

# Fast Algebraic State and Parametric Estimation in AC Machines

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**Abstract.** *An algebraic method for fast, on line estimation of unknown parameters and unmeasured observable states, using only inputs and outputs is applied to estimate the synchronous machine parameters and the asynchronous machine rotor flux. The entire purpose is to gather some input output information so that we can identify unknown parameters and states. The estimation process should not depend on any of the initial conditions and the real parameter values should be reached in a relatively short time interval in order to make possible its use in real time. Evaluation of the unmeasured rotor flux of an asynchronous machine is based on derivatives calculation and can be reliably achieved in a quite short amount of time.*

**Keywords.** *Parameter estimation, state estimation, derivatives calculation.*

## 1. Introduction

The theory of asymptotic observers for linear systems was started in the pioneering work of Kalman, Luenberger and many other important contributions. For an observable system, represented in state space, the state estimation problem is intimately related to the computation of time derivatives of the output signals, in a sufficient number [1].

The Parameter estimation method is mainly based on differential algebra and operational calculus [7], [9], [10].

This paper includes two main parts, the first part is devoted to the estimation of the permanent magnet synchronous machine (PMSM) parameters such as the direct and quadrature stator inductance and the stator resistance. The required state variables for the estimation process are the direct and quadrature stator currents, the angular

speed and the stator voltages which can be obtained from on-line measures or from numerical simulation results.

The second part is dedicated to use an algebraic method, of non-asymptotic nature, for the estimation of states using only first order time derivatives of the direct and quadrature currents.

Such method, based on derivatives calculus, is applied to estimate the unmeasured rotor flux of an asynchronous machine. The required state variables for the state estimation process are mainly the stator currents and voltages which can be obtained from on-line measures or from numerical simulation results.

Each of the two parts comprising this paper will include the state model representation of a given AC machine, the algebraic parametric or state estimation method to be applied and the corresponding simulation results and discussions.

## 2. Algebraic Parametric Estimation of PMSM's Parameters

### 2.1. State Model of the PMSM

A PMSM is basically an ordinary AC machine with windings distributed in the stator slots so that the flux created by the stator current is approximately sinusoidal. A PMSM can be thought as a synchronous machine with constant excitation current. The parameters to be estimated are the inductance components and the stator resistance. Of these parameters, the stator resistance is temperature dependent and thus could require on line estimation. Whereas, the inductances do not depend on the temperature and therefore off line estimation is sufficient.

Consider the estimation of the direct inductance  $L_d$ , the quadrature inductance  $L_q$  and stator resistance  $R_s$  from the permanent magnet synchronous machine described by the following differential system accordingly to Park's model [11]:

$$\begin{cases} v_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_r L_q i_q \\ v_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d + \omega_r \Phi_r \end{cases} \quad (1)$$

Where  $v_d, v_q$  are stator voltages,  $i_d, i_q$  are the direct and quadrature currents respectively.  $L_d$ , the direct inductance,  $L_q$ , the quadrature inductance;  $R_s$  is the stator resistance,  $\omega_r$  is the angular speed and  $\Phi_r$  is the rotor flux.

The parameters values used in the synchronous machine are  $R_s = 1.78 \Omega$ ,  $L_q = 0.0485 H$ ,  $L_d = 0.0342 H$ ,  $J = 1.7 kg.m$ ,  $\Phi_r = 0.9566 wb$ ,  $p = 10$ .

The state variables  $i_d$ ,  $i_q$  and  $\omega_r$  are obtained through classical resolution of differential system of the synchronous machine which operates under a conventional Field Oriented Control method for a speed reference equal to 140 rd/s and a direct stator reference current  $i_{dc} = 0$ .

Direct and quadrature currents  $i_d$ ,  $i_q$  are given in Figure 1. The angular speed  $\omega_r$  is shown in Figure 2.

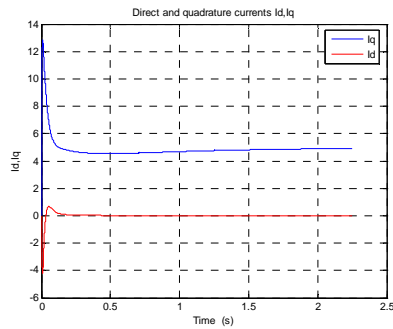


Fig. 1. Direct and quadrature currents

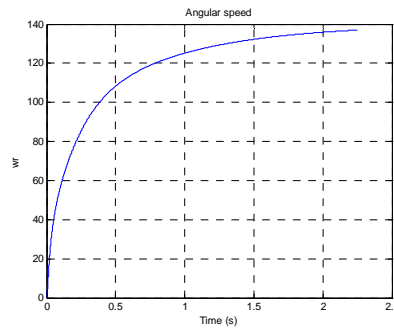


Fig. 2: Angular speed

### 2.2. Parametric Estimation Method

Aiming to find a good approximation of the parameters  $L_d$ ,  $L_q$  and  $R_s$  we have attempted to apply an algebraic method recently presented in [3][7][10]. Such method is based on differential algebra introduced in control theory and operational calculus which is the most classical tool among control and mechanical engineers.

