International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 3, N° 2, December 2009, pp. 996–1011.

New Nonlinear Output Feedback Controller for Stabilizing the Colpitts Oscillator

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Abstract. *This paper deals with new asymptotic stability conditions and asymptotic output feedback control laws, stabilizing nonlinear continuous systems, using the arrow form matrix properties. The proposed approach is, essentially, based on the determination of the controller gains, guaranteeing, in the same time, the achievement of an arrow form instantaneous characteristic matrix and making easier the stability study. One numerical example, consisting of the nonlinear Colpitts oscillator, is considered to illustrate the efficiency of the developed results.*

Keywords. *Nonlinear continuous systems; Colpitts oscillator; Output feedback control; Stability; Arrow form matrix.*

1. Introduction

The study of the stability property of an open or closed loop, linear or nonlinear, plant is generally based on the assumption that mathematical model of the system under consideration is known.

Linear system stability study generally leads to necessary and sufficient conditions and does not depend on the system representation. The task is different for nonlinear systems with or without uncertainties, for which only sufficient conditions are given, then their stability domains depend on the choice of both description of the studied system and the used stability method [1], [2], [3], [4].

In state feedback nonlinear control, full state information can be measured and is available as initial condition for predicting the future system behaviour. In many applications, however, the system state can not be fully measured, and only output information is directly available for feedback stabilization of nonlinear systems [5], [6], [7], [8], [9], [10], [11], [12].

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

In such a way, and by using the practical Borne-Gentina stability criterion [13], [14], applied to continuous systems, this paper sets out to establish a new output feedback stabilizing approach for nonlinear continuous systems. The proposed approach, which constitutes an extension of the used state feedback stabilizing approach formulated in our previous work [15], is carried out through the determination of control laws, guaranteeing the asymptotic stabilisability property by making the matrix description of the controlled system under the arrow form [1], [2], [3], [15], [16], [17], [18], [19], [20], [21], [22].

This arrow form representation was used in previous works on asymptotic and global stability of nonlinear systems [1] and, recently, on chaotic systems synchronization [19]. It appears very suitable for two-level hierarchical systems with many nonlinearities and can be extended without any difficulty to multilevel hierarchical system description.

Indeed, the main purpose in this work is to design an adaptive output feedback controller for stabilizing the nonlinear Colpitts oscillator, which has been extensively used in electronic devices and communication systems for years [23], [24], [25], especially in radio frequencies applications as a source of sinusoidal waveforms with low harmonic content.

The remainder of this paper is structured as follows. In Section 2, the studied systems are presented and previous work on the stabilisability analysis, using the Benrejeb arrow form matrix, is, briefly, recalled. The new sufficient stabilisability conditions are stated in Section 3 and followed, in Section 4, by an example of the Colpitts oscillator to illustrate the proposed design effectiveness.

2. Efficiency of the sufficient stabilisability conditions using the Benrejeb arrow form matrix

The quantum advance in stability theory that allowed the analysis of arbitrary differential equations is due to Lyapunov, who introduced the basic idea and the definitions of stability that are in use today and proved many of the existing fundamental theorems [26].

This section is devoted to the review of some results already existing, exploiting a particular matrix description; namely, the Benrejeb arrow form matrix [1], [18], in order to give sufficient stabilisability conditions of nonlinear continuous systems.

Let the nonlinear system assumed to be described in the following forced form:

$$\begin{cases} y^{(n)} + \sum_{i=0}^{n-1} a_i(.)y^{(i)} = u(t) \\ u(t) = -\begin{bmatrix} k_0(.) & \dots & k_{n-1}(.) \end{bmatrix} y(t) \end{cases} \tag{1}$$

where $y \in \mathbb{R}$ denotes the output of the studied system and $a_i(.)$, $\forall i = 0,1,\dots,n-1,$ are nonlinear coefficients.

