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A Mixed Linear Program for a Multi-Part Cyclic Hoist Scheduling Problem

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Abstract. This paper deals with a single-hoist cyclic scheduling problem in electroplating systems. A comprehensive mixed linear programming model, that can be used to find the optimal sequence minimising the cycle time for two jobs or maximizing the throughput of the production system, is proposed. To improve the throughput in the process, a refined model is developed by allowing the hoist to be stopped with charge. After describing the problem to solve, the basic model and the refined model, benchmarks and examples are given to illustrate the performances of proposed models.

Keywords: Multi-Product Cyclic Hoist Scheduling Problem; Electroplating System; Mixed Linear Programming Model

1 Introduction

This paper studies the 2-degree cyclic scheduling of two different part jobs, where during one cycle, two different jobs are introduced into the production line and two others are living it. This problem is widely encountered in scheduling problem, such as in manufacturing and pipeline architecture where a large production is required. This is particularly the case in electroplating facilities, where a hoist is programmed to perform a move sequence.

In this work we consider the Cyclic Hoist Scheduling Problem (CHSP), in which some set of activities are to be repeated an indefinite number of time. In this set of activities, the cycle time is minimised and processing time constraints as well as transport travelling time constraints are satisfied. This class of problem has proven to be an NP-complete problem [1].

Since the first model given by Philips and Unger [2] several researchers have been interested to the cyclic scheduling problem [4-5] and a large number of mathematical

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models [6-7-8-9-10-11-12-13-14] and heuristic algorithms [15] were developed. Nevertheless, few works have been interested to the multi-product case. Ptuskin in [16], for example, considers the problem that parts are processed in the same sequence, with various processing times and a sequence of different parts periodically is entering the system. The sequence is considered to be known, and the date of each part has to be computed. This problem is decomposed in several mono-product sub-problems and the solution corresponds to a common period.

Mateo in [17] develops a branch and bound procedure which builds the sequence of movements progressively. Each level of the search tree consists of adding one tank and thus, the stages to be done on it. A linear program is then solved at each node to check the consistency of the constraint system.

In this study, a mixed linear programming model is elaborated to solve single hoist scheduling problem with multi-product part jobs.

The paper is organized as follow. Section 2 provides a description of the single hoist scheduling problem. The single hoist cyclic scheduling problem with multiple products is modelled in section 3. Illustrative examples are presented in section 4. Conclusions are given in section 5.

2 Problem statements

The considered problem is a single hoist multi-product scheduling problem. It consists in finding the optimal move sequence of the hoist that minimize the cycle period. This sequence is containing two different part-jobs with the same tank route and individual window constraints. It must respect the following particularities:

- Two kinds of products are to be treated in equal quantity.
- Every product type must be treated through the same baths.
- Each processing time have to be included between a minimum and maximum durations.
- Each one tank must receive at most one carrier at a given time.
- Hoist can not move more than one carrier in the same time.
- Between two successive moves in the sequence, hoist must have enough time to travel empty.
- Stock is not authorized between two soaking operations.

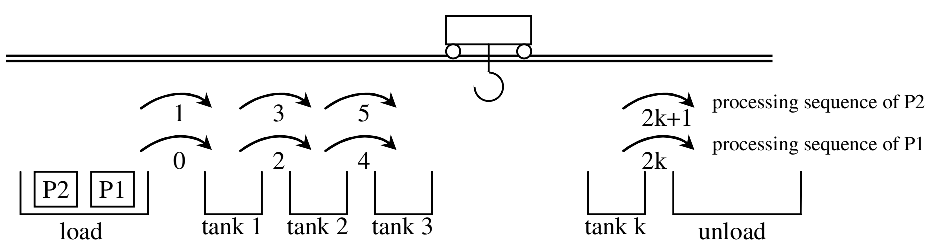

Fig. 1. Diagram of a hoist production line with k+2 tanks and 2k+2 stages

Fig. 1 shows an example of an electroplating line, with a single hoist, k+2 tanks, and 2k+2 stages used to process two different types of products P1 and P2, every cyclic period. At each cycle, there are two different jobs entering the line and two others are living.