Firstly the state variables are supposed to satisfy the linear time varying differential system (1).

Two attempts will be performed to estimate the PMSM parameters; the first case uses only the first equation of the algebraic differential system whereas the second case takes into account only the second equation of such system.

Starting from the first case we have:

$$\frac{di_d}{dt} = A_0 \cdot i_d + B_0 \cdot v_d + B_1 \omega_r i_q \tag{2}$$

Where

$$A_0 = \frac{-R_s}{L_d} = -52.0468; B_0 = \frac{1}{L_d} = 29.2398; B_1 = \frac{L_q}{L_d} = 1.4118$$

The equation (2) includes a nonlinear term  $\omega_r \dot{i}_q$ , therefore it is not possible to use operational calculus. It is necessary to linearize this equation or to have an 'equivalent' linear equation. We have introduced a new state variable obtained from the direct product of the angular speed and the quadrature current defined as:

$$x_q = \omega_r \dot{i}_q \quad (3)$$

Thus equation (2) is rewritten as:

$$\frac{di_d}{dt} = A_0 i_d + B_0 v_d + B_1 x_q \quad (4)$$

Translated into the operational domain it becomes:

$$sI_d - i_{d0} = A_0 I_d + B_0 V_d + B_1 X_q \quad (5)$$

's' being the Laplace variable.

$I_d$ ,  $I_q$  and  $V_d$  are the Laplace transforms of  $i_d$ ,  $i_q$  and  $v_d$  respectively.  $A_0$ ,  $B_0$  and  $B_1$  are the system parameters to be estimated and  $i_{d0}$  is the initial direct current value. The latter being unknown, we start by eliminating it. For that one should differentiate the members from the equation (5) with respect to  $s$ , we then obtain:

$$sI_d^{(1)} + I_d = A_0 I_d^{(1)} + B_0 V_d^{(1)} + B_1 X_q^{(1)} \quad (6)$$

In order to determine the unknown parameters  $A_0$ ,  $B_0$  and  $B_1$ , the equation (6) should be differentiated as many times as the unknown parameters number, in this case three times.

The resulting differential system is:

$$(sI_d)^{(i)} = A_0 I_d^{(i)} + B_0 V_d^{(i)} + B_1 X_q^{(i)}, \quad i = 1, 2, 3 \quad (7)$$

Where the exhibitor  $(i)$  indicates the derivation order wrt to  $s$ .

It is worth noting that the original equation corresponding to  $i = 0$ , does not form part of the system because it includes unknown initial conditions that should be avoid by differentiating once.

A multiplication by  $s$  means a derivation wrt  $t$ . Obviously a derivation is not a numerically robust operation, therefore one can divide the system of equations (7) by a factor  $s^\gamma$ , where  $\gamma$  is a constant integer higher than 2 in order to abolish all the derivative terms.

$$\frac{I_d^{(i)}}{s^{\gamma-1}} + i \frac{I_d^{(i-1)}}{s^\gamma} = A_0 \frac{I_d^{(i)}}{s^\gamma} + B_0 \frac{V_d^{(i)}}{s^\gamma} + B_1 \frac{X_q^{(i)}}{s^\gamma}, \quad i = 1, 2, 3 \quad (8)$$

Since the derivatives are eliminated the differential system includes only integral terms of the form:

$$I_{dij} = \left( \frac{I_d^{(i)}}{s^{\gamma-j}} \right), V_{dij} = \left( \frac{V_d^{(i)}}{s^{\gamma-j}} \right), X_{qij} = \left( \frac{x_q^{(i)}}{s^{\gamma-j}} \right); \quad i = 0, 1, 2, 3, \quad j = 0, 1.$$

