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A robust unknown input multiobserver against disturbances for nonlinear systems: Experimental results on a batch esterification process

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***Abstract*—This paper deals with the design of robust unknown input multiobserver for nonlinear systems. A discrete uncoupled multiomdel is adopted as a suitable structure in nonlinear systems modeling. An unknown input multiobserver (UIMO) is synthesized based on this retained structure to achieve an accurate and a robust state estimation even in the presence of disturbances.To guarantee the multiobserver stability, sufficient conditions are reformulated in terms of linear matrix inequalities (LMIs) and equalities constraints with which the multiobserver gains are calculated. A real time application on an esterification reactor is proposed to prove the performance of the proposed robust multiobserver in terms of accuracy and robustness.**

***Index Terms*—Nonlinear system, uncoupled multimodel, robust unknown input multiobserver, disturbances, esterification chemical reactor.**

I. Introduction

Unknown inputs such as disturbances (which can be a measurement noise or a slowly varying signal) and uncertainties of modeling can affect the system dynamic behavior [1], [2]. In the fact, a state estimation degradation is considerably observed [3]. The robust and accurate state estimation of systems becomes an important research field to carry out robust control, robust fault diagnosis and fault tolerance. Many studies are proposed and different approaches are developed to design an observer that aims to estimate unmeasurable states [4]. To improve the observer design regardless the presence of unknown inputs, an unknown input observer (UIO) is considered as an efficient observer that reconstructs state variables with a good accuracy [5], [6]. Unknown input observers have been designed for linear and nonlinear systems and they have been exploited in monitoring and fault tolerance [7], [8]. In [9], authors have designed a robust unknown input observer to achieve a robust sensor fault detection and isolation for a linear time-invariant system and for a nonlinear CSTR. For a wind turbine system, actuator fault was compensated in [10] using a fault tolerant control scheme consisting of unknown input observer. An UIO is conceived for a wind turbine system using linear parameter varying model to estimate state and achieve robust fault detection in [11]. As an alternative to partial decoupling [12] or an attenuation [13] disturbances approaches, the proposed unknown input observer affords an exact decoupling. A perfect decoupling of disturbances (measurement noise or slowly varying signal) is the main purpose of the present work. The design of an efficient observer rely narrowly on the use of an accurate model. The previous mentioned works are proposed for nonlinear systems linearized only around operating points. Whereas, the obtained model seems not able to represent the global system behavior outside these zones. Indeed, the approximative assumptions respected in an operating regime can not be applicable in the complete operating space scared of losing the generality of the method.

To cope with this issue, an interesting approach named multimodel approach [14], [15] is proposed to handle nonlinear systems. Exploiting this powerful tool in modeling, it is easy to represent systems with an accurate simple form. With this approach, the nonlinear system is decomposed to a set of regimes corresponding to linear local models with simple structure. Several techniques are developed to determine the multimodel structure [14], [16], [17]. Two different multimodel structures are mentioned coupled and uncoupled structure. The uncoupled one is characterized by an independent partial models each having its own state vector. As a result, heterogeneous partial models are built. Thanks to its flexibility of modeling, the partial models structures are adapted to the complexity of the local models in each operating zone.

Various works such as [1], [5], [18] are interested in the design of unknown input observer for nonlinear systems where specific conditions are established for the existence of the proposed observer. Avoiding the loss of generality of observer synthesis, the uncoupled multimodel approach overcomes this problems. Inspired by the classical decoupling approach and motivated by the flexibility of the uncoupled multimodel structure via a simple use of linear models, new necessary conditions are made for the observer existence and consequently its easier design. In addition, observation problems for discrete uncoupled multimodel with unknown inputs affecting both state and output have received less attention that motivates this work. The main purpose of this paper interests in the robust unknown input multiobserver design for nonlinear systems. Firstly, based on an uncoupled state multimodel, an unknown input multiobserver is synthesized. Thereafter, the multiobserver can be easily designed and stability conditions are given in a set of LMIs with equalities constraints satisfactory. Thus, the designed multiobserver can estimate robustly and with a good accuracy states.

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The present paper is structured as follows. For the representation of nonlinear systems, an uncoupled multimodel structure is given in section II. In the following section, a robust unknown input multiobserver against disturbances (measurement noise or slowly varying signal) is designed to perform state estimation of the nonlinear system modeled by the retained uncoupled multimodel structure. A real experimentation on a batch esterification process is presented in section IV to improve the multiobserver performances in terms of robustness despite the presence of disturbances. A conclusion finishes this paper.