The problem treated is characterised as follow.

- The line is composed of k+2 tanks where the first tank and the last one are respectively the load and the unload station.
- In every tank, there are two soaking operations to be done, one for every product type.
- Job of product 1 is transported by the hoist from the first stage (stage 0), into the following even stage (2, 4,…, 2k) and the second type of product is transported by the hoist from the first stage (stage 1), into the following odd ones (3, 5,…, 2k+1).
- The processing time of job j in tank k, is given by t_k for the first product if k is even and for the second product if k is odd; some time points are given in fig. 2.

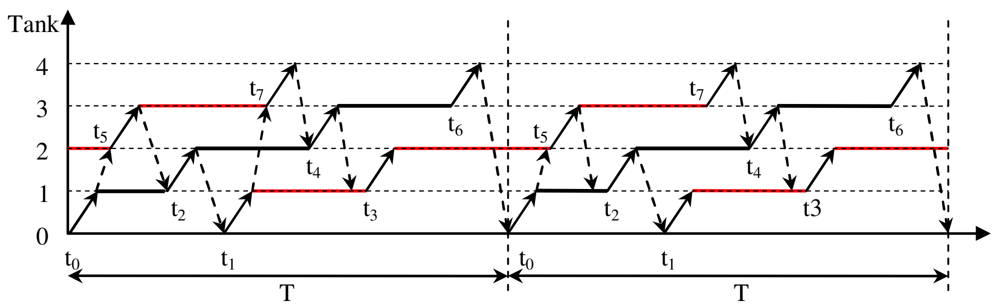

Fig. 2. Example of time way diagram corresponding to the considered problem

2.1 Basic model

Problem parameters

Let define the following notation.

n = the number of processing stages for the two products.
s_i = the tank used to achieve the process of stage i, i = 0, 1, 2, …, n+2.
a_i = the minimum processing time in stage i, i = 2, …, n.
b_i = the maximum processing time in stage i, i = 2, …, n.
d_i = the time needed for a hoist to move a carrier from tank i to tank i+1, i = 0,…, n+2.
$c_{i,j}$ = the time needed for one hoist to move empty from tank i to tank j, i, j = 0,...., n+2.
M = a very big number (represents the value $+\infty$).
H = the sequence of hoist movements: $H=(h_{[0]}, h_{[1]},\ldots,h_{[n]})$, with h_i is the hoist travelling from the stage i to the following stage i+2, for job P1 if i is even and for job P2 if i is odd.

These notations are used in Jiyin Liu, Yun Jiang and Zhili Zhou model [14].

Decision variables

T = the period of the cycle.

t_i = the starting time, during one cycle, of move from stage i to stage i+2: h_i, i = 0,.., n.

$$y_{i,j}= \begin{cases} 1 & \text{if } t_i < t_j \\ 0 & \text{otherwise} \end{cases} \qquad i,j = 1, 2, \ldots n, i < j.$$

In the model there is a total of n+2 continuous and n(n-1)/2 binary variables to find.

The model

Using the above notations, the model is formulated as the following linear equalities and inequalities:

$$\text{Minimize } T \tag{1}$$

Subject to:

$$d_{i-2} + a_i - M(1 - y_{i-2,i}) \le t_i - t_{i-2} \;. \qquad i = 2,\ldots,n\text{-}1, \tag{2}$$

$$t_i - t_{i-2} \le d_i + b_i + M(1 - y_{i-2,i}) \;. \qquad i = 2,\ldots,n\text{-}1, \tag{3}$$

$$d_{i-2} + a_i - My_{i-2,i} \le t_i + T - t_{i-2} \;. \qquad i = 2,\ldots,n\text{-}1, \tag{4}$$