Their corresponding quantities in time domain are  $I_{dij}$ ,  $V_{dij}$  and  $X_{qij}$  respectively and can be written as:

$$i_{dij}(t) = \frac{(-1)^i}{(\gamma - j - 1)!} \int_0^t (t - \tau)^{\gamma-j-1} \tau^i i_d(\tau) d\tau \quad (9)$$

$$x_{qij}(t) = \frac{(-1)^i}{(\gamma - j - 1)!} \int_0^t (t - \tau)^{\gamma-j-1} \tau^i x_q(\tau) d\tau \quad (10)$$

$$v_{dij}(t) = \frac{(-1)^i}{(\gamma - j - 1)!} \int_0^t (t - \tau)^{\gamma-j-1} \tau^i v_d(\tau) d\tau \quad (11)$$

Then, we have to solve the following algebraic system of three equations and three unknown estimated parameters  $\hat{A}_0, \hat{B}_0$  and  $\hat{B}_1$  :

$$\begin{cases} i_{d00} + i_{d11} = \hat{A}_0 i_{d10} + \hat{B}_0 v_{d10} + \hat{B}_1 x_{q10} \\ 2i_{d10} + i_{d21} = \hat{A}_0 i_{d20} + \hat{B}_0 v_{d20} + \hat{B}_1 x_{q20} \\ 3i_{d20} + i_{d31} = \hat{A}_0 i_{d30} + \hat{B}_0 v_{d30} + \hat{B}_1 x_{q30} \end{cases} \quad (12)$$

### 2.3. Simulation Results

The algebraic linear system (12) can be written as:

$$P \cdot \Theta = Q \quad (13)$$

Where

$$P = \begin{pmatrix} i_{d10} & v_{d10} & x_{q10} \\ i_{d20} & v_{d20} & x_{q20} \\ i_{d30} & v_{d30} & x_{q30} \end{pmatrix}; Q = \begin{pmatrix} i_{d00} + i_{d11} \\ 2i_{d10} + i_{d21} \\ 3i_{d20} + i_{d31} \end{pmatrix} \text{ and } \Theta = \begin{pmatrix} \hat{A}_0 \\ \hat{B}_0 \\ \hat{B}_1 \end{pmatrix}$$

The parameters  $\Theta$  are said to be linearly identifiable with respect to  $x$  if, and only if [5][6]:

- $P_{i,j}, Q_j \in \text{span}_{k_0(s)[\frac{d}{ds}]}(\mathbf{1}, y), i, j = 1, \dots, r$  and
- $\det(P) \neq 0$

$\text{span}_{k_0(s)[\frac{d}{ds}]}(\mathbf{1}, y)$  is the set of all linear combinations of  $(\mathbf{1}, y)$  where the

coefficients are differential operators with respect to  $s$  and  $y \in \{i_d, v_d, x_q\}$ .

If these conditions are satisfied, we obtained the estimates of system parameters by solving equation (13).

The numerical simulation results presented here are run with the obtained data of the direct and quadrature currents,  $i_d, i_q$  and the angular speed  $\omega_r$ .

The traces of the temporal evolution of the estimated parameters  $\hat{A}_0, \hat{B}_0$  and  $\hat{B}_1$ , which are linearly identifiables, are given in figure 3.a. Such parameters are closes to their real values. the PMSM's parameters  $L_d, L_q$  and  $R_s$  cannot be obtained directly from the algebraic system (13), thus its are said to be weakly linearly identifiables. These estimated parameters converge perfectly to their real values. according to figures 3b-c-d.

The estimation results are recapitulated in table 1.

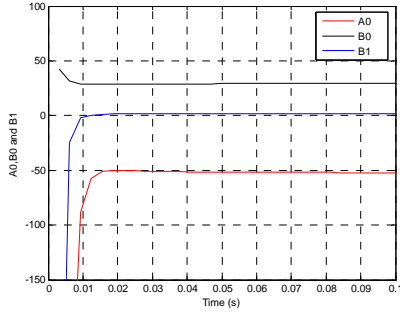
**Table 1: parametric estimation results:1<sup>st</sup> case**

	$A_0$	$B_0$	$B_1$	$L_d$	$L_q$	$R_s$
Real values	-52.0468	29.2398	1.4118	0.0342	0.0485	1.78
Estimated values	-52.103	29.1363	1.4131	0.0343	0.0484	1.787

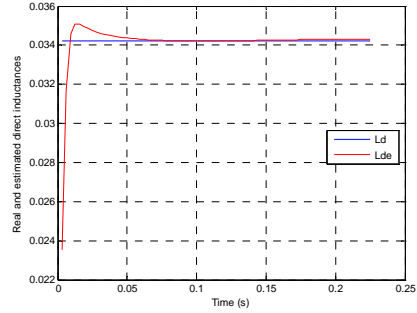
From figure 3.a, it is shown that at  $t = 20$  ms, the system parameters have already reached real values or very close to them.