II. Discrete uncoupled state multimodel structure

It is well known that nonlinear systems are complex by nature. Their complexity make definitely modeling and controlling more difficult. In fact, several researches have been proposed and many methods and tools have been developed to surmount this problem. The multimodel approach is one of the most efficient techniques which has been used to represent faithfully nonlinear systems. Moreover, multimodel structure is functional for a large range of nonlinear systems. It represents a very interesting mathematical representation that describes the real nonlinear process as a set of local linear models connected via weighting functions. The main feature of the uncoupled multimodel is that partial models can have different dimensions. Unlike the classically used multimodel structures where partial models complexity is remained constant independently of the nonlinear system complexity, the retained uncoupled multimodel structure is well known by its flexibility of modeling and with which the structures of partial models are adapted to the complexity of the local model in each operating zone. The uncoupled structure is the greatest adapted technique for modeling systems that guarantees simplicity-precision compromise and decreases the number of parameters to be identified.

Let a discrete disturbed uncoupled state multimodel described by the following equation [15], [19], [20]:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u(k) + R_i w(k) \\ y_i(k) = C_i x_i(k) + F_i w(k) \\ y_{MM}(k) = \sum\limits_{i=1}^{N_m} \mu_i(\xi_{k-1}) y_i(k) \end{cases} \tag{1}$$

where N_m is the number of partial models, $x_i(k) \in \Re^{n_i}$ and $y_i(k) \in \Re^p$ represent respectively the state vector and the measured output vector of the i^{th} partial model.

$y_{MM}(k) \in \Re^p$ and $u(k) \in \Re^m$ are the multimodel output vector and the input vector. $w(k) \in \Re^d$ is a disturbance (measurement noise or slowly varying signal).

$A_i \in \Re^{n_i \times n_i}$, $B_i \in \Re^{n_i \times m}$, $C_i \in \Re^{p \times n_i}$, $R_i \in \Re^{n_i \times d}$ and $F_i \in \Re^{n_i \times p}$ are known and constant matrices.

$\mu_i(\xi_{k-1})$ are the activation functions which are associated with each operating zone and they ensure the transition between partial models. The activation functions fulfill the following constraints:

$$\begin{cases} \sum\limits_{i=1}^{N_m} \mu_i(\xi_{k-1}) = 1 \\ 0 \le \mu_i(\xi_{k-1}) \le 1 \end{cases} \quad \forall\, i = 1 \dots N_m \tag{2}$$

with ξ_{k-1} is the decision variable which can be chosen as the nonlinear system input.

We define an augmented state vector so as to rewrite the multimodel structure in a compact form:

$$x_{c_f}(k) = \begin{bmatrix} x_1^T(k) & \cdots & x_i^T(k) & \cdots & x_{N_m}^T(k) \end{bmatrix}^T \in \Re^n, \quad n = \sum_{i=1}^{N_m} n_i \tag{3}$$

The compact form of the uncoupled state multimodel is considered:

$$\begin{cases} x_{c_f}(k+1) = A_{c_f} x_{cf}(k) + B_{c_f} u(k) + R_{c_f} w(k) \\ y_{MM}(k) = C_{c_f}(k) x_{c_f}(k) + F_{c_f} w(k) \end{cases} \tag{4}$$

where

$$A_{cf} = \begin{bmatrix} A_1 & 0 & \cdots & & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & & A_i & & \\ & & & \ddots & 0 \\ 0 & \cdots & & 0 & A_{N_m} \end{bmatrix} \in \Re^{n \times n},$$

$$B_{c_f} = \begin{bmatrix} {B_1}^T & \cdots & {B_i}^T & \cdots & {B_{N_m}}^T \end{bmatrix}^T \in \Re^{n \times m},$$

$$R_{c_f} = \begin{bmatrix} {R_1}^T & \cdots & {R_i}^T & \cdots & {R_{N_m}}^T \end{bmatrix}^T \in \Re^{n \times d},$$

$$F_{c_f} = \sum_{i=1}^{N_m} \mu_i(\xi_{k-1}) F_i,\ C_{c_f}(k) = \sum_{i=1}^{N_m} \mu_i(\xi_{k-1}) \tilde{C}_{cf_i}$$

where $\tilde{C}_{c_{f_i}}$ is a constant block matrix defined by:

$$\tilde{C}_{c_{f_i}} = \begin{bmatrix} 0 & \cdots & C_i & \cdots & 0 \end{bmatrix} \in \Re^{p \times n}$$

This paper aims to design an unknown input multiobserver for the nonlinear system described by (4) as so to estimate robustly state variable with good accuracy.

