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H∞ Tracking observer-based control for T-S uncertain fuzzy models

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Abstract. *This paper deals with the tracking control for nonlinear systems described by uncertainty Takagi-Sugeno (T-S) fuzzy models. An H∞ robust observer based control is proposed for guaranteeing tracking performances of closed loop nonlinear systems. The design conditions obtained using Lyapunov approach are given in terms of solvability as a set of Linear Matrix Inequities (LMIs). To illustrate the effectiveness of the proposed H∞ tracking controller, a numerical simulation is given.*

Keywords *T-S model, uncertainties, tracking control, observer, Lyapunov method, LMI.*

1. Introduction

The tasks of stabilization and tracking are two typical control problems. In general, tracking problems are more difficult than stabilization problems. For nonlinear systems described by Takagi-Sugeno (TS) uncertain fuzzy models [10], the stability analysis and stabilization problems has been studied extensively by many researchers and many significant advances have been achieved. In [4] [8] [9] [11] and [12] stability sufficient conditions of closed loop systems are given when all state variables are available, whereas, observer based control of uncertain TS fuzzy models is studied [5][6][7][11][13]. The advantage of these results is that the stability analysis and controller and observer gains design can be converted into convex optimization problems in terms of LMIs which can be solved efficiently. On the other hand, tracking control designs are also important issues for practical applications, for example, in robotic tracking control, missile tracking control and attitude tracking control of aircraft [1][2][5][9]. However, there are few studies concerning with tracking control design based on the TS fuzzy model in presence of uncertainties and when all state variables are not available. In general, the resolution of this problem becomes very complexes and difficult to resolve. For example in [2], Tseng and Chen have pro-

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posed fuzzy control design method for T-S fuzzy systems without parametric uncertainties. So the robustness of the whole control tracking control system can not be guaranteed. In [14] the parametric uncertainties have been considered but the proposed results are conservatives. In this work, we propose the new stabilization conditions to reduce the conservatism results proposed in [14] for TS uncertain fuzzy systems with extern disturbances and when all the state variables are not available.

The paper is organized as follows. The first section, T-S fuzzy model with parametric uncertainties is employed to represent a nonlinear system, then an H∞ adaptive fuzzy observer-based tracking control scheme is introduced to reject all extern disturbances and to reduce tracking error as small as possible. Sufficient conditions of stability for T-S uncertain model in closed loop are proposed in this second section. An algorithm of linearization is then proposed to determine sequentially and in two steps synthesis variables.

Notation: a symmetric positive matrix is defined as $P > 0$. We define also $I_n = \{1, 2, .., n\}$ and $\sum_{i<j}^{n} x_i x_j = \sum_{i=1}^{n}\sum_{i<j}^{n} x_i x_j$. The symbol $*$ represents the transpose of symmetric matrix.

2. Problem Formulation

The T-S fuzzy model [3] has been proved to be a very good representation for some class of nonlinear dynamic systems. It's a piecewise interpolation of several linear models through membership functions (for more details see [3] and [4]).

The objective is to consider parametric uncertainties and extern disturbance in the system for modeling the behaviors of complex nonlinear dynamic systems.

The T-S uncertain model is represented as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{n} \mu_i(z(t))\big((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\big) + w \\ y(t) &= \sum_{i=1}^{n} \mu_i(z(t)) C_i x(t) \end{aligned} \tag{1}$$

where n is the number of local models, $x(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^q$ is the system output vector, $A_i \in \mathbb{R}^{p.p}, B_i \in \mathbb{R}^{p.m}, C_i \in \mathbb{R}^{q \cdot p}$ are system matrix, input matrix and output matrix, respectively, ΔA_i and ΔB_i are time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the plant model, $w \in \mathbb{R}^p$ denotes unknown but bounded disturbance.

$\mu_i(z(t))$ represents membership function of i[th] local model. Some basic properties are:

$$\begin{cases} \sum_{i=1}^{n} \mu_i(z(t)) = 1 \\ \mu_i(z(t)) \geq 0 \quad \forall\, i:1,..,n \end{cases} \tag{2}$$

$z(t) = \left[z_1(t), ..., z_n(t) \right]^T$ is the premise variable vector supposed measurable.

We assume that the uncertain matrices ΔA_i and ΔB_i are admissibly norm-bounded and structured.

$$\Delta A_i = D_i F_i(t) E_{1i}, \quad \Delta B_i = D_i F_i(t) E_{2i}, \quad F_i(t)^T F_i(t) < I \tag{3}$$

where D_i, E_{1i}, E_{2i} are known real constant matrices of appropriate dimension, and $F_i(t)$ is an unknown matrix function, I is the identity matrix of appropriate dimension.