$$t_i + T - t_{i-2} \le d_{i-2} + b_i + My_{i-2,i} \;. \qquad i = 2,\ldots,n\text{-}1, \tag{5}$$

$$t_j - t_i \ge d_i + c_{s_{i+2},s_i} - M(1 - y_{i,j}) \;. \qquad i, j = 1,\ldots,n\text{-}1, \quad i < j \tag{6}$$

$$t_i - t_j \ge d_j + c_{s_{j+2},s_i} - My_{i,j} \;. \qquad i, j = 1,\ldots,n\text{-}1, \quad i < j \tag{7}$$

$$t_i \ge d_0 + c_{s_2,s_i} \;. \qquad i = 2,\ldots,n\text{-}1, \tag{8}$$

$$T - t_i \ge d_i + c_{s_{i+2},s_0} \;. \qquad i = 2,\ldots,n\text{-}1, \tag{9}$$

$$y_{i,i+2} + y_{i+1,i+3} \ge 1 \;. \qquad i = 2,\ldots,n\text{-}4, \text{ if } s_i = s_{i+1}, \tag{10}$$

$$y_{i+2,i+1} + y_{i+3,i} \ge 3 - y_{i,i+2} - y_{i+1,i+3} \;. \qquad i = 2,\ldots,n\text{-}4, \text{ if } s_i = s_{i+1}, \tag{11}$$

$$1 - y_{i,j} \ge y_{j,i} \;. \qquad i, j = 1,\ldots,n\text{-}1, \quad i < j \tag{12}$$

$$y_{j,i} \ge 1 - y_{i,j} \;. \qquad i, j = 1,\ldots,n\text{-}1, \quad i < j \tag{13}$$

$$t_i \geq 0 \ . \qquad i = 1,...,n\text{-}1, \qquad \textbf{(14)}$$

$$y_{i,j} \in \{0,1\} \ . \qquad i, j = 1,...,n\text{-}1\ , \quad i < j \qquad \textbf{(15)}$$

$$T \geq 0 \ . \qquad \textbf{(16)}$$

In this model, the objective is to minimize the cycle time T (1). Constraints (14)-(16) are positive and binary constraints. Constraints (12)-(13) ensure that $y_{i,j}$ and $y_{j,i}$ are defined correctly (i.e., for i < j if $y_{j,i}$=1 then $y_{i,j}$= 0 (and *vice versa*)). Constraints (17)-(18) means that the first move from the loading tank to the first stage is equal to zero and match to the first move in the cycle. The other constraints can be classified into three classes. In fact, the first class constraints (constraints (2)-(5)) are related to tanks and ensure that the processing time at each stage respects a specific time window (maximum and minimum processing time). The second class of constraints (constraints (6)-(9)) is associated to hoist and guarantee that there is enough time to travel. The third class and the last one (constraints (10)-(11)), ensures the feasibility of the cyclic sequence, analysed below.

These last two constraints are added to the model to guarantee that there is no more than one carrier in each tank.
If two jobs are using the same tanks (k and k+1) during there process sequences, one of the following 3 cases, (a), (b) and (c) has to be considered.

Case (a): If the two processes are in the same cycle boundary

- The first carrier (containing the first type of product) enters tank k, when the product is treated it leaves this tank for the next stage and when this second stage is achieved it moves away and it will be the turn of the second carrier (containing the second type of product). Thus, the feasible sequence is (i, i+2, i+1, i+3).
- It is the same sequence than follow but the second carrier is treated firstly (the second product will be treated in priority) and then the feasible sequence is (i+1, i+3, i, i+2).

Case (b): if one of the processes (i+2 or i+3) crosses the cycle boundary

- In the stage i+2 the first carrier is moved into tank k+1, later in a cycle and it will be moved away early in the next cycle. And then the feasible sequence is defined by (i+2, i+1, i+3, i).
- Similarly, the second carrier is moved into tank k+1 for the stage i+3, later in a cycle and it will be moved away early in the next cycle. And then the feasible sequence is defined by (i+3, i, i+2, i+1).