Thus a short integration interval is seemingly enough to have a good parameter estimation so that the results can be used in real time. It is possible by means of such

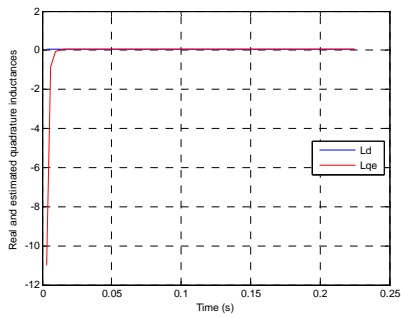
method to perform online parameter estimation that can be suitable for some control problems.



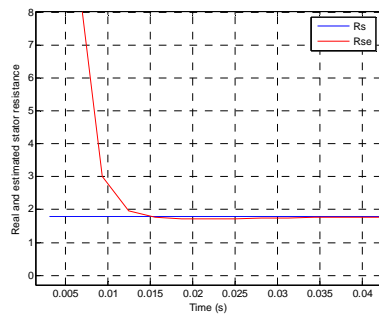
**Fig.3a.** Parameter estimation: 1<sup>st</sup> case



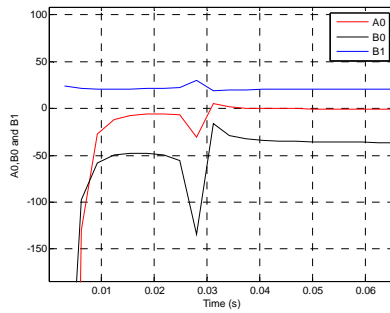
**Fig.3b.** Estimation of  $L_d$



**Fig.3c.** Estimation of  $L_q$



**Fig.3d.** Estimation of  $R_s$



**Fig. 4.** Parameter estimation: 2<sup>nd</sup> case

For different angular speeds, the estimation method gives fitting results, some numerical simulations were run for a low angular speed namely 20rd/s, the obtained real parameters values are likewise closes to their estimates ones.

The second equation of the differential system equations (1) is rewritten as:

$$\frac{di_q}{dt} = -\frac{L_d}{L_q}\omega_r i_d - \frac{R_s}{L_q}i_q + \frac{1}{L_q}(v_q - \omega_r \Phi_r) \quad (14)$$

Such equation includes a non linear term  $\omega_r i_d$ , assuming that  $x_d = \omega_r i_d$  and  $x_q = v_q - \omega_r \Phi_r$  are the new state variables to be introduced in equation (14)

which yields a linear differential equation in variables  $i_d, x_d, x_q$ :

$$\frac{di_q}{dt} = -\frac{L_d}{L_q}x_d - \frac{R_s}{L_q}i_q + \frac{1}{L_q}x_q \quad (15)$$

Generating a linear algebraic system of three equations devoid of any initial conditions and solving it as in the former case, the principal goal is to seek for a good approximation of the new linear identifiable parameters:

$$A_0 = \frac{-L_d}{L_q} = -0.7051; B_0 = \frac{-R_s}{L_q} = -36.701; B_1 = \frac{1}{L_q} = 20.6185$$

The estimated parameters converge to their real values as shown in figure 4 and reads as:  $\hat{A}_0 = -0.7102$ ;  $\hat{B}_0 = -37.0831$  and  $\hat{B}_1 = 20.9395$ .

Finally, the PMSM's parameters  $L_d$ ,  $L_q$  and  $R_s$ , which are weakly linearly identifiable, are deduced from  $\hat{A}_0$ ,  $\hat{B}_0$  and  $\hat{B}_1$ . The estimation results are summarized in table 2.

**Table 2: parametric estimation results: 2<sup>nd</sup> case**

	$A_0$	$B_0$	$B_1$	$L_d$	$L_q$	$R_s$
Real values	-0.7051	-36.701	20.6185	0.0342	0.0485	1.78
Estimated values	-0.7102	-37.0831	20.9395	0.0339	0.0477	1.768

### 3. Algebraic State Estimation in an Asynchronous Machine

#### 3.1. State Model of Asynchronous Machine

Mathematical models are of fundamental importance in understanding any physical system. In the fixed  $(\alpha, \beta)$  reference frame linked to the stator, by choosing the stator current components and rotor flux components, the Concordia model is used to describe the dynamics of state variables and angular speed controlled by stator voltage [11].



