

## Graph Cut Based Segmentation of Brain Tumor From MRI Images

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**Abstract.** *In this paper, the image segmentation is considered as a graph partition problem and global criterion which measures both the total dissimilarity among the different groups and the total similarity inside them is proposed. An efficient method based on a generalized eigenvalue treatment is used to optimize this criterion in order to segment images. The method is applied to segment brain tumors from MRI (Magnetic Resonance Imaging) images, then providing automatically the information about tumor for helping the diagnostic. The results obtained by the proposed method are encouraging.*

**Résumé.** *Dans ce papier, la segmentation d'images est vue comme un problème de partition de graphes et le critère global qui mesure à la fois la dissimilarité totale parmi les différentes groupes et la similarité totale est proposé. Une méthode efficace basée sur un traitement de valeurs propres généralisées est utilisée pour optimiser ce critère dans le but de segmenter les images. La méthode est appliquée pour segmenter les tumeurs de cerveau issus d'images à résonance magnétique, apportant ainsi automatiquement les informations sur des tumeurs afin d'aider le diagnostic. Les résultats obtenus par la méthode sont très encourageants.*

**Keywords.** Graph, graph cut, image segmentation, MRI images

**Mots-clés.** Graphe, Coupe de graphes, segmentation d'images, Images de résonance magnétique.

### 1. Introduction

Many years ago, Wertheimer pointed out the importance of perceptual grouping and organization in vision and listed several key factors, such as similarity, proximity, and good continuation, which lead to visual grouping. However, even to nowadays, many computational issues of perceptual grouping remain unsolved [1]. Many methods have been proposed for the divisions of an image into several subsets, the problems can be summed as the following questions: (1) How can we pick the “right” one? (2) What is the criterion to be optimized? (3) Is there an efficient algorithm for carrying out the optimization? In order to solve these problems, the method related to the graph theoretic formulation of grouping is

proposed in [2]. This method is very interesting in the field of medical image segmentation, for detecting and characterizing smart objects, seizing the semantic content (i.e. geometric characteristics of the organs, such as location of the tumor). That is why we choose this graph theoretic formulation for tumor segmentation.

Accurate and robust brain tissue segmentation from Magnetic Resonance Imaging (MRI) is a very important issue in many applications of medical image system for quantitative studies and particularly in the study of some brain disorders. One example is to analyze and estimate quantitatively the growth process of brain tumors, and to evaluate effects of some pharmaceutical treatments in clinic. A lot of studies of brain segmentation have been carried out and are reported in the literature. The methods based on elastic registration using elastic matching techniques, or deformable models [13] have proven the efficiency for small and local shape changes, especially for normal tissue segmentation. The methods based on statistical models, such as Gaussian intensity models [14], explicit models [15], Markov random field models [16] work well in case of normal tissue segmentation. In the pathological cases, the methods based on supervised or unsupervised classification integrating anatomical templates [17][18] have shown their robustness. Level set methods are also used for brain tumor segmentation [19][20] with some successes. Mancas and Gosselin [21] used the iterative watersheds to segment the brain tumor with a given initialization.

In this paper, we propose a region segmentation based on a graph cuts theoretic criterion to measure the advantages of image partition. This criterion formulated as a generalized eigenvalue problem will be introduced in the section 2. In Section 3 a detailed explanation to the steps of grouping algorithm is presented. The experiments of the algorithm are shown in section 4. We discuss the future work and get a conclusion in section 5.

## 2. The Graph Cut

Graph cut is the process of partitioning directly or indirectly a graph into disjoint subsets. The concept of optimality of the cuts here is introduced by associated eigenvalue to each cut. Problems of this kind have been well studied within the field of graph theory, but for the optimization it is more difficult. Graph cut methods have been successfully applied to stereo, image restoration, texture synthesis, image segmentation. In the following we will give a brief overview of graph cuts for image segmentation as well as introduction to some basic definitions [3][4].