Consider a reference model as follows:

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{4}$$

Where A_r is an asymptotically stable matrix, $r(t)$ is bounded reference input and x_r is the reference state which represents the desired trajectory for $x(t)$.

Define the tracking error as:

$$e_r(t) = x(t) - x_r(t) \tag{5}$$

The objective is to design a T-S fuzzy model-based controller, which stabilizes the fuzzy system (1) when disturbance is zero and achieves the H∞ performance related to tracking error as follows

$$\int_0^{t_f} \left(x(t) - x_r(t) \right)^T Q \left(x(t) - x_r(t) \right) dt \leq \rho^2 \int_0^{t_f} \overline{w}^T \overline{w} dt \tag{6}$$

with

$$\overline{w}(t) = \begin{bmatrix} w(t)^T & r(t)^T \end{bmatrix}^T \tag{7}$$

t_f : Terminal time of control.

Q : Positive definite weighting matrix.

ρ : prescribed attenuation level.

The equation (6) guarantees that the effect of any $\overline{w}(t)$ on tracking error must be attenuated below a desired level ρ .

2.1. TS Fuzzy observer

Several works consider that all the state variables are available. However in practice this assumption often does not hold. In this situation we need to estimate state vector x from output y for feedback control.

Consider the T-S observer as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{n} \mu_i(z(t))\left(A_i\hat{x}(t) + Bu(t) + L_i(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) &= \sum_{i=1}^{n} \mu_i(z(t))C_i\hat{x}(t)\end{aligned} \quad (8)$$

where $L_i \in \mathbb{R}^{p.q}$ is the constant observer gain to be determined.

Define observation error as

$$e(t) = x(t) - \hat{x}(t) \quad (9)$$

The aim is to determine an observer based control law for reducing as small as possible the difference between the desired state x_r and the state of the plant x , so we define the observer-based fuzzy controller in the form

$$u(t) = \sum_{i=1}^{n} \mu_i(z(t))K_i\left(\hat{x}(t) - x_r(t)\right) \quad (10)$$

where $K_i \in \mathbb{R}^{m.p}$ is the constant controller gain to be determined.

From (1), (8), (9) et (10), we obtain

$$\begin{aligned}\dot{x}(t) &= \sum_{i}^{n}\sum_{j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\begin{pmatrix}\left(A_i + B_i K_j\right)x(t) - B_i K_j e(t) - B_i K_j x_r(t) + \\ \left(\Delta A_i + \Delta B_i K_j\right)x(t) - \Delta B_i K_j e(t) - \Delta B_i K_j x_r(t)\end{pmatrix} + w \\ &= \sum_{i}^{n}\sum_{j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left(A_i + \Delta A_i\right)x(t) + \left(B_i + \Delta B_i\right)K_j\left(\hat{x}(t) - x_r(t)\right) + w\end{aligned} \tag{11}$$

Using (9), the expression (11) becomes

$$\dot{x}(t) = \sum_{i}^{n}\sum_{j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\begin{pmatrix}\left(A_i + B_i K_j + \Delta A_i + \Delta B_i K_j\right)x(t) - \\ \left(B_i K_j + \Delta B_i K_j\right)e(t) - \left(B_i K_j + \Delta B_i K_j\right)x_r(t)\end{pmatrix} + w \tag{12}$$

And the estimation error:

$$\dot{e}(t) = \sum_{i}^{n}\sum_{j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\begin{pmatrix}\left(A_i - L_i C_j - \Delta B_i K_j\right)e(t) + \\ \left(\Delta A_i + \Delta B_i K_j\right)x(t) - \Delta B_i K_j x_r(t)\end{pmatrix} + w \tag{13}$$

The augmented system composed of (1), (4) and (13), on using (10), can be expressed in the following form:

$$\dot{\bar{x}}(t) = \sum_{i}^{n}\sum_{j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left(A_{ij} + \Delta A_{ij}\right)\bar{x}(t) + E\bar{w}(t) \tag{14a}$$

with $\bar{w}(t)$ is given in (7) and

$$\bar{x}(t) = \begin{bmatrix} e(t)^T & x(t)^T & x_r(t)^T \end{bmatrix}^T \tag{14b}$$

$$A_{ij} = \begin{pmatrix} A_i - L_i C_j & 0 & 0 \\ -B_i K_j & A_i + B_i K_j & -B_i K_j \\ 0 & 0 & A_r \end{pmatrix} \tag{14c}$$

$$\Delta A_{ij} = \begin{pmatrix} -\Delta B_i K_j & \Delta A_i + \Delta B_i K_j & -\Delta B_i K_j \\ -\Delta B_i K_j & \Delta A_i + \Delta B_i K_j & -\Delta B_i K_j \\ 0 & 0 & 0 \end{pmatrix} \tag{14d}$$

$$E = \begin{pmatrix} I & 0 \\ I & 0 \\ 0 & I \end{pmatrix} \qquad (14e)$$

2.2. An H∞ observer-based tracking control

The main result on fuzzy observer-based tracking control for the T-S fuzzy system with norm-bonded uncertainties is summarized in the following theorem.