It should be mentioned that, as shown in appendix, the system (1) can be described by the state space equations:

$$\dot{z}(t) = A_c(.)z(t) \tag{2}$$

where z represents the state vector, such that $z = \begin{bmatrix} y & y' & \dots & y^{(n-1)} \end{bmatrix}^T$, $z \in \mathbb{R}^n$ and the matrix $A_c(.)$ is in the Benrejeb arrow form, with:

$$A_c(.) = \begin{bmatrix} \alpha_1 & & & \beta_1 \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \beta_{n-1} \\ \gamma_1(.) & \dots & \gamma_{n-1}(.) & \gamma_n(.) \end{bmatrix} \tag{3}$$

$$\beta_i = \prod_{\substack{i,j=1 \\ i \neq j}}^{n-1} \left(\alpha_i - \alpha_j \right)^{-1} \tag{4}$$

$$\gamma_i(.) = -P_{A_c}\left(.,\alpha_i\right) \ \forall i = 1,2,\dots,n-1 \tag{5}$$

$$\gamma_n(.) = -\left(a_{n-1}(.) + k_{n-1}(.)\right) - \sum_{i=1}^{n-1} \alpha_i \tag{6}$$

and α_i, $\forall i = 1,2,\dots,n-1,$ distinct arbitrary constant parameters.

$P_{A_c}\left(.,\lambda\right)$ is the instantaneous characteristic polynomial of the matrix $A_c(.)$ [18] which can be written as:

$$P_{A_c}\left(.,\lambda\right) = \lambda^n + \sum_{i=0}^{n-1} \left(a_i(.) + k_i(.)\right)\lambda^i \tag{7}$$

The particular matrix form (3) allows, having the non constant elements of the free state matrix located in the last row, to establish a stability criterion, relatively to the nonlinear analyzed system.

In fact, it is shown [1] that the equilibrium state of the nonlinear closed loop system (2) is asymptotically stable if:

$$\alpha_i < 0, \ \alpha_i \neq \alpha_j, \ \forall i, \ j = 1,\dots,n-1, \ i \neq j \tag{8}$$

i. there exist a small strictly positive parameter ε, such that:

$$\gamma_n(.) - \sum_{i=0}^{n-1} \left|\beta_i \gamma_i(.)\right| \alpha_i^{-1} \leq -\varepsilon \tag{9}$$

It is noted that, in the specific case, where the $(n-1)$ products $\beta_i \gamma_i(.)$, $\forall i = 1,\dots,n-1,$ are non-negative, the aforementioned condition (9) can be reduced and stated, by means of the instantaneous characteristic polynomial of the matrix $A_c(.)$, in the following manner:

$$P_{A_c}\left(.,0\right) \geq \varepsilon \tag{10}$$

which constitutes a verification of the validity of the linear Aizerman conjecture [27]. These conditions, associated to aggregation techniques based on the use of vector norms [14], have led to stability domains for a class of Lur'e Postnikov systems whereas for example, Popov stability criterion failed [1].
Then, it seems interesting to look for control approaches putting characteristic matrices in arrow form to elaborate, easily, stabilisability conditions and to formulate them analytically which make them so useful.

3. New output feedback approach for stabilizing nonlinear continuous systems

3.1. Proposed stabilizing approach – Basic idea

A generalization to the case when the model cannot fit the controllable form (1) is presented afterwards.
Following the obtained stability conditions, explicitly expressed and based on a specific state space description, this section is concerned with the synthesis of nonlinear control systems via output feedback.
Specifically and throughout this paper, we focus our attention on the class of nonlinear dynamical systems assumed to be described, in the state space, by the following system of first order differential equations:

$$\begin{cases} \dot{x}(t) = A(x,t)x(t) + B(x,t)u(t) \\ y(t) = C(x,t)x(t) \end{cases} \tag{11}$$

where x is the n state vector, u the m control input vector, y the l output vector, $A(x,t)$ the $n \times n$ instantaneous characteristic matrix, $A(x,t) = \left\{a_{ij}(x,t)\right\}$, $B(x,t)$ the $n \times m$ control matrix, $B(x,t) = \left\{b_{ij}(x,t)\right\}$, and $C(x,t)$ the $l \times n$ output matrix, $C(x,t) = \left\{c_{ij}(x,t)\right\}$.
It is straight forward to note that the basis of the proposed stabilisability analysis is the controllable form of the state space equation. Moreover, the feedback controllers presented in this part, are designed under the assumption of accessibility of all the process outputs for measurement.
At this stage, our primordial interest is restricted to the determination of the control stabilizing law, by output feedback, of the following form:

$$u(t) = -K(x,t)y(t) \tag{12}$$

which leads, by substituting u in the first equation of system (11) above and by considering the second equation of the same system, to the unforced closed loop system characterized by the following state space description:

$$\dot{x}(t) = A_f(x,t)x(t) \tag{13}$$

with:

$$A_f(x,t) = A(x,t) - B(x,t)K(x,t)C(x,t) \tag{14}$$

$K(x,t)$ is the $m \times l$ instantaneous gain matrix, $K(x,t) = \left\{k_{ij}(x,t)\right\}$.

Thus, the synthesis problem is to find an instantaneous output feedback gain matrix $K(x,t)$ such that the closed loop system matrix $A_f(x,t)$ is under the arrow form. Then, as shown in [14], the stability study of the closed loop system (13) can be easily treated, using the practical Borne-Gentina stability criterion [13].

3.2. Proposed output feedback control design

In this section a solution to the output feedback stabilization problem is proposed based on the application of the practical Borne-Gentina stability criterion combined with the use of the arrow form matrix properties. In particular, it is shown that the closed loop system can be described as an interconnection of subsystems, which recall the structure characterizing the class of two-level hierarchical systems.

The determination of the output feedback controller's structure (12), intended to the stabilization of the dynamic continuous nonlinear dynamical system described, in the state space, by (11), is led by imposing the achievement of an arrow form matrix, characterizing the closed loop system defined by (13).

The imposed arrow form of (14) suggests that we have to find an adequate instantaneous gain matrix $K(x,t)$ so that the following equations hold:

$$a_{ij}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{ir}(x,t)k_{rs}(x,t)\right)c_{sj}(x,t) = 0 \;\; \forall i,\, j = 1,\dots,n-1 \text{ for } i \neq j \tag{15}$$

Relations (15) arise $(n-1)(n-2)$ conditions of (ml) unknown parameters $k_{rs}(x,t),\ \forall r = 1,\dots,m$ and $\forall s = 1,\dots,l.$

So, a necessary condition, required in designing a stabilizing output feedback controller and leading to the existence of a solution relatively to this control system, is that the number of equations to resolve must be less than or equal to the number of unknown parameters such as:

$$ml \geq (n-1)(n-2) \tag{16}$$

Theorem. *The system described by (11) and verifying (15) and (16), is stabilized by the output feedback control law (12) if the instantaneous characteristic matrix* $A_f(x,t)$, *defined by (14), is such that:*

i. the nonlinear elements are located in either one row or one column,

ii. the first $(n-1)$ *diagonal elements are such that:*

$$a_{ii}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{ir}(x,t)k_{rs}(x,t)\right)c_{si}(x,t) < 0 \;\; \forall i = 1,\ldots,n-1 \tag{17}$$

iii. there exist $\varepsilon > 0$, *such that:*

$$\begin{aligned} & a_{nn}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{nr}(x,t)k_{rs}(x,t)\right)c_{sn}(x,t) \\ & -\left(\sum_{i=1}^{n-1}\left(\left| \begin{array}{l} \left(a_{ni}(x,t) - \sum_{s=1}^{l}\left(\sum_{j=1}^{m} b_{nr}(x,t)k_{rs}(x,t)\right)c_{si}(x,t)\right) \\ \times\left(a_{in}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{ir}(x,t)k_{rs}(x,t)\right)c_{sn}(x,t)\right) \end{array} \right|\right) \right. \\ & \left. \times\left(a_{ii}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{ir}(x,t)k_{rs}(x,t)\right)c_{si}(x,t)\right)^{-1} \right) \le -\varepsilon \end{aligned} \tag{18}$$