III. Robust Uncoupled state MultiObserver design based on disturbed uncoupled Multimodel structure

An unknown input multiobserver is considered as the suitable solution to an accurate estimation for systems with unknown inputs. It is distinguished by its robustness, thanks to the supplementary decoupling principle, against disturbances (measurement noise or slowly varying signal).

The multimodel approach highlights a set of simple and linear local models, thanks to the convexity property of the weighting functions, affording a possible and an easier synthesis of observers extended from linear cases to the nonlinear ones whatever the complexity of the considered systems.

To estimate accurately state variables, the compact structure of the proposed robust uncoupled state multiobserver is described by the following equation:

$$\begin{cases} z_{c_f}(k+1) = N_{c_f} z_{cf}(k) + G_{c_f} u(k) + L_{c_f} y_{MM}(k) \\ \hat{x}_{c_f}(k) = z_{c_f} - E_{c_f} y_{MM}(k) \\ \hat{y}_{MM}(k) = C_{c_f}(k) \hat{x}_{c_f}(k) + F_{c_f} \hat{w}(k) \end{cases} \tag{5}$$

with $z_{c_f}(k) \in \Re^n$ and $\hat{x}_{c_f}(k) \in \Re^n$ are the state vector and the estimated state vector of the proposed multiobserver.

$\hat{y}_{MM}(k) \in \Re^p$ is the output vector reconstructed by the multiobserver.
$N_{c_f} \in \Re^{n\times n}$, $G_{c_f} \in \Re^{n\times m}$, $L_{c_f} \in \Re^{n\times p}$ and $E_{c_f} \in \Re^{n\times p}$ are known, appropriately dimensioned and will be determined later.

- *Convergence analysis*:

To study the multiobserver stability, let define the state estimation error:

$$e_x(k) = x_{c_f}(k) - \hat{x}_{c_f}(k) \tag{6}$$

It can be rewritten otherwise:

$$e_x(k) = P_{c_f}(k)x_{c_f}(k) - z_{c_f}(k) + E_{c_f}F_{c_f}w(k) \tag{7}$$

with

$$P_{c_f}(k) = \mathrm{I}_n + E_{c_f}C_{c_f}(k) \in \Re^{n\times n} \tag{8}$$

Refereing to (7), its dynamics is easily computed and is expressed as follows:

$$\begin{aligned} e_x(k+1) = P_{c_f}(k+1)x_{c_f}(k+1) - z_{c_f}(k+1) \\ +E_{c_f}F_{c_f}w(k+1) \end{aligned} \tag{9}$$

From (7), it can be deduced that:

$$\begin{aligned} -N_{c_f}z_{c_f}(k) = N_{c_f}e_x(k) - N_{c_f}P_{c_f}(k)x_{c_f}(k) \\ -N_{c_f}E_{c_f}F_{c_f}w(k) \end{aligned} \tag{10}$$

Substituting (4), (5) and (10) into (9) yields:

$$\begin{aligned} e_x(k+1) = N_{c_f}e_x(k) + \left[P_{c_f}(k+1)B_{c_f} - G_{c_f}\right]u(k) \\ + \left[P_{c_f}(k+1)A_{c_f} - N_{c_f} - K_{c_f}C_{c_f}(k)\right]x_{c_f}(k) \\ + \left[P_{c_f}(k+1)R_{c_f} - K_{c_f}F_{c_f}\right]w(k) \\ +E_{c_f}F_{c_f}w(k+1) \end{aligned} \tag{11}$$

where

$$K_{c_f} = L_{c_f} + N_{c_f}E_{c_f}; \quad K_{c_f} \in \Re^{n\times p}$$

The main purpose is that the estimated states accurately converge to the real one despite the presence of disturbances. In fact, sufficient conditions, by decoupling disturbances, are established for the existence of the proposed multiobserver:

$$\begin{cases} P_{c_f}(k+1)A_{c_f} - N_{c_f} - K_{c_f}C_{c_f}(k) = 0 \\ P_{c_f}(k+1)B_{c_f} - G_{c_f} = 0 \\ P_{c_f}(k+1)R_{c_f} - K_{c_f}F_{c_f} = 0 \\ E_{c_f}F_{c_f} = 0 \end{cases} \tag{12}$$