For demonstration, we use the following lemma.

Lemma 1: For any matrices X and Y with appropriate dimensions, the following property holds for any positive scalar ε

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

Theorem 1: if there exist symmetric and positives definite matrices $P > 0$ and $Q > 0$, R_{ii}, R_{ij}, some matrices K_i and L_i, a positive scalars $\varepsilon_{ij} > 0$ such that the following matrices inequalities are satisfied , then for a prescribed ρ^2 , H∞ tracking control performance in (6) is guaranteed via fuzzy observer-based controller (10)

$$\begin{pmatrix} S_{ii} & PE & \bar{E}_{ii}^T & P\bar{D}_i \\ E^T P & -\rho^2 I & 0 & 0 \\ \bar{E}_{ii} & 0 & -\varepsilon_{ii} I & 0 \\ \bar{D}_i^T P & 0 & 0 & -\varepsilon_{ii}^{-1} I \end{pmatrix}_{(i=1,\dots,n)} < 0 \qquad (15a)$$

$$\begin{pmatrix} S_{ij} & * & * & * & * & * \\ E^T P & -\frac{\rho^2}{2} I & * & * & * & * \\ \bar{E}_{ij} & 0 & -\varepsilon_{ij} I & * & * & * \\ \bar{E}_{ji} & 0 & 0 & -\varepsilon_{ji} I & * & * \\ \bar{D}_j^T P & 0 & 0 & 0 & -\varepsilon_{ij}^{-1} I & * \\ \bar{D}_j^T P & 0 & 0 & 0 & 0 & -\varepsilon_{ji}^{-1} I \end{pmatrix}_{(i<j)} < 0 \qquad (15b)$$

$$\begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{12}^T & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1n}^T & R_{2n}^T & \cdots & R_{nn} \end{pmatrix} < 0 \tag{15c}$$

where

$$S_{ii} = A_{ii}^T P + PA_{ii} + \bar{Q} + R_{ii} \tag{16a}$$

$$S_{ij} = A_{ij}^T P + PA_{ij} + A_{ji}^T P + PA_{ji} + 2\bar{Q} + R_{ij} + R_{ij}^T \tag{16b}$$

$$\bar{D}_i = \begin{pmatrix} D_i & 0 & 0 \\ 0 & D_i & 0 \\ 0 & 0 & D_i \end{pmatrix} \tag{17}$$

$$\bar{E}_{ij} = \begin{pmatrix} -E_{2i}K_j & E_{1i} + E_{2i}K_j & -E_{2i}K_j \\ -E_{2i}K_j & E_{1i} + E_{2i}K_j & -E_{2i}K_j \\ 0 & 0 & 0 \end{pmatrix} \tag{18}$$

$$\bar{Q} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Q & -Q \\ 0 & -Q & Q \end{pmatrix} \tag{19}$$

A_{ij}, E are defined in (14c)-(14e)

Proof: Consider the Lyapunov function candidate

$$V\left(\bar{x}(t)\right) = \bar{x}(t)^T P\bar{x}(t), \; P > 0 \tag{20}$$

The time derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(\bar{x}(t)) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\bar{x}(t)^T \begin{pmatrix} A_{ij}^T P + PA_{ij} + \\ \Delta A_{ij}^T P + P\Delta A_{ij} \end{pmatrix} \bar{x}(t) \\ + \bar{w}(t)^T E^T P\bar{x}(t) + \bar{x}(t)^T PE\bar{w}(t) \end{aligned} \tag{21}$$

which can be rewritten as follows

$$\dot{V}(\bar{x}(t)) = \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\bar{x}(t)^T\begin{pmatrix} A_{ij}^T P + PA_{ij} + \\ \Delta A_{ij}^T P + P\Delta A_{ij}\end{pmatrix}\bar{x}(t)$$
$$-\left(\frac{1}{\rho}E^T P\bar{x}(t) - \rho\bar{w}(t)\right)^T\left(\frac{1}{\rho}E^T P\bar{x}(t) - \rho\bar{w}(t)\right)$$
$$+\frac{1}{\rho^2}\bar{x}^T(t)E^T PPE\bar{x}(t) + \rho^2\bar{w}(t)^T\bar{w}(t) \quad (22)$$
$$\le \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\bar{x}(t)^T\begin{pmatrix} A_{ij}^T P + PA_{ij} + \\ \Delta A_{ij}^T P + P\Delta A_{ij}\end{pmatrix}\bar{x}(t)$$
$$+\frac{1}{\rho^2}\bar{x}^T(t)E^T PPE\bar{x}(t) + \rho^2\bar{w}(t)^T\bar{w}(t)$$