### 2.1. Spectral clustering

Given a set of data points  $x_1, \dots, x_n$ , the goal of clustering is to divide the data points into several groups such that points in the same group are similar and points in different groups are dissimilar to each other [5]. The data can be represented in form of a graph  $G = (V, E, W)$ ,  $V$  denotes the nodes representing the data points  $x_i$ ,  $E$  the edges weighted by a weighted adjacency matrix  $W = (w_{ij})_{i, j=1, \dots, n}$ . Each edge between two vertices  $v_i$  and  $v_j$  carries a non-negative weight. If  $w_{ij} = 0$  this means

that the vertices  $v_i$  and  $v_j$  are not connected. The degree of a node  $v_i \in V$  is defined

as  $d_i = \sum_{j=1}^n w_{ij}$ . Note that, this sum only runs over all nodes adjacent to  $v_i$ . The

degree matrix  $D$  is defined as the diagonal matrix with the degrees  $d_1, \dots, d_n$  on the diagonal. A partition of similarity graphs is in fact based on modeling the local neighborhood relationships between the data points. Different similarity graphs have been proposed in spectral clustering. Here we use the  $\varepsilon$ -neighborhood graph. All connections with distances below a threshold  $\varepsilon$  are set to 1:

$$\sum_{\langle i,j \rangle} w_{ij} = 1 \quad \{ \langle i,j \rangle | d_{ij} \in \varepsilon \} \tag{1}$$

When the input data are represented in form of a similarity graph with the adjacency matrix  $W$ , the simplest and most direct way to segment the image is to construct a bipartition of the graph. One most common objective function the normalized Cut Ncut [11] is usually used to solve this problem. The criterion to be minimized for separating an image into  $A$  and  $\bar{A}$  is defined as follows:

$$Ncut(A, \bar{A}) = \left( \frac{1}{Vol A} + \frac{1}{Vol \bar{A}} \right) \sum_{i \in A, j \in \bar{A}} w_{i,j} \tag{2}$$

where  $Vol A = \sum_{i \in A} d_i$

and  $A$  and  $\bar{A}$  are the subsets of  $V$ ,  $A \cup \bar{A} = V$ ,  $A \cap \bar{A} = \phi$ .

Minimizing Ncut means finding a cut of relatively small weight between two subsets with strong internal connections. It is shown that optimizing the Ncut criterion is NP hard. In [3], the Laplacian matrix is used to solve the optimizing problem.

In the case of  $k$ -partition, the clustering consists of choosing the partition  $A_1, \dots, A_k$  which minimizes:

$$cut(A_1, \dots, A_k) = \sum_{i=1}^k cut(A_i, \bar{A}_i) \tag{3}$$

### 2.2. Laplacians and their properties

The spectral algorithm presented here is based on eigenvectors of Laplacians, which is the combination of the weight and the degree matrices. The unnormalized graph Laplacian matrix is defined as  $L = D - W$ . The eigenvectors of the matrix can be normalized to norm 1 in two ways:

$$L_{sym} = D^{-\frac{1}{2}} L D^{\frac{1}{2}} = I - D^{-\frac{1}{2}} W D^{\frac{1}{2}} \quad (4)$$

$$L_{rw} = D^{-\frac{1}{2}} L = I - D^{-1} W \quad (5)$$

They have some following properties:

1.  $L_{sym}$  and  $L_{rw}$  are positive semi-definite and have n non-negative eigenvalues  $0 \leq \lambda_0 \leq \lambda_1 \leq \dots \leq \lambda_{n-1}$ .
2. The multiplicity of the eigenvalue 0 of the both  $L_{sym}$  and  $L_{rw}$  is equal to the number of connected components  $A_1, \dots, A_k$  in the graph [7].

The Ncut algorithm focuses on the second smallest eigenvalue  $\lambda_0$  and its corresponding eigenvector. When the partition is optimal,  $Ncut = \lambda_0$ .

### 3. Clustering algorithms for tumor segmentation

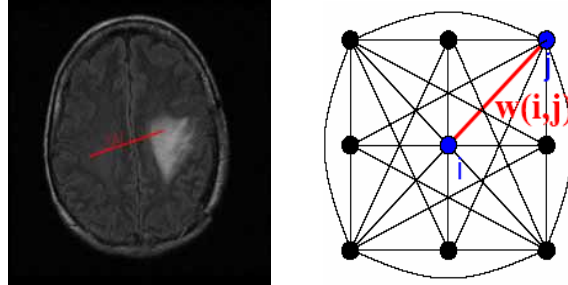
We present in this section the implementation algorithm for extracting the tumor areas in MRI brain images. The algorithm is composed of the following steps:

Step1: Construct a weighted graph from the image pixels  $G = (V, E, W)$  by taking each pixel and connecting each pair of pixel in an edge [10] [12]:

$$w_{ij} = e^{-\frac{\|F(i)-F(j)\|_2^2}{SI^2}} * \begin{cases} e^{-\frac{\|X(i)-X(j)\|_2^2}{SX^2}} & si \|X(i) - X(j)\|_2 < r \\ 0 & other \end{cases} \quad (6)$$