$$\frac{dx}{dt} = Ax + Bu + F(x) \tag{16}$$

Where  $x = (i_{s\alpha} \ i_{s\beta} \ \varphi_{r\alpha} \ \varphi_{r\beta})^T$  and  $u = [v_{s\alpha} \ v_{s\beta}]^T$

$$A = \begin{bmatrix} a_1 & 0 & a_3 & a_4 \\ 0 & a_1 & -a_4 & a_3 \\ a_5 & 0 & a_6 & a_7 \\ 0 & a_5 & -a_7 & a_6 \end{bmatrix}; \quad B = \frac{1}{\sigma l_s} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

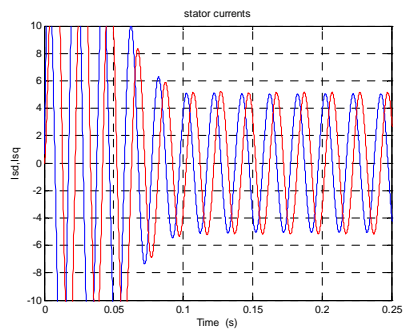
with

$$\begin{cases} a_1 = \frac{Rs + (\frac{M}{\sigma l_s})^2 Rr}{\sigma l_s}; \\ a_3 = \frac{M}{\sigma l_s \cdot l_r T r}; \quad a_4 = \frac{M \cdot \omega r}{\sigma l_s \cdot l_r}; \\ a_5 = \frac{M}{T r}; \quad a_6 = -\frac{1}{T r}; \quad a_7 = -\omega r \end{cases}$$

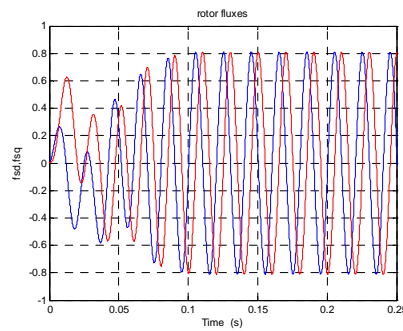
$i_{s\alpha}, i_{s\beta}$  being the stator currents;  $\varphi_{r\alpha}, \varphi_{r\beta}$ , the rotor flux components.

The computation of differential equation (16) leads to obtain the state variables waveforms in the temporal domain. For instance, for a starting asynchronous machine supplied via a sinusoidal voltage, the stator current components  $i_{s\alpha}, i_{s\beta}$  are shown in figure 5, and those of the rotor flux are given in figure 6. The angular speed trace is given in figure 7.

For the state estimation section, only stator currents  $i_{s\alpha}, i_{s\beta}$  and stator voltages  $v_{s\alpha}, v_{s\beta}$  will be used for the approximate rotor flux estimation whereas the obtained rotor flux will stand for reference signals to which will be compared their corresponding estimates.



**Fig. 5.** Stator currents waveforms



**Fig. 6.** Rotor flux waveforms

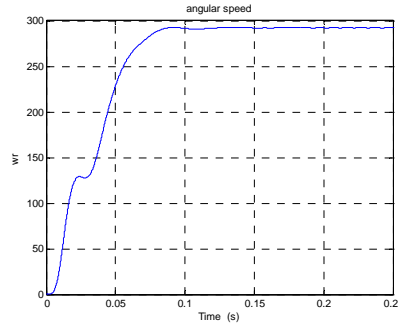


Fig. 7. Angular speed

### 3.2. State Estimation Method

The state estimation is mainly based on the output derivatives calculation. In former studies an important mathematic background is established in order to estimate derivatives of first or upper orders [1][2][4][8].

First of all, consider an unspecified signal  $x(t)$  having the following convergent Taylor expansion:

$$x(t) = \sum_{i \geq 0} c_i \frac{t^i}{i!}, \text{ with } t = 0, c_i \in \mathbb{C} \quad (17)$$

The truncated Taylor expansion given as:

$$x_N(t) = \sum_{i=0}^N c_i \frac{t^i}{i!}, \text{ N positive integer} \quad (18)$$

satisfies the differential equation:

$$\frac{d^{N+1}}{dt^{N+1}} x_N(t) = 0 \quad (19)$$

Translated in the operational field it yields:

$$s^{N+1} X_N(s) = s^N x_N(0) + s^{N-1} x_N^{(1)}(0) + \dots + x_N^{(N)}(0) \quad (20)$$

Where  $X_N(s)$  is the Laplace transform of  $x_N(t)$ .

