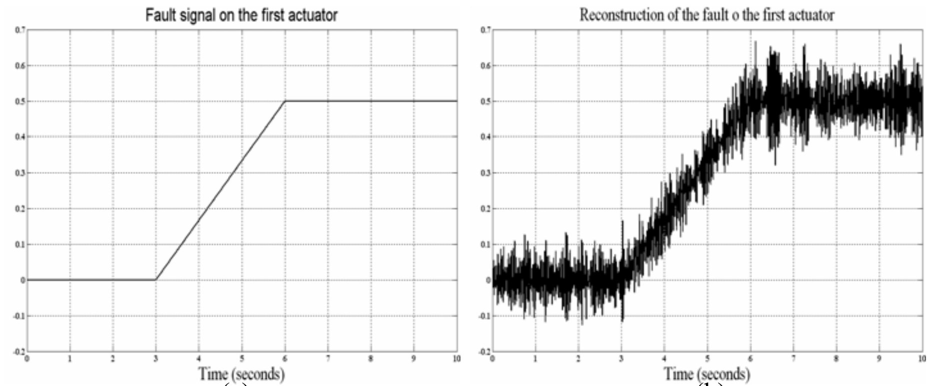
Figure 5. **(a):** Fault signal on the first sensor (solid line -)
(b): Reconstruction of the fault sensor 1 (dashed line --)

5.3.2 Simulations in the presence of measurement noise

The following figures are from simulations of identical scenarios to those considered above except white noise of standart deviation of 10^{-4} has been added to the output signal so that the measured signal which is used by the fault detection observer is corrupted.

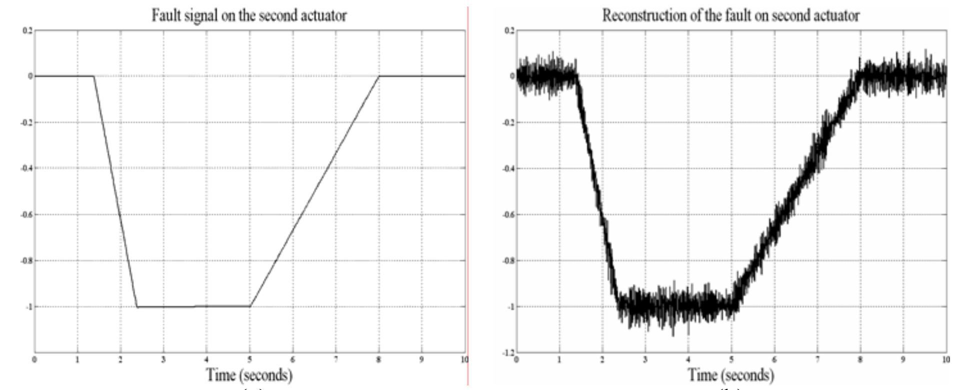
From Eq. (37) it can be seen that theoretically the derivative of the noise appears in the output error channel and hence constitutes a large disturbance. Thus arbitrarily large values of ρ would be needed to sustain a sliding motion.

As before in Figures 6, 7 and 8 the observer replicates the fault, except noise is now overlaid on the reconstruction signal.



(a) (b)

Figure 6. **(a):** Fault signal on the first actuator
(b): Reconstruction of the fault actuator 1



(a) (b)

Figure 7. **(a):** Fault signal on the second actuator
(b): Reconstruction of the fault actuator 2

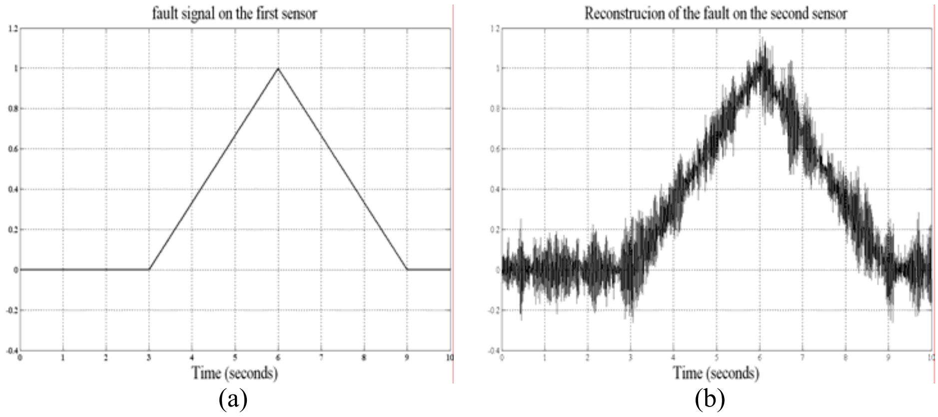


Figure 8. **(a):** Fault signal on the first sensor
(b): Reconstruction of the fault sensor 1

6. Conclusion

This paper has described the design of a sliding mode observer using Linear Matrix Inequalities (LMIs). The design approach was formulated as a convex optimization problem which can be efficiently solved using standart LMIs tools. The observer design methodology was built on the work of Edwards & Tan [5]. The implementation of this observer is useful for detecting and reconstructing actuator and sensor faults. A simulation based on a lateral axis model of an L-1011in cruise flight conditions has demonstrated its effectiveness in achieving fault detection and isolation.

7. References

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