Proof. The choice of a comparison system having a characteristic matrix $M\left(A_f(x,t)\right)$, relatively to the vector norm $p(z) = \left[|z_1| \quad \ldots \quad |z_n|\right]^T$, $z = \left[z_1 \quad \ldots \quad z_n\right]^T$, leads to the following system of differential equations:

$$\dot{z}(t) = M\left(A_f(x,t)\right)z(t) \tag{19}$$

The elements $m_{ij}(x,t)$ of $M\left(A_f(x,t)\right)$ are deduced from the ones of the matrix $A_f(x,t)$ by substituting the off-diagonal elements by their absolute values, which can be written as:

$$\begin{cases} m_{ii}(x,t) = a_{f_{ii}}(x,t) \;\; \forall i = 1,\ldots,n \\ m_{ij}(x,t) = \left|a_{f_{ij}}(x,t)\right| \;\; \forall i,\, j = 1,\ldots,n,\; i \ne j \end{cases} \tag{20}$$

When the nonlinearities are isolated in either one row or one column of $M\left(A_f(x,t)\right)$, it comes the following sufficient stabilization conditions:

$$(-1)^h M\left(A_f(x,t)\right)\begin{pmatrix} 1 & 2 & \dots & h \\ 1 & 2 & \dots & h \end{pmatrix} > 0 \ \forall h = 1, 2, \dots, n \tag{21}$$

The system (11) is then stabilized by (12) if the matrix $M\left(A_f(x,t)\right)$ is the opposite of an $M-$ matrix, or equivalently, by application of the practical Borne-Gentina stability criterion [14], the following inequalities corresponding to (21) are satisfied:

$$\begin{cases} a_{f_{ii}}(x,t) < 0 \ \forall i = 1, 2, \dots, n-1 \\ (-1)^n \det\left(M\left(A_f(x,t)\right)\right) > 0 \end{cases} \tag{22}$$

The development of the first member of the last inequality of (22):

$$\begin{aligned} &(-1)^n \det\left(M\left(A_f(x,t)\right)\right) = \\ &(-1)\left(a_{f_{nn}}(x,t) - \sum_{i=1}^{n-1}\left(\left|a_{f_{ni}}(x,t)a_{f_{in}}(x,t)\right|\right)a_{f_{ii}}^{-1}(x,t)\right)(-1)^{n-1}\prod_{j=1}^{n-1} a_{f_{jj}}(x,t) \end{aligned} \tag{23}$$

achieves easily the proof of the theorem.

Corollary. *The system described by (11) and verifying (15) and (16), is stabilized by the output feedback control law (12) if the instantaneous characteristic matrix* $A_f(x,t)$, *defined by (14), is such that:*

i. the nonlinear elements are located in either one row or one column,

ii. the first diagonal elements $a_{f_{ii}}(x,t)$, $\forall i = 1, \dots, n-1$, *are strictly negative,*

iii. the off-diagonal elements are such that:

$$\left(\begin{aligned} &\left(a_{ni}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{nr}(x,t)k_{rs}(x,t)\right)c_{si}(x,t)\right) \\ &\times\left(a_{in}(x,t) - \sum_{s=1}^{l}\left(\sum_{r=1}^{m} b_{ir}(x,t)k_{rs}(x,t)\right)c_{sn}(x,t)\right) \end{aligned}\right) \geq 0 \ \forall i = 1, \dots, n-1 \tag{24}$$

iv. there exist $\varepsilon > 0$, *for which the instantaneous characteristic polynomial* $P_{A_f}(x,t)$ *is such that:*

$$P_{A_f}(x,t,\lambda)\Big|_{\lambda=0} = \det\left(\lambda\mathbb{I} - A_f(x,t)\right)\Big|_{\lambda=0} \geq \varepsilon \tag{25}$$

Proof. The proof of the corollary is inferred of the previous theorem one by taking into account the new assumption iii. conditions, which guarantee, through a simple transformation, the identity of the matrix $A_f(x,t)$ and its overvaluing $M\left(A_f(x,t)\right)$, and, finally, by pointing out that:

$$(-1)^n \det\left(M\left(A_f(x,t)\right)\right) = P_{A_f}\left(x,t,0\right) \tag{26}$$

4. Application to the control of the nonlinear Colpitts oscillator

To demonstrate that the proposed output feedback scheme recovers the performance of the corresponding state feedback controller, the control of a continuous nonlinear Colpitts oscillator as presented in [23], is considered, in this last part of the paper.
The simplest configuration of the Colpitts oscillator is represented on Fig. 1. (a).
The circuit uses a Bipolar Junction Transistor (BJT) as the gain element. The resonant network consists of the inductor L, in series with the resistor R, and a pair of capacitors C_1 and C_2.