Satisfying (12), the dynamic of the state estimation error is reduced to:

$$e_x(k+1) = N_{c_f}e_x(k) \tag{13}$$

From (13), it can be seen that the disturbances (measurement noise or slowly varying signal) impact is suppressed thanks to the decoupling principle. The unknown input multiobserver estimates robustly state variables.
Then, the stability of the robust proposed multiobserver relies on the matrix N_{c_f}. Indeed, the state estimation error converges asymptotically towards zero, if N_{c_f} is a Hurwitz matrix. Afterwards, the multiobserver gains are determined:

$$\begin{cases} N_{c_f}(k+1) = P_{c_f}(k+1)A_{c_f} - K_{c_f}C_{c_f}(k) \\ G_{c_f}(k+1) = P_{c_f}(k+1)B_{c_f} \\ L_{c_f}(k+1) = K_{c_f} - N_{c_f}(k+1)E_{c_f} \end{cases} \tag{14}$$

Disturbances (measurement noise or slowly varying noise) can affect the dynamic behavior of systems and performs a degradation in state estimation. An exact decoupling method afforded by the robust uncoupled state multiobserver removes its effect. The decoupling approach allows to ensure a good compromise between dynamic performances to the state estimation error and accuracy of reconstruction even in presence of unknown inputs.
Thereafter, based on the Lyapunov approach, a particular study is addressed to provide exponential convergence conditions of the estimation error near zero. The unknown input multiobserver is then designed and sufficient conditions are given in terms of Linear Matrix Inequalities (LMIs) within equalities constraints.

Theorem 1. *The state estimation error tends exponentially to zero, if there exists S_{c_f} and W_{c_f} and a symmetric positive definite matrix $X = X^T > 0$ of appropriate dimensions such that the following LMIs are satisfied:*

$$\begin{bmatrix} (1-2\alpha)X & (XA_{c_f} + S_{c_f}\tilde{C}_{c_{f_i}}A_{cf} - W_{c_f}\tilde{C}_{c_{f_j}})^T \\ XA_{c_f} + S_{c_f}\tilde{C}_{c_{f_i}}A_{c_f} - W_{c_f}\tilde{C}_{c_{f_j}} & X \end{bmatrix} > 0$$
$$\forall \; i,j = 1...N_m \tag{15}$$

while the equalities constraints hold:

$$\begin{cases} XR_{c_f} + S_{c_f}\tilde{C}_{c_{f_i}}R_{c_f} = W_{c_f}F_{c_f}; \\ E_{c_f}F_{c_f} = 0; \end{cases} \quad \forall i = 1...N_m \tag{16}$$

α is the decay rate that serves to quantify the convergence speed of the estimation error. It must obey $0 < \alpha < 0.5$.
The unknown input multiobserver gains are given by:

$$\begin{cases} K_{c_f} = X^{-1}W_{c_f} \\ E_{c_f} = X^{-1}S_{c_f} \\ G_{c_f}(k+1) = \left(\mathrm{I}_n + E_{c_f}C_{c_f}(k+1)\right)B_{c_f} \\ N_{c_f}(k+1) = \left(\mathrm{I}_n + E_{c_f}C_{c_f}(k+1)\right)A_{c_f} - K_{c_f}C_{c_f}(k) \\ L_{c_f}(k+1) = K_{c_f} - N_{c_f}(k+1)E_{c_f} \end{cases} \tag{17}$$

Proof: The exponential stability of the multiobserver (5) designed for the disturbed uncoupled multimodel (4) is guaranteed if there exists a Lyapunov function $V(k) > 0$ and $\alpha > 0$ such that:

$$\begin{aligned} \exists X = X^T > 0;\ \alpha > 0;\ \Delta V(k) + 2\alpha V(k) < 0; \\ \Delta V(k) = V(k+1) - V(k) \end{aligned} \tag{18}$$

The candidate Lyapunov function is defined and given by:

$$V(k) = e_x(k)^T X e_x(k) \tag{19}$$

Substituting (19) and (13) into (18) yields:

$$\begin{aligned} \left[P_{c_f}(k+1)A_{c_f} - K_{c_f}C_{c_f}(k)\right]^T X \\ \left[P_{c_f}(k+1)\ A_{c_f} - K_{c_f}C_{c_f}(k)\right] + (2\alpha - 1)X < 0 \end{aligned} \tag{20}$$

Inequality (20), including (8) and thanks to the convex sum properties, can be rewritten in other way:

$$\begin{aligned} \left[A_{c_f} + E_{c_f}\tilde{C}_{c_{f_i}}A_{c_f} - K_{c_f}\tilde{C}_{c_{f_j}}\right]^T X \\ \left[A_{c_f} + E_{cf}\tilde{C}_{c_{f_i}}A_{c_f} - K_{c_f}\tilde{C}_{c_{f_j}}\right] + (2\alpha - 1)X < 0 \end{aligned} \tag{21}$$