The derivatives at initial time  $x_N^{(i)} \triangleq \frac{d^i}{dt^i} x_N(t)|_{t=0}$  are thus obtained starting from the following system of linear equations:

$$s^{-\nu} \frac{d^m}{ds^m} \{x_N(0)s^N + \dots + x_N^{(N-1)}(0)s + x_N^{(N)}(0)\} = s^{-\nu} \frac{d^m}{ds^m} \{s^{N+1} X_N\} \quad (21)$$

$m = 0, \dots, N, \nu \geq N + 1$ . This system being triangular with diagonal elements not no one, the parameters  $x_N^{(i)}(0), i = 0, \dots, N$ , therefore the coefficients  $a_0, \dots, a_N$  are linearly identifiable.

Aiming to solve (6) in time domain we replace the terms of equation system expressed in operational domain by their analogues in time domain. For instance,

$\frac{c}{s^k}, k \geq 1, c \in \mathbb{C}$  corresponds to  $c \frac{t^{k-1}}{(k-1)!}, t \geq 0$  in time domain and

$\frac{1}{s^k} \frac{d^n x}{ds^n}$  to the reiterated integral of order  $k$ .

$$\int_0^t \int_0^{t_{k-1}} \dots \int_0^{t_1} (-1)^n \tau^n x(\tau) dt_{k-1} \dots dt_1 d\tau = \frac{(-1)^n}{(k-1)!} \int_0^t (t-\tau)^{k-1} \tau^n x(\tau) d\tau \quad (22)$$

Using the linear system (21), the derivatives of order  $n=1, \dots, N$  a original time, can be computed simultaneously. Nevertheless, the matrix derived from such linear system of  $N+1$  equations is in general ill-conditioned, and yields therefore poor estimates.

Aiming to overcome this problem we opt to use an independent estimator for each order of derivation [8].

For instance, in order to determine the  $n^{\text{th}}$  order derivative one should annihilate the remaining coefficients  $x_N^{(j)}(0), j \neq n$  by multiplying (21) by a linear differential operators of the form:

$$\Pi_k^{N,n} = \frac{d^{n+k}}{ds^{n+k}} \cdot \frac{1}{s} \cdot \frac{d^{N-n}}{ds^{N-n}} \quad k \geq 0 \quad (23)$$

Such operator leads to the following estimator of  $x^{(n)}(0)$  :

$$\frac{x_N^{(n)}(0)}{s^{\nu+n+k+1}} = \frac{(-1)^{n+k}}{(n+k)!(N-n)!} \frac{1}{s^\nu} \Pi_k^{N,n} (s^{N+1} X) \quad (24)$$

$\nu$  has the form  $\nu = N + 1 + \mu, \mu \geq 0$

In this paper we need only first order derivatives so we have the following calculation formulae:

Taking  $N = 1$ , so  $\nu = \mu + 2$  we have in operational domain :

$$\frac{1}{s^{\mu+k+4}} x_0^{(1)}(k, \mu) = \frac{(-1)^{k+1}}{(k+1)!} \left( \frac{x^{(k+1)}}{s^{\mu+1}} + (k+1) \frac{x^{(k)}}{s^{\mu+2}} \right) \quad (25)$$

Translated in time domain we have:

$$x_0^{(1)}(k, \mu) = \frac{\mu+2}{T} \binom{\mu+k+3}{k+1} \int_0^1 p(\tau) \tau^k (1-\tau)^\mu x(T\tau) d\tau \quad (26)$$

Where :

$p(\tau) = (\mu+k+2)\tau - (k+1)$ , and  $T$  is the estimation time.

### 3.3. Rotor Flux Estimation of an Asynchronous Machine

From differential system (16) we focus our interest only on the two first equations which include the first derivatives of the stator currents components  $i_{s\alpha}$ ,  $i_{s\beta}$  :

$$\begin{cases} \frac{di_{s\alpha}}{dt} = a_1 i_{s\alpha} + a_3 \varphi_{r\alpha} + a_4 \varphi_{r\beta} + b_1 v_{s\alpha} \\ \frac{di_{s\beta}}{dt} = a_1 i_{s\beta} - a_4 \varphi_{r\alpha} + a_3 \varphi_{r\beta} + b_1 v_{s\beta} \end{cases} \quad (27)$$

where  $b_1 = \frac{1}{\sigma \ell_s}$

The stator currents  $i_{s\alpha}$ ,  $i_{s\beta}$  and the stator voltages  $v_{s\alpha}$ ,  $v_{s\beta}$  are supposed to be known either from on line measures or from numerical simulation results.