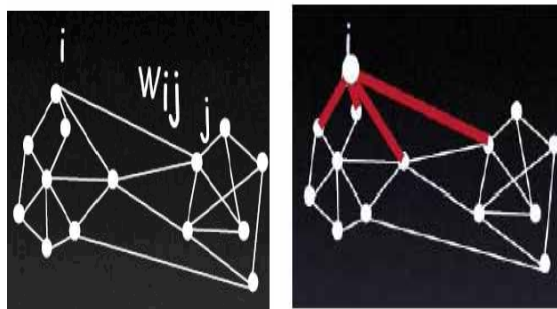
where  $F(i)$  is the gray level value at pixel i,  $X(i)$  the position of pixel I in image,  $W_{ij}$  measures the similarity between two pixels i and j,  $r$  the threshold of distance from i to j,  $SI$  a constant relating to the gray level and  $SX$  a constant relating to the distance.

Figure 1 shows an illustration of the weight  $w_{ij}$  which measures the likelihood of pixel i and j within a neighborhood and the map of connection design.



**Figure 1:** Two pixels  $i, j$  on the original image (left), affinity between the two pixels and its connection in a neighboring region (right).

Step2: divide the graph into segments, with the similar pixel in the same segments, and dissimilar in different segments (Figure. 2), then find the similarity matrix  $W = |w_{ij}|$  and the degree matrix  $D = |d_i| = \sum_i w_{i,j}$ .



**Figure 2:** Illustration of a graph (left) and its division (right).

These two matrices  $W$  and  $D$  are very important for solving the generalized eigenvector which minimizes the graph cut [3].

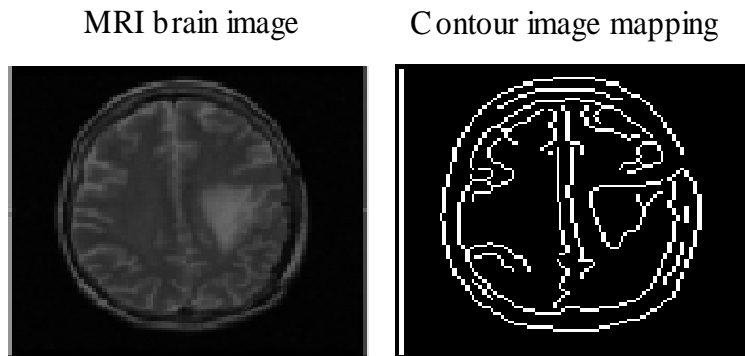
Step3: Use the Laplacian matrix  $L = D - W$  and  $(D - W)y = \lambda Dy$  to calculate the eigenvalues and eigenvectors.

Step 4: Repeat bipartition recursively. Stop if Ncut value is larger than a prespecified value.

#### 4. Experiments

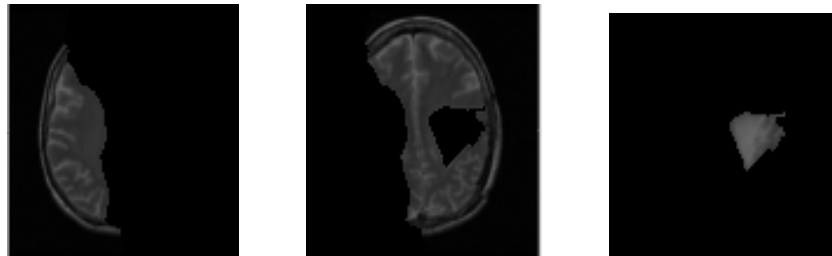
MRI images are acquired on a 1.5T GE (General Electric Co.) machine using an axial T1-weighted sequence with a voxel size of  $1 \times 1 \times 1.2 \text{ mm}^3$ . Figure 3 shows one slice of the MRI brain data and the result of contour image segmentation by using Canny edge detection method. This result shows that it is difficultly to identify the localization

and quantify consequently the size of the tumor. Therefore a graph cut based method is proposed in this work.



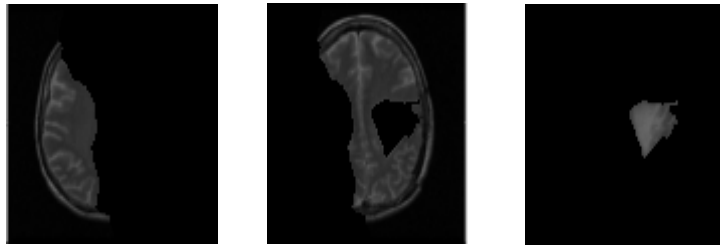
**Figure 3:** Original MRI brain image (left) and contour detection with Canny operator (right).