Case (c): if the two processes i+2 and i+3 cross the cycle boundary

- It is impossible because two carriers will be in the same tank and then the cyclic sequence will be infeasible.

All this cases can be seen in fig. 3.

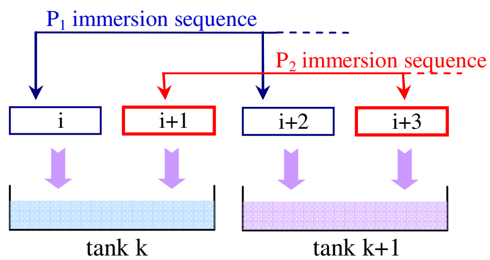

Fig. 3. Stages using the same consecutive tanks for different jobs

2.2 Refined model

In some manufactories, the hoist have a little impact in the production process, it is used for packaging and storing, but in this class of problem (HSP) the sequence how to move the hoist is important and can affect the productivity and therefore the performance of the company in term of throughput and quality. Thus, by analysing the hoist move in load, a new variable (w_i), defined as a slack time between the actual time taken and the minimum time required for the move i, is introduced. In other term, the hoist is allowed to be stopped with charge [14].

To take this variable into account, constraint (17) is added to the model and constraints (2)-(9) are substituted with the following ones (constraints 18-25):

$$w_i \geq 0 . \qquad i = 0,\ldots,n-1, \tag{17}$$

$$d_{i-2} + a_i - M(1 - y_{i-2,i}) \leq t_i - t_{i-2} - w_{i-2} . \qquad i = 2,\ldots,n-1, \tag{18}$$

$$t_i - t_{i-2} - w_{i-2} \leq d_i + b_i + M(1 - y_{i-2,i}) . \qquad i = 2,\ldots,n-1, \tag{19}$$

$$d_{i-2} + a_i - My_{i-2,i} \leq t_i + T - t_{i-2} - w_{i-2} . \qquad i = 2,\ldots,n-1, \tag{20}$$

$$t_i + T - t_{i-2} - w_{i-2} \leq d_{i-2} + b_i + My_{i-2,i} . \qquad i = 2,\ldots,n-1, \tag{21}$$

$$t_j - t_i \leq d_i + w_i + C_{s_{i+2},s_j} - M(1-y_{i,j}) . \qquad i, j = 1,\ldots,n-1, \quad i < j \tag{22}$$

$$t_i - t_j \leq d_j + w_j + C_{s_{j+2},s_i} - My_{i,j} . \qquad i, j = 1,\ldots,n-1, \quad i < j \tag{23}$$

$$t_i \leq d_0 + w_0 + C_{s_2,s_i} . \qquad i = 2,\ldots,n-1, \tag{24}$$

$$T - t_i \leq d_i + w_i + C_{s_{i+2},s_0} . \qquad i = 2,\ldots,n-1, \tag{25}$$

2.3 Complexity

The proposed integer linear model is function of the number of jobs and work stations. For two jobs, in the basic model, there are 2(n-2)(n+2)+(3n-8) processing station constraints and material handling constraints. Then, the total number of constraints is $O(n^2)$. Moreover, there are two types of decisions variables. There are a total of n+1 continuous, t_i and T and (n-1)(n-2)/2 binary variables $y_{i,j}$ to find.

By considering that hoist is allowed to be stopped with charge, the number of processing station constraints and the material handling constraints in the refined model is increased by n and thus the total number of constraints is still $O(n^2)$. And in this refined model there is a total of 2n+1 continuous, t_i, w_i and T and (n-1)(n-2)/2 binary variables $y_{i,j}$ to find.

3 Numerical examples

In the aim to illustrate the efficiency of the model and the refined model, two examples are defined and a comparison is also made between the models, the refined model and benchmarks examples found in literature.