The ideal current generator I_0 helps to maintain constant biasing emitter current.
The nonlinear element is the BJT, for which the simplified model shown on Fig. 1. (b). consists of a nonlinear voltage-controlled resistance R_E and a linear current-controlled current source I_E [24].

One should note that the base current I_B can be neglected as compared to the emitter current I_E, which means that the current gain is supposed to be infinite. Moreover, the current source I_0 can be replaced by a resistor in series connection with a negative direct-current voltage generator.

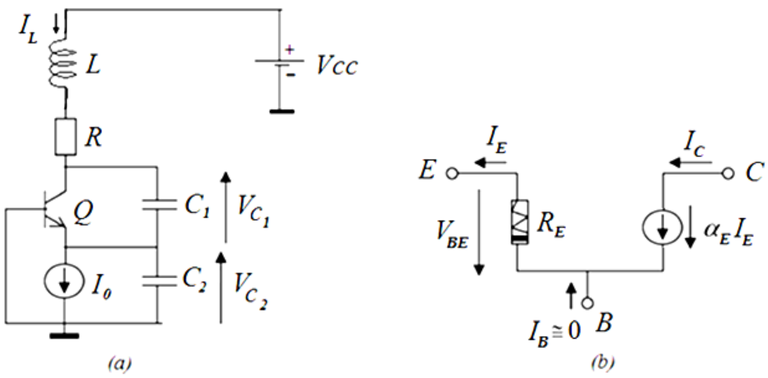

Fig. 1. Schematic model of the Colpitts oscillator [23]:
(a). The physical realization of the Colpitts oscillator
(b). The model of the transistor Q

Taking into account the BJT model of Fig. 1. (b)., the state equations of the circuit of Fig. 1. (a). are the following:

$$\begin{cases} C_1\dot{V}_{C_1}(t) = -\alpha_E I_E(t) + I_L(t) \\ C_2\dot{V}_{C_2}(t) = I_L(t) - I_0 \\ L\dot{I}_L(t) = -V_{C_1}(t) - V_{C_2}(t) - RI_L(t) + Vcc \end{cases} \tag{27}$$

α_E is the common-base forward short-circuit current gain that we further assume to be ideal $(\alpha_E = 1)$ because of the neglected base current. The tension-courant $V - I$ characteristic of the nonlinear resistor R_E, corresponding, normally, to the emitter-base diode, is defined as usual by:

$$I_E(t) = I_S\left(\exp\left(\frac{eV_{BE}(t)}{K_B T}\right) - 1\right) \tag{28}$$

where e is the elementary charge, K_B the Boltzmann constant and T the absolute temperature. At room temperature, the thermal voltage $V_T = \dfrac{K_B T}{e}$ is approximately equal to $26\ mV$. Since $V_{BE}(t) = -V_{C2}(t)$, it comes:

$$I_E(t) = I_S\left(\exp\left(-\frac{V_{C_2}(t)}{V_T}\right) - 1\right) = f\left(V_{C_2}(t)\right) \tag{29}$$

The previous first order differential equations (27) can be rewritten under the following matrix description:

$$\dot{x}(t) = A(.)x(t) + B(.)u(t) \tag{30}$$

with: $x(t) = \begin{bmatrix} V_{C1}(t) & V_{C2}(t) & I_L(t) \end{bmatrix}^T$

$$A(.) = \begin{bmatrix} 0 & \dfrac{I_S \exp\left(\dfrac{V_{C2}(t)}{V_T}\right)}{V_{C2}(t)} & \dfrac{1}{C_1} \\ 0 & 0 & \dfrac{1}{C_2} \\ \dfrac{-1}{L} & \dfrac{-1}{L} & \dfrac{-R}{L} \end{bmatrix},\ B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \dfrac{-1}{C_2} & 0 \\ 0 & 0 & \dfrac{1}{L} \end{bmatrix}$$