Consider that:

$$W_{c_f} = XK_{c_f}, \qquad S_{c_f} = XE_{c_f} \tag{22}$$

Knowing that X is a symmetric matrix keeping the following property:

$$X^{-1}X = \mathrm{I} \tag{23}$$

Inequality (21) is rewritten:

$$\begin{array}{c}(1-2\alpha)\,X - \left[XA_{c_f} + S_{c_f}\tilde{C}_{c_{f_i}}A_{cf} - W_{c_f}\tilde{C}_{c_{f_j}}\right]^T X^{-1} \\ \left[XA_{c_f} + S_{c_f}\tilde{C}_{c_{f_i}}A_{cf} - W_{c_f}\tilde{C}_{c_{f_j}}\right] > 0; \\ \forall\ i,j = 1\ldots N_m\end{array} \tag{24}$$

Apply the Schur complement to (24), LMIs (15) are obtained. Consider equalities constraints:

$$\begin{cases} P_{c_f}(k+1)R_{c_f} - K_{c_f}F_{c_f} = 0 \\ E_{c_f}F_{c_f} = 0 \end{cases} \tag{25}$$

Constraints (25) are retained, taking into account (22), equalities constraints (16) are obtained.
LMIs (15) and equalities constraints (16) are efficiently solved based on a dedicated numerical tools, thereafter the proposed multiobserver is synthesized. ■

IV. Real time validation of the proposed robust multiobserver on an esterification process

In practice, nonlinear processes are usually subject to unavoidably disturbances (measurement noise or slowly varying signal). Hence, an experimental validation on chemical reactor is presented to prove the performance of the proposed multiobserver in robust state estimation against disturbances. This section is devoted to an experimental validation on a chemical olive oil esterification process producing an ester with high added value.

A. Batch chemical process description

A chemical process (figure 1) is used to esterify olive oil. The plant is consisting of a stirred tank equipped by a jacket where a heat exchange is provided between a cooling fluid and the reaction mixture. The fluid flow rate of the heating cooling reactor across the jacket is constant. An external servo system that implies a plate heat exchange with electric resistors, can regulate the fluid temperature within the jacket. Electric resistors are used to provide the fluid heating whereas its cooling which is also nominated tap of water is provided by a plate exchanger. Many temperature sensors aimed at measuring the chemical process temperatures and the inlet and the outlet jacked temperature are implemented for this reason. An esterification reaction is carried out into a chemical reactor used in batch mode to give an ester extensively used for cosmetic products' manufacture [21].
The esterification reaction between the crude olive oil namely containing free fatty acid and alcohol such as the 1-butanol takes place to extract an ester. This reaction is described by the following equation [22], [23]:

$$Acid + Alcohol \rightleftharpoons Ester + Water$$

The ester's yield may be increased when water is removed from the reaction by vaporisation. The esterification experiments characteristic temperature profile is the following: Firstly the reaction mixture is heated from the ambient temperature to a specific temperature corresponding to the esterification reaction temperature, thereafter the temperature is kept constant until the end of the reaction (the absence of water is deduced when it is no more water is dripping out of the condenser). Lastly, the temperature of the olive oil esterification reaction is decreased and the batch reactor is cooled back to the room temperature.

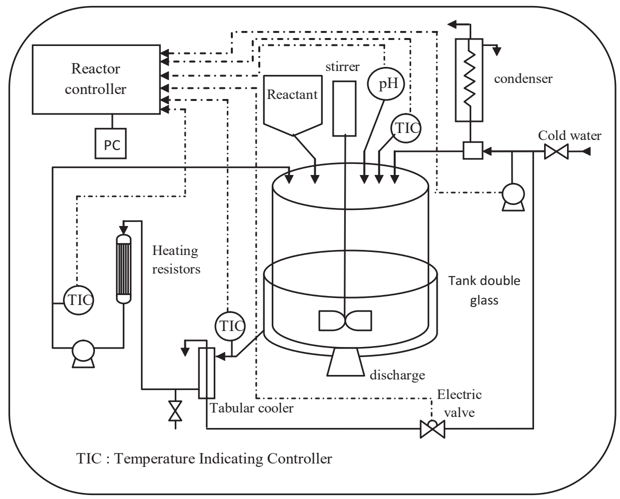

Fig. 1. Batch esterification reactor scheme.