The modal and the refined model are solved using commercial software, CPLEX, on a Pentium 4 with 3GHZ frequency processor.

3.1 Example 1

This example is similar to the one given by Mateo in [13]. It is used here to prove the efficiency of elaborated basic model. In this example, two products must be soaked in a line with 5 tanks including the loading and the unloading stations. The time spent soaking for every part product is given by table 1.
The time needed for the hoist to move empty between tanks of successive treatments is 10 time unit (t.u.). With load, the time needed for the hoist to move is 15 t.u.

Table 1. Data of example 1

Stage (i)	Job	Tank (s_i)	a_i	b_i	d_i
0	1	0	-	-	15
1	2	0	-	-	15
2	1	1	40	100	15
3	2	1	35	105	15
4	1	2	20	80	15
5	2	2	25	80	15
6	1	3	25	75	15
7	2	3	35	100	15

The optimal solution for example 1 is given in table 2.

Table 2. Results of example 1

T	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
280	0	165	60	245	120	25	200	85

The optimal cycle length is 280 t.u. as shown in figure 4 and it is equal to the cycle time find by [2].

Using the refined model, the same result is obtained.

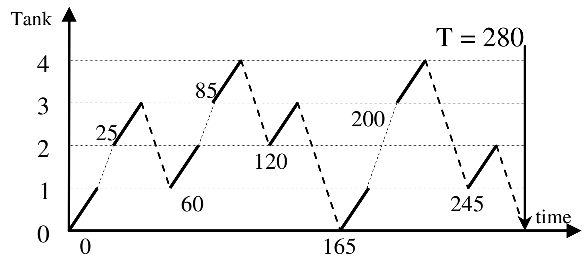

Fig. 4. Transportation time of example 1

Continuous lines show the loaded hoist travelling time between two successive stages, while discontinuous arcs show the unloaded hoist travelling time between different tanks.

Table 3 provides the minimum cycle time found in literature, obtained from the basic model and from the refined one for the cases of 5, 6, 7, 8, 9 and 10 tanks. These instances were used by Mateo in [17].

Table 3. Comparison with Mateo benchmarks

Problem number	tank number (k)	T_{min} *literature*	T_{min} *basic model*	T_{min} *refined model*
1	5	368	368	368
2	6	450	450	450
3	7	446	446	446
4	8	645	645	645
5	9	692	692	692
6	10	737	737	737

Where T_{min} *literature* is the best cycle time known in literature, T_{min} *basic model* is the best cycle time found by the basic model and T_{min} *refined model* is the best cycle time found by the refined model.

These results show that the best solution found in the literature *(T_{min} literature*) is reached by the use of the basic and the refined models and the obtained results have been confirmed by more than 150 benchmarks.

The same results are obtained by the refined model. This is due to the fact that, time required to the hoist to travel between two tanks is closed to the processing time and time windows are not enough large to provide flexibility to the model and thus to reduce the cycle time.

3.2 Example 2

This example is used here to prove the efficiency of the elaborated refined model. In this example, two products must be soaked in a line with 5 tanks including the loading and the unloading stations. The time spent soaking for every part product is given by table 4.
The time needed for the hoist to move empty between tanks of successive treatments is 5 t.u. With load, the time needed for the hoist to move is 10 t.u.

If the slack time is forced to be zero, by using the basic model, the two carriers enter to the line during a cycle and they are treated each in 308 t.u. as given by fig. 5. Thus, the mean period between two carriers is 154 t.u., never the less, with non-zero slack time, the optimal cycle time is compacted to 272 t.u. as shown on fig. 6, and the mean period is then reduced by 18 t.u.