and: $u(t)=\begin{bmatrix} I_S & I_0 & V_{CC}\end{bmatrix}^T$

Using the circuit parameters:

$$\begin{array}{l} L=100\ \mu H,\ R=45\ \Omega,\ C_1=47\ nF,\ C_2=47\ nF \\ Vcc=5\ V,\ V_T=26\ mV,\ I_0=5\ mA,\ I_S=50\ mA \end{array} \tag{31}$$

and starting from the initial values of states $\begin{bmatrix} V_{C1}(0) & V_{C2}(0) & I_L(0)\end{bmatrix}=\begin{bmatrix}5 & 9 & 8\end{bmatrix}^T$ yields to the evolutions shown in Fig. 2.

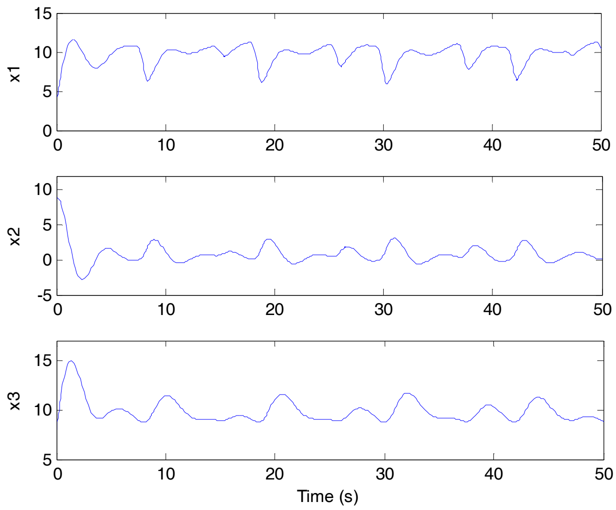

Fig. 2. Dynamics of the state variables V_{C1}, V_{C2} and I_L of the Colpitts oscillator before activating the controller

As stated, before, the controller, under output feedback, drives the system states to a neighbourhood of the origin desired set-point, that is to say, the system will be asymptotically stable.

In order to achieve our goal, an adequate non constant output feedback controller has to be determined so that all the hypotheses of the above-mentioned theorem are satisfied.

Now, by considering the output signals $y(t)=\begin{bmatrix} V_{C1}(t) & V_{C2}(t)\end{bmatrix}^T$ given by:

$$y(t)=Cx(t) \tag{32}$$

$$C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

The application of the above approach to the system described by the equations (30) and (32), consists in the determination of an adequate output feedback structure, of the form:

$$u(t)=-K(.)y(t) \tag{33}$$

with:

$$K(.) = \begin{bmatrix} k_{11}(.) & k_{12}(.) \\ k_{21}(.) & k_{22}(.) \\ k_{31}(.) & k_{32}(.) \end{bmatrix}$$

so that the closed loop system described by (34):

$$\dot{x}(t) = \left(A(.) - BK(.)C \right) x(t) \tag{34}$$

with:

$$A(.) - BK(.)C = \begin{bmatrix} -k_{11}(.) & \dfrac{I_S \exp\left(\dfrac{V_{C2}(t)}{V_T}\right)}{V_{C2}(t)} - k_{12}(.) & \dfrac{1}{C_1} \\ -k_{21}(.) & -k_{22}(.) & \dfrac{1}{C_2} \\ \dfrac{-1}{L} - k_{31}(.) & \dfrac{-1}{L} - k_{32}(.) & \dfrac{-R}{L} \end{bmatrix}$$

being characterized by an arrow form matrix. That is to say, it is sufficient to choose $k_{12}(.)$ and $k_{21}(.),$ such that some off-diagonal elements are zero:

$$\begin{cases} k_{12}(.) = \dfrac{I_S \exp\left(\dfrac{V_{C2}(t)}{V_T}\right)}{V_{C2}(t)} \\ k_{21} = 0 \end{cases} \tag{35}$$