Batch esterification process can be considered as a Single-Input Single-Output plant. The output is the chemical reactor temperature $T_R(°C)$ and the heating power $Q(W)$ is considered as an input.

B. Multimodel identification of the Batch chemical process

The oil olive esterification reactor is a complex process that has a nonlinear behavior. The nonlinear dynamic of the chemical reactor is complicated and requires an efficient model. Exploiting the interesting multimodel representation, the entire system behavior is decomposed into a set of local models with simple structures. The obtention of partial models can be performed by multiple methods [14], [16], [22], [24]. Indeed, a multimodel identification procedure gives an uncoupled multimodel representation of the chemical process containing three local models.

$$A_1 = \begin{bmatrix} 0.3368 & 0.4019 \\ 0.3503 & 0.6693 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.1072 \\ 0.1532 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0.0726 \\ 0.0984 \end{bmatrix},\ F_1 = 0.0775$$

$$A_2 = \begin{bmatrix} 0.0951 & 0.3785 \\ 0.3554 & 0.6997 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.0920 \\ 0.3931 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 0.0092 \\ 0.0083 \end{bmatrix},\ F_2 = 0.0031$$

$$A_3 = \begin{bmatrix} 0.2283 & 0.3673 \\ 0.3500 & 0.7278 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.1796 \\ 0.1062 \end{bmatrix}, C_3 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.0777 \\ 0.0328 \end{bmatrix},\ F_3 = 0.0904$$

The centers are $c_1 = 0.8$, $c_2 = 0.7$, $c_3 = 0.5$ and the dispersion is the following $\sigma = 0.07$.

C. Multimodel validation phase

The input signal which is the heating power $Q(W)$ as well as the reactor and the multimodel outputs are drawn in figure 2.

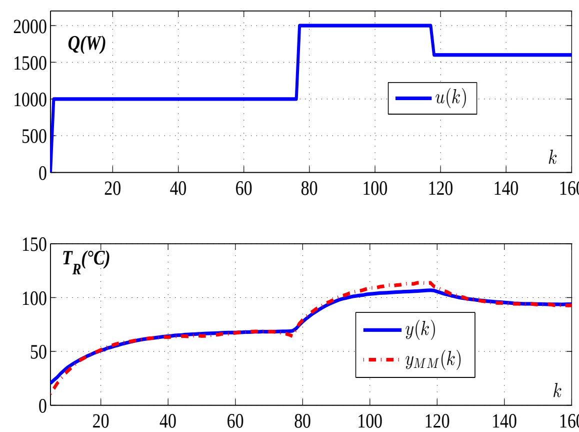

Fig. 2. Evolutions of input signal, the reactor and the multimodel outputs.

It can be noted that the multimodel output describes the real reactor behavior with acceptable precision.
To evaluate, again, the accuracy of the obtained multimodel, the mean square error (MSE) and the variance accounted for (VAF) two indices of performance have been calculated.
The mean square error is given as follows:

$$MSE = \frac{1}{\mathbb{N}_e} \sum_{k=1}^{\mathbb{N}_e} y_{MM}(k) - y(k)$$

$\mathbb{N}_e$ is the number of measures and $y(k)$ is the real system output.
The variance accounted for has the following expression:

$$VAF = \max \left\{ 1 - \frac{\text{var}\left\{ Y(k) - Y_{MM}(k) \right\}}{\text{var}\left\{ Y(k) \right\}}, 0 \right\} \times 100\%$$

where $Y_{MM}(k) = \left(\; y_{MM}(1), \; y_{MM}(2), \; \ldots, \; y_{MM}(\mathbb{N}_e) \; \right)$ and $Y(k) = \left(\; y(1), \; y(2), \; \ldots, \; y(\mathbb{N}_e) \; \right)$ and *var* stands for the signal variance.
In this case, VAF is equal to 97.93% however the MSE is 0.1468. From these indices, we can deduce a good adequation between reactor output and the multimodel one and show the good performance of the proposed representation.
The relative error $er(\%)$ between the reactor output $y(k)$ and the multimodel output $y_{MM}(k)$ is illustrated in figure 3. We note that the considered error does not exceed 10% over the full operating zones (heating, reaction and cooling).

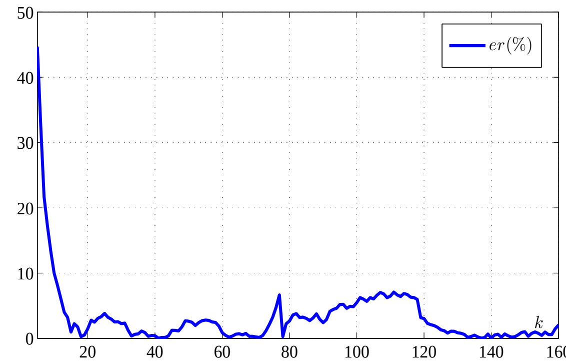

Fig. 3. The relative error evolution.