We firstly study the influence of the parameters used in the algorithm. Experiment results are realized by using different coefficients values of the graph cut to the impact of them. There are five coefficients to be adjusted in the segmentation process (see EQ .6). The choice of them is very important. In figure 4-a, it shows three partitions derived from the proposed method with a set of following parameters  $SI = 4$ ;  $SX = 4$ ;  $r = 3$ ;  $sNcut = 0.03$ ;  $sArea = 100$ . The two thresholding values  $sNcut$  for  $Ncut$  and  $sArea$  for the size of region are used to stop the graph cut process. The tumor location is presented in last partition.



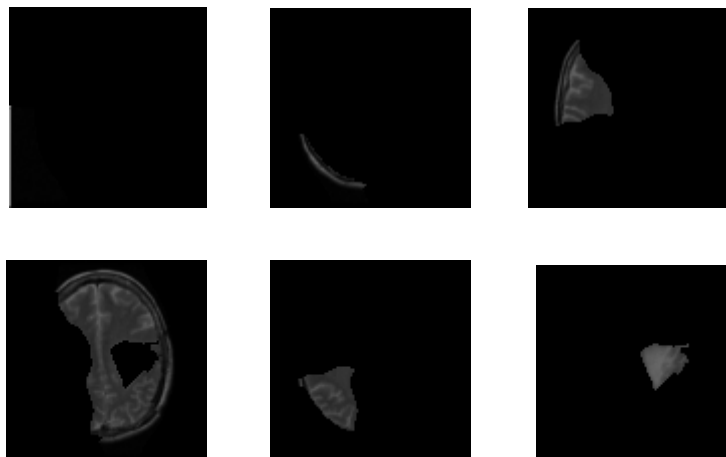
**Figure 4-a:** Graph cut image segmentation and tumor location with  $SI = 4$ ;  $SX = 4$ ;  $r = 3$ ;  $sNcut = 0.03$ ;  $sArea = 100$ .

In figure 4-b, we have modified coefficient  $SX$  to be greater than the initial value, we observe the same partition number as the first results. The last partition contains also tumor location.



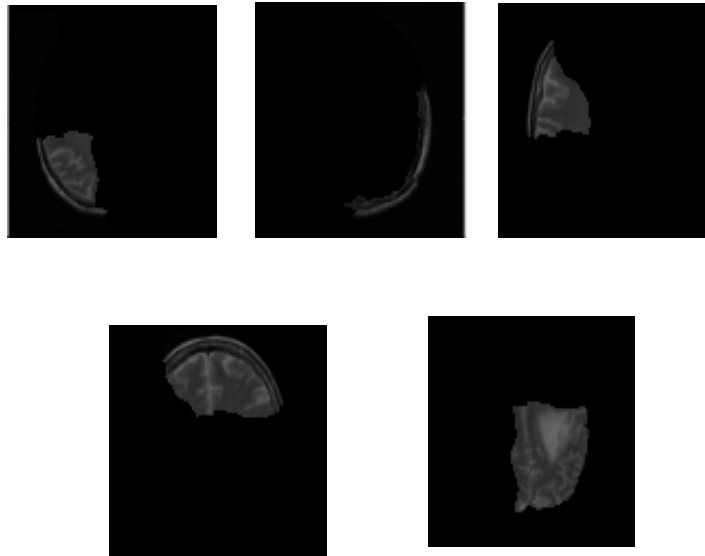
**Figure 4-b:** Graph cut image segmentation and tumor location with  $SI = 4$ ;  
 $SX = 10$ ;  $r = 3$ ;  $sNcut = 0.03$ ;  $sArea = 100$ .

In figure 4-c, we have modified the threshold  $sNcut$ . The segmentation results give six partitions. The global result is changed. However the tumor is segmented and localized in sixth partition.



**Figure 4-c:** Graph cut image segmentation and tumor location with  $SI = 4$ ;  
 $SX = 4$ ;  $r = 3$ ;  $sNcut = 0.07$ ;  $sArea = 100$ .

In figure 4-d, we modified the  $sArea$  value to get greater than the initial value in order to increase likelihood regions. Segmentation cannot unfortunately give the tumor location. Therefore, this threshold is sensitive to the final results.



**Figure 4-d:** Graph cut image segmentation and no tumor location with  $SI = 4$ ;  $SX = 4$ ;  $r = 3$ ;  $sNcut = 0.03$ ;  $sArea = 200$ .