Actually, to ensure asymptotic stability of the controlled system (34), let choose the two parameters of regulation $k_{11}(.)$ and $k_{22}(.),$ allowing to satisfy the assumptions (17) of the above-mentioned theorem, so, such as:

$$\begin{cases} k_{11} = 1 \\ k_{22} = 2 \end{cases} \tag{36}$$

At this stage, it remains to tune the parameters of regulation $k_{31}(.)$ and $k_{32}(.),$ so that the sufficient condition (18), explicitly expressed, in this specific case, by:

$$\frac{-R}{L} + \left| \left(\frac{-1}{L} - k_{31}(.) \right) \left(\frac{1}{C_1} \right) \right| + 0.5 \left| \left(\frac{-1}{L} - k_{32}(.) \right) \left(\frac{1}{C_2} \right) \right| < 0 \tag{37}$$

being true.
To simplify the previous inequality, consider:

$$k_{31} = \frac{-1}{L} \tag{38}$$

Hence, to fulfil the modified condition (37), $k_{32}(.)$ has to satisfy the inequality:

$$k_{32}(.) < \frac{2RC-1}{L} \tag{39}$$

Among several choices, let choose this gain parameter $k_{32}(.)$, as follows:

$$k_{32} = \frac{2(RC-1)}{L} \tag{40}$$

The simulation results show the convergence properties of the considered states of the Colpitts oscillator, Fig. 3.

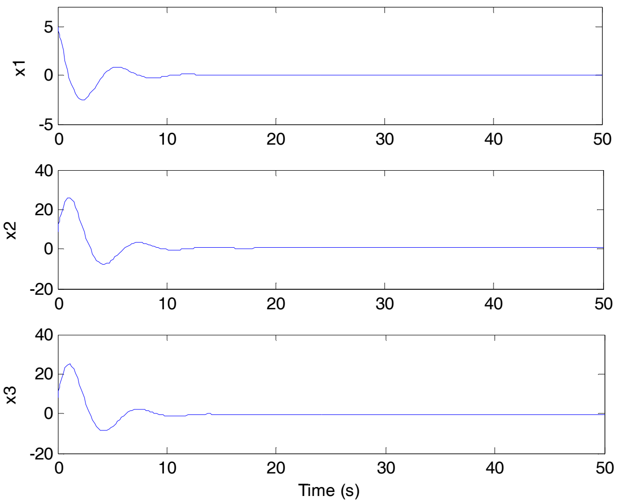

Fig. 3. Regulation of the voltages V_{C1} and V_{C2} and the current I_L signals of the Colpitts oscillator

Conclusion

The key of the control developments, throughout this work, is an appropriate design of an output feedback controller in order to achieve one particular matrix description; namely the arrow form, to which practical Borne-Gentina stability criterion can be applied.
It was rigorously established that the proposed output feedback controllers enforce stability in the nonlinear closed loop system.
The effectiveness of the proposed control design tools have been evinced by applying it to the Colpitts oscillator that, successfully, achieves global state regulation.

Appendix [18]

Consider the instantaneous characteristic matrix $\hat{A}(.)$, in Companion form, written as:

$$\hat{A}(.)=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & & 0 \\ 0 & & \cdots & 0 & 1 \\ -a_0(.)-k_0(.) & & \cdots & & -a_{n-1}(.)-k_{n-1}(.) \end{bmatrix}$$

A change of base under the form [18]

$$P=\begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & \\ \alpha_1^2 & \alpha_2^2 & \cdots & \alpha_{n-1}^2 & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \cdots & \alpha_{n-1}^{n-1} & 1 \end{bmatrix}$$

with $\alpha_i \neq \alpha_j,\ \forall i \neq j,$ allows the new state matrix, denoted by $A_c(.)$:

$$A_c(.)=P^{-1}\hat{A}(.)P$$

to be in the Benrejeb arrow form [1,18], defined by (3).

References

1. Benrejeb, M., Borne, P., Laurent, F: Sur une application de la représentation en flèche à l’analyse des processus, RAIRO Automatique/Systems Analysis and Control 16 (1982) 133–146.
2. Benrejeb, M., Abdelkrim, M. N.: On order reduction and stabilization of TSK nonlinear fuzzy models by using arrow form matrix, Systems Analysis Modelling Simulation 43 (2003).
3. Benrejeb, M., Gasmi, M.: On the use of an arrow form matrix for modelling and stability analysis of singularly perturbed nonlinear systems, Systems Analysis Modelling Simulation 40 (2001).
4. Borne, P., Benrejeb, M.: On the representation and the stability study of large scale systems, International Journal of Computers Communications and Control 3 (2008) 55–66.
5. Syrmos, V. L., Abdallah, C., Dorato, P., Grigoriadis, K.: Static output feedback: a survey, Automatica 33 (1997).