The current derivatives are accordingly estimated at any time of interval estimation by means of equation (26). Therefore we obtain an algebraic linear system with two equations and two unknown variables such as the rotor flux components  $\varphi_{r\alpha}$ ,  $\varphi_{r\beta}$ .

$$\begin{cases} a_3 \varphi_{r\alpha} + a_4 \varphi_{r\beta} = \frac{di_{s\alpha}}{dt} - a_1 i_{s\alpha} - b_1 v_{s\alpha} \\ -a_4 \varphi_{r\alpha} + a_3 \varphi_{r\beta} = \frac{di_{s\beta}}{dt} - a_1 i_{s\beta} - b_1 v_{s\beta} \end{cases} \quad (28)$$

Thus, for  $a_3^2 + a_4^2 \neq 0$ , the system solutions are expressed as:

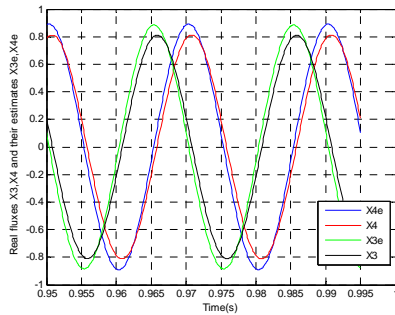
$$\begin{cases} \varphi_{r\alpha} = \frac{a_3 \frac{di_{s\alpha}}{dt} + a_4 \frac{di_{s\beta}}{dt} - a_3 a_1 i_{s\alpha} + a_4 a_1 i_{s\beta} - a_3 b_1 v_{s\alpha} + a_4 b_1 v_{s\beta}}{a_3^2 + a_4^2} \\ \varphi_{r\beta} = \frac{a_4 \frac{di_{s\alpha}}{dt} + a_3 \frac{di_{s\beta}}{dt} - a_4 a_1 i_{s\alpha} - a_3 a_1 i_{s\beta} - a_4 b_1 v_{s\alpha} - a_3 b_1 v_{s\beta}}{a_3^2 + a_4^2} \end{cases} \quad (29)$$

The used computer variables  $X3, X4, X3e$  and  $X4e$  correspond to real and estimated rotor flux  $\varphi_{rd}, \varphi_{rq}, \varphi_{rde}$  and  $\varphi_{rqe}$  respectively.

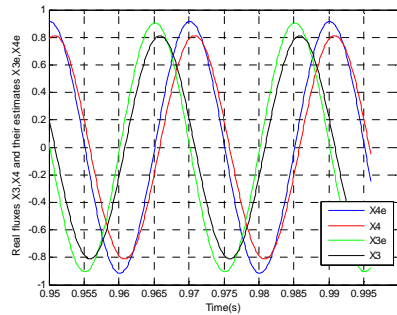
It is obvious to remark from equation (26), that the rotor flux estimation depends on the integers  $k$ , parametrizing the differential operator  $\Pi_k^{N,n}$  and  $\mu$  characterizing the derivative elimination stage.

For different values combinations of  $(k, \mu) \in \{0, 1, 2\} \times \{0, 1, 2\}$  the real rotor flux and their estimates are given in figures 8 a-p.

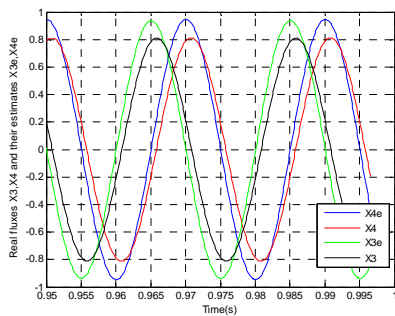
A state estimation method based on an independent estimator leads to acceptable results for certain couples of values of  $(k, \mu)$  namely for  $(k, \mu) = (1, 0), (1, 1)$  (see figures 8.d, 8.i), one has the least delay between the real and the estimated state variables .