D. Experimental validation of the proposed robust multiobserver

The real and estimated states and their estimation errors are depicted in figures 4, 5, 6, 7, 8 and 9.

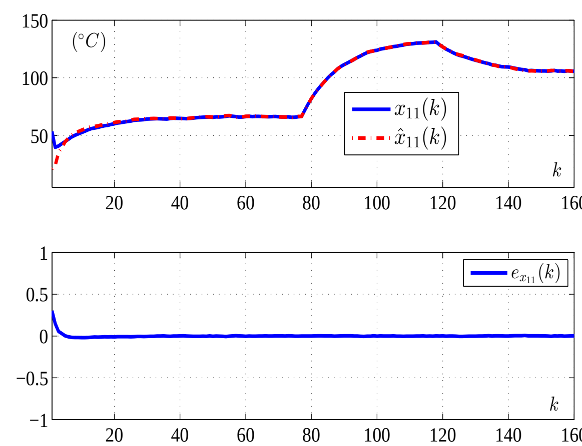

Fig. 4. First partial model: Evolution of state 1, its estimate and the estimation error.

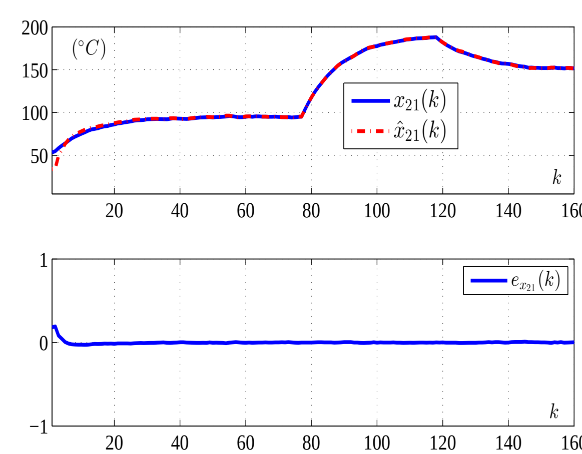

Fig. 5. First partial model: Evolution of state 2, its estimate and the estimation error.

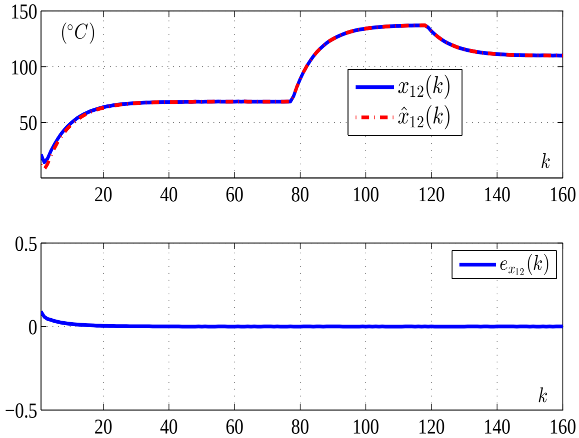

Fig. 6. Second partial model: Evolution of state 1, its estimate and the estimation error.

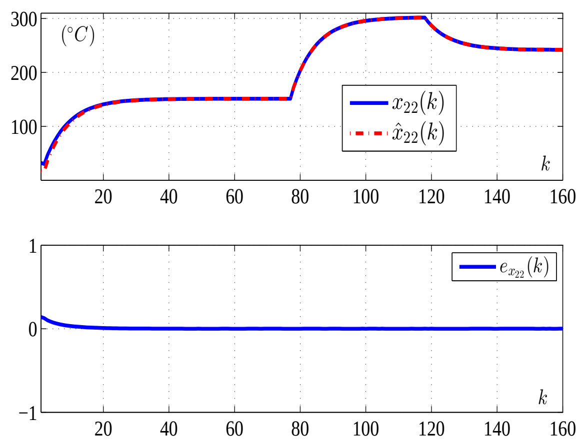

Fig. 7. Second partial model: Evolution of state 2, its estimate and the estimation error.