Moreover, in term of throughput, during 500 000 t.u., in the first case, 1623 jobs for each product are obtained however, in the second case 1838 jobs for each product are obtained, it means that throughput is improved by 11,7 %.
Furthermore, the optimal sequence H_1 = (0, 7, 2, 4, 1, 6, 3, 5), obtained from table 5 and illustrated by figure 5, is different from the optimal sequence H_2 = (0, 5, 2, 7, 4, 1, 6, 3), obtained from table 6 and illustrated by fig. 6.
It can be seen from fig. 5, that sequence H_2 is not feasible without using the slack time. In fact, the fifth stage has to be achieved in the best at 86 t.u and in the worst at 91 t.u.
In the best, the second carrier attends the unload station at 96 t.u. and the first stage at 111 t.u., thus the first carrier exceeds the maximal required time. In the worst, the hoist moves the first carrier from the first stage to the second one and it attends the fifth stage at 110 t.u. and consequently, the second carrier exceeds the upper bound.

Table 4. Data of example 2

Stage (i)	Job	Tank (s_i)	a_i	b_i	d_i
0	1	0	-	-	10
1	2	0	-	-	10
2	1	1	80	96	10
3	2	1	76	91	10
4	1	2	41	46	10
5	2	2	21	26	10
6	1	3	61	66	10
7	2	3	61	66	10

The optimal solution for example 2 is given in tables 5 and 6.

Table 5. Results of example 2 using the basic model

T	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7
308	0	166	90	252	141	283	212	51

Table 6. Results of the example 2 using the refined model

T	t_0	t_1	t_2	t_3	t_4	t_5	T_6	t_7
272	0	166	90	252	141	15	212	105
	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7
	0	0	0	0	0	14	0	0

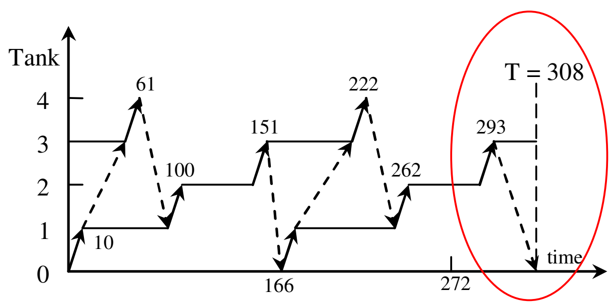

Fig. 5. Best hoist movements sequence for example 2 using the basic model

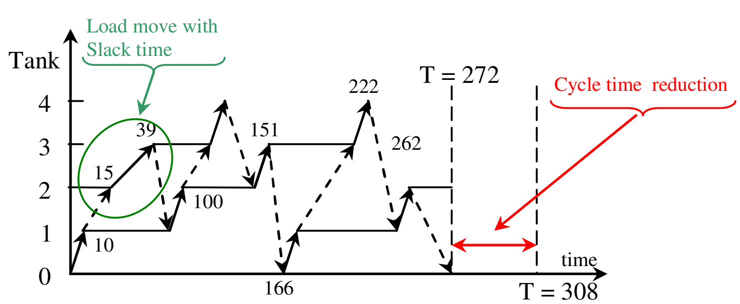

Fig. 6. Best hoist movements sequence for example 2 using the refined model

Using the elaborated, basic model, the time consumed by the hoist to carry out a complete sequence of movements of two jobs is 308 t.u. Nevertheless, by considering the refined model and for 14 t.u. (39-15-10) of slack time, the cycle time is reduced to 272 t.u.

4 Conclusions

In this paper, the single hoist 2-degree cyclic scheduling for different part jobs problems is studied and a mixed integer linear programming model is proposed to solve this problem. By considering the illustrative example 1 and using a set of more than 150 test problems, the obtained computational results have shown the efficiency of the elaborated basic model to find satisfying solutions for the considered problem.
Then, a refined model is elaborated with the assumption that the hoist is allowed to be stopped with charge.
This refined model is applied to the illustrative example 2 in order to show how, by considering the slack time, the cycle time can be reduced and production performance can be considerably improved.

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