6. Geromel, J. C., Peres, P. L. D., Souza, S. R.: Convex analysis of output feedback control problems: robust stability and performance, IEEE Trans. Automat. Control 40 (1996).
7. Gu, G.: On the existence of linear optimal control with output feedback, SIAM J. Control Optim. 28 (1990).
8. Iwasaki, T., Skelton, R. E., Geromel, J. C.: Linear quadratic suboptimal control with static output feedback, System Control Lett. 23 (1994).
9. Kucera, V., De Souza, C.: A necessary and sufficient condition for output feedback stabilizability, Automatica 31 (1995).
10. Perez, F., Abdallah, C. Dorato, P., Docampo, D.: Algebraic tests for output stabilizability, Proc. IEEE Conf. on Decision and Control (1993).
11. Syrmos, V. L., Lewis, F.: A bilinear formulation for the output feedback problem in linear systems, IEEE Trans. Autom. Control 39 (1994).
12. Toker, O., Ozbay, H.: On the NP-hardness of solving bilinear matrix inequalities and simultaneous stabilization with static output feedback (1995).
13. Gentina, J. C., Borne, P., Laurent, F.: Stabilité des systèmes continus non linéaires de grande dimension, RAIRO (1972) 69–77.
14. Grujic, L. T., Gentina, J. C., Borne, P.: General aggregation of large scale systems by vector Lyapunov functions and vector norms, International Journal of Control 24 (1976) 529–550.
15. Hammami, S., Ben Saad, K., Benrejeb, M.: Sur une nouvelle approche de stabilisation de processus continus non linéaires multivariables, Editions de l'Académie Roumaine (2009) 137–141.
16. Grujic, L. T.: General stability of large scale systems, IFAC on Large Scale Systems Theory and Applications, Udine (1976).
17. Benrejeb, M., Borne, P.: On a algebraic stability criterion for nonlinear processes. Interpretation in the frequency domain, Proc. of MECO Congress, Athens 2 (1978).
18. Borne, P., Vanheeghe, P., Duflos, E.: Automatisation des processus dans l'espace d'état, Editions Technip, Paris (2007).
19. Hammami, S., Ben Saad, K., Benrejeb, M.: On the synchronization of identical and non-identical 4-D chaotic systems using arrow form matrix, Chaos, Solitons & Fractals 42 (2009) 101–112.
20. Benrejeb, M., Hammami, S., Ben Saad, K.: Nouvelles conditions de stabilisation par retour d'état de processus multivariables hiérarchisés à deux niveaux, JTEA (2008).
21. Benrejeb, M., Hammami, S.: New approach of stabilization of nonlinear continuous monovariable processes characterized by an arrow form matrix, SENDA (2008).
22. El Kamel, A., Borne, P., Ksouri Lahmari, M. Benrejeb, M.: On the stability of nonlinear multimodel systems, Stability And Control: Theory and Applications 2 (1999).

23. Fotsin, H. B., Daafouz, J.: Adaptive synchronization of uncertain chaotic colpitts oscillators based on parameter identification, Physics Letters A 339 (2005) 304–315.
24. Maggio, G. M., De Feo, O., Kennedy, M. P.: Nonlinear analysis of the Colpitts oscillator and applications to design, IEEE Transactions on Circuits and Systems – I: Fundamental Theory and Applications 46 (1999) 1118–1130.
25. Kennedy, M. P.: Chaos in Colpitts oscillator, IEEE Transactions on Circuits and Systems – I 41 (1994) 771–774.
26. Kuzmina, L. K.: Lyapunov's theory methods in stability problems of singular systems, Nonlinear Analysis: Theory, Methods & Applications (2009) (in press).
27. Grujic, L. T.: On absolute stability and the Aizerman conjecture, Automatica 17 (1981).