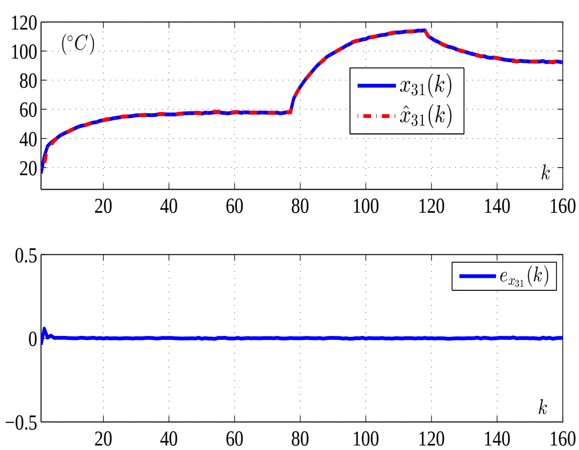

Fig. 8. Third partial model: Evolution of state 1, its estimate and the estimation error.

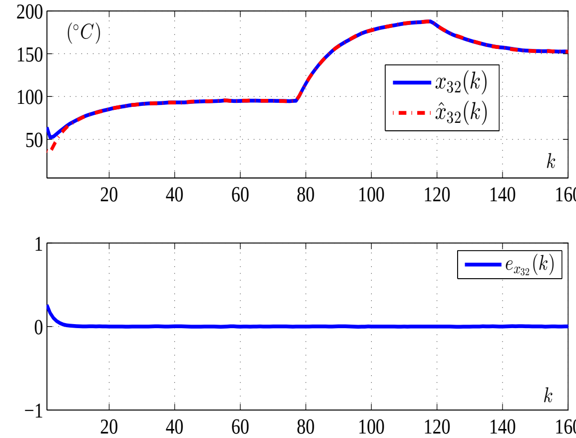

Fig. 9. Third partial model: Evolution of state 2, its estimate and the estimation error.

From the drawn figures, we can deduce that the estimated states converge to the real ones with a good accuracy. Thus, a rapid and accurate state estimation is achieved with a good satisfactory using the proposed robust unknown input multiobserver even despite changes of reaction.
The proposed robust uncoupled state multiobserver reconstructs the multimodel output with a good precision and which is depicted in figure 10.

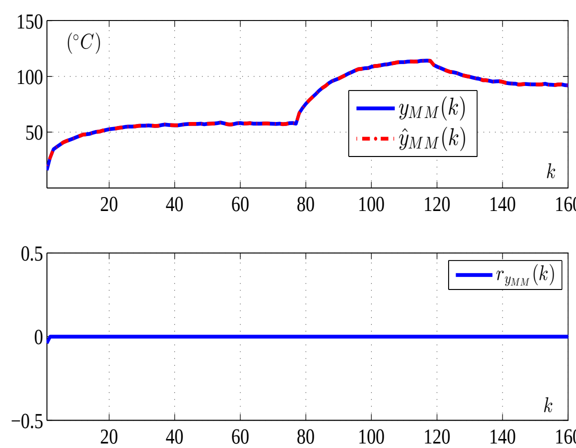

Fig. 10. Evolution of multimodel output, its estimate and the estimation error.

The estimated disturbance can be deduced and it is drawn in figure 11.

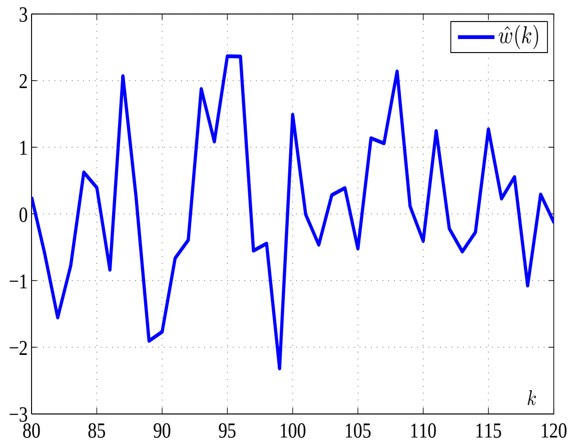

Fig. 11. Evolution of the estimated disturbance in the reaction phase.

V. Conclusion

An accurate and a robust state estimation of nonlinear processes modeled by a discrete uncoupled state multimodel is presented in the present framework. A robust unknown input multiobserver, according to an uncoupled state multimodel, is designed to reconstruct accurately state variables regardless of disturbances (measurement noise or slowly varying signal). The robust uncoupled state multiobserver stability conditions are formulated in a set of Linear Matrix Inequalities within equalities constraints. A real time experimentation on a batch chemical reactor is illustrated to highlight the good performance of the robust proposed multiobserver in terms of accuracy and robustness.

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