## 5. Conclusion

In this paper, we propose brain tumor segmentation method based on the graph cut. It measures the similarity within the regions and dissimilarity between ones. We show a generalized eigenvalue system to solve our segmentation. Five coefficients in our algorithms are tested and studied to obtain a good segmentation in particular tumor location. This approach based graph cut seems well adapted to the problem of anatomic variability encountered. In the future work, we will apply the algorithm to a large data to valid it, and study a way to choose adaptively the parameters. Our next framework will be focus on the correlation between the desired segmentation and the eigenvectors of the affinity matrix. The obtained results must be equally validated by an expert radiologist in order to verify the efficiency of detection and tracking tumor imaging.

## References

- [1] Meila.M and Shi.J. Learning segmentation by random walk. NIPS, 873-879, 2000.
- [2] Benjamin.A. Spectral graph clustering. UPC, Jan 2007.



- [3] Jitendra.M and Shi.J. Normalized cuts and image segmentation, IEEE Trans. Pattern. Anal. Mach. Intell., vol 22(8) Aug 2000.
- [4] Mickael.P. Introduction aux graph cuts en vision par ordinateur, Aug 2006.
- [5] Ulrike von.L. A tutorial on spectral clustering. MPIBC, Aug 2006.
- [6] Chung F.R.K. Spectral graph theory. Providence, RI, 1997.
- [7] Meila, M and Shi, J. A random walks view of spectral segmentation. *AISTATS 2001*.
- [8] Ng, A. Y, Jordan,M., and Weiss, On spectral clustering: analysis and an algorithm. *T. Dietterich, S. Becker, and Z. Ghahramani Advances in Neural Information Processing Systems*, 14, 2001.
- [9] Wikipedia. Graph (mathematics) — wikipedia, the free encyclopedia. ([Online; accessed 9-January-2007]).
- [10] Boykov and Jolly. Interactive graph cuts for optimal boundary and region segmentation of objects in n-d images. 2001.
- [11] Leo Grady and Gareth Funka-Lea. Multi-label image segmentation for medical applications based on graph-theoretic electrical potentials. *ECCV 2004 Workshops CVAMIA and MMBIA*, number LNCS3117 in *Lecture Notes in Computer Science*, pages 230–245, Prague, Czech Republic, May 2004. Springer.
- [12] Mickael Péchaud, Introduction aux GraphCuts en Vision par Ordinateur, 30 août 2006.
- [13] Pitiot, A., Toga, A.W., Thompson, P.M. Adaptive Elastic Segmentation of Brain MRI via Shape Model Guided Evolutionary Programming. *IEEE Trans. on Med. Imag.* 21, pp. 910–923, 2002.
- [14] Styner, M., Brechb`uhler, C., Sz`ekely,G., Gerig,G. Parametric estimate of intensity inhomogeneities applied to MRI. *IEEE Trans. On Med. Imag.* 19, pp. 153–165, 2000.
- [15] Leemput, K.V., Maes, F., Vandermeulen, D., Suetens, P. A Unifying Framework for Partial Volume Segmentation of Brain MR Images. *IEEE Trans. On Med. Imag.* 22, pp. 105–119, 2003.
- [16] Ruan, S., Moretti, B., Fadili, J., Bloyet, D. Fuzzy Markovian Segmentation in Application of Magnetic Resonance Images. *Computer Vision and Image Understanding*. 85, pp.54–69, 2002.
- [17] Warfield, S. K., Kaus, M., Jolesz, F.A., Kikinis, R. Adaptive, Template Moderated, Spatially Varying Statistical Classification. *Medical Image Analysis*. 4(1), pp.43–55, 2000.
- [18] Su Ruan, Stéphane Lebonvallet, Abderrahim Merabet, Jean-Marc Constans, Tumor segmentation from A Multispectral MRI images by using support vector machine classification, *IEEE-ISBI2007*, Washington USA, April, 2007.
- [19] Ho, S., Bullitt, E., and Gerig, G. Level set evolution with region competition: Automatic 3- D segmentation of brain tumors. *International*

1063 IJ-STA, Volume 3, N°2, December, 2009.

Conference on Pattern Recognition, **1**, pp.532–535, 2002.

- [20] Stephane Lebonvallet, Sonia Khatchadourian, and Su Ruan, Automated tumor segmentation using level set method, International Conference on Computer Vision Theory and Applications, Barcelona Spain, March, 2007.
- [21] Mancas, M. and Gosselin, B. Iterative watersheds and fuzzy tumor visualization. Proceedings of the IEEE Visualization Conference, 2003.