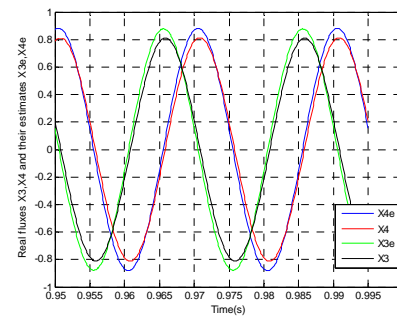
(a)  $k = 0 ; \mu = 0$



(b)  $k = 0 ; \mu = 1$

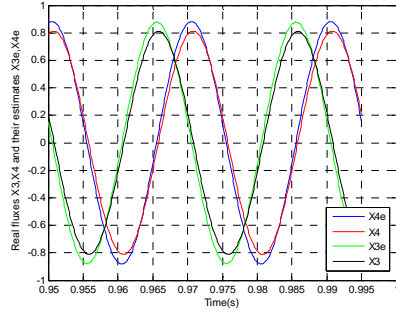


(c)  $k = 0 ; \mu = 2$

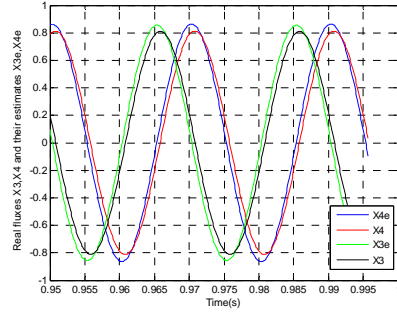


(d)  $k = 1 ; \mu = 0$

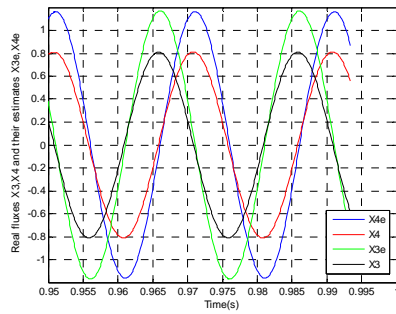
(continued)



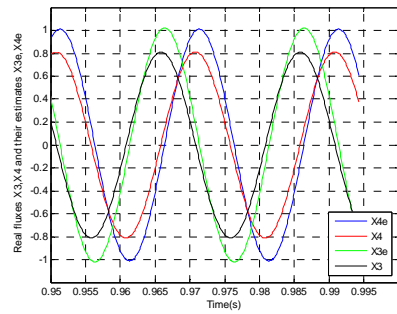
(e)  $k = 1 ; \mu = 1$



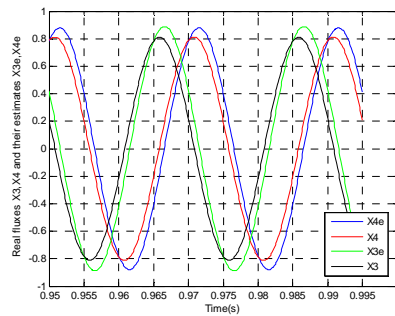
(f)  $k = 1 ; \mu = 2$



(g)  $k = 2 ; \mu = 0$



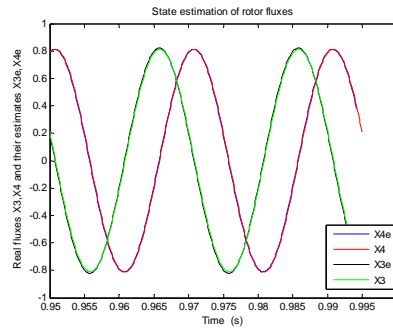
(h)  $k = 2 ; \mu = 1$



(i)  $k = 2 ; \mu = 2$

**Fig. 8.** State estimation of asynchronous machine rotor flux for different values combinations of  $(k, \mu) \in \{0, 1, 2\} \times \{0, 1, 2\}$

For these cases real and estimated rotor flux values are very close to each other. Using the state estimation method we have to choose adequately the values of  $(k, \mu)$  in order to obtain the unmeasured state variables estimate with satisfactory accuracy.



**Fig. 9.** Rotor flux estimation with anticipate adjustment

If the delay between the real state variable and its estimate as well as their amplitudes ratio are known with certainty, one could apply an anticipate correction and have the real signals and their estimates superimposed as in figure 9.

#### 4. Conclusion

The parameter estimation method has the advantage of being completely independent of the initial conditions and it only requires the measurements of the input and output variables. This estimation process is so fast that it allows on line implementation for certain control problems.

The technique of fast algebraic state estimation based on derivative calculation applied to evaluate the unmeasured rotor flux of an asynchronous machine permits to obtain satisfactory results. The estimation results lead to highlight the importance of the individual estimator and the fast convergence of the estimated variables to their original ones.

Such algebraic method for approximate state estimation in the control of a real life linear can be reliable to evaluate some unattainable variables through the knowledge of the accessible output variables.

The algebraic treatment of many problems in AC machines control as well as problems in other fields pave the way to set up computationally implementable schemes, or algorithms characterized by fast computations.

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