Applying Lemma 1 to $\bar{x}(t)^T\left(\Delta A_{ij}^T P + P\Delta A_{ij}\right)\bar{x}(t)$, we have

$$\dot{V}(\bar{x}(t)) \le \sum_{i=1}^{n}\sum_{j=1}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\bar{x}(t)^T\begin{pmatrix} A_{ij}^T P + PA_{ij} + \\ \varepsilon_{ij}^{-1}\bar{E}_{ij}^T\bar{E}_{ij}^T + \varepsilon_{ij}PD_i\bar{D}_i P + \\ \frac{1}{\rho^2}\bar{E}^T PP\bar{E}\end{pmatrix}\bar{x}(t) \quad (23)$$
$$+\rho^2\bar{w}(t)^T\bar{w}(t)$$

where $\bar{D}_i$ and $\bar{E}_{ij}$ are defined in (17) and (18).

Applying the classical decomposition of equation (23), direct terms ($i = j$) and indirect terms ($i \neq j$), and using the result in [4], we obtain from conditions (15-a, b, c)

$$\dot{V}(\bar{x}(t)) \le -\bar{x}(t)^T\bar{Q}\bar{x}(t) - \bar{x}(t)^T R_{ii}\bar{x}(t) + \rho^2\bar{w}(t)^T\bar{w}(t) \quad (24)$$

Taking account the structure of the matrices $\bar{Q}$ (19) and R_{ii} (15c), we obtain after integration of (24) between $t=0$ and $t=t_f$

$$V(t=tf) - V(t=0) \le -\int_0^{t_f} e_r(t)^T Qe_r(t)dt + \rho^2\int_0^{t_f}\bar{w}^T\bar{w}dt \quad (25)$$

which guarantees

$$\int_0^{t_f} e_r(t)^T Qe_r(t)dt \le \rho^2\int_0^{t_f}\bar{w}^T\bar{w}dt, \quad Q > 0 \quad (26)$$

where e_r *(t)* defined in (5)
So, H∞ control performance is achieved with a prescribed ρ^2. ■

The drawback of theorem 1 is that the design variables are nonlinear. To resolve this non convex problem, we propose in the next section a sequential algorithm.

2.3. Linearisation of the conditions of synthesis

Considering the Lyapunov matrix P as follows:

$$P = \begin{pmatrix} P_{11} & * & * \\ 0 & P_{22} & * \\ 0 & 0 & P_{33} \end{pmatrix}; \quad R_{ii} = \begin{pmatrix} q_{11}^{ii} & * & * \\ q_{21}^{ii} & q_{22}^{ii} & * \\ q_{31}^{ii} & q_{32}^{ii} & q_{33}^{ii} \end{pmatrix}; \quad R_{ij} = \begin{pmatrix} q_{11}^{ij} & q_{12}^{ij} & q_{13}^{ij} \\ q_{21}^{ij} & q_{22}^{ij} & q_{23}^{ij} \\ q_{31}^{ij} & q_{32}^{ij} & q_{33}^{ij} \end{pmatrix} \quad (27)$$

We can note that (15a) and (15b) imply

$$S_{ii} < 0 \quad (28a)$$
$$S_{ij} < 0 \quad (28b)$$

where S_{ii} and S_{ij} are defined in (16a), (16b)

Replacing (27a) and (27b) in (15a), (15b) and (15c), by introducing new variables $Z_i = P_{11}L_i$ and using Schur complement, (28a) and (28b) are equivalent at the following LMIs :

$$\Sigma_{ii} < 0 \quad (29a)$$
$$\Sigma_{ij} + \Sigma_{ji} < 0 \quad (29b)$$

where

$$\Sigma_{ij} = \begin{pmatrix} M_{11} & * & * \\ M_{21} & M_{22} & * \\ M_{21} & M_{32} & M_{33} \end{pmatrix} \quad (30)$$

and

$$M_{11} = A_i^T P_{11} + P_{11} A_i - C_j^T Z_i^T - Z_i C_j + q_{11}^{ij}$$
$$M_{21} = -P_{22} B_i K_j + q_{21}^{ij}$$
$$M_{22} = A_i^T P_{22} + K_j^T B_i^T P_{22} + P_{22} A_i + P_{22} B_i K_j + Q + q_{22}^{ij}$$
$$M_{31} = q_{31}^{ij}$$
$$M_{32} = -K_j^T B_i^T P_{22} - Q + q_{32}^{ij}$$
$$M_{33} = A_r^T P_{33} + P_{33} A_r + Q + q_{33}^{ij}$$

However, conditions (29) are always nonlinear on parameters $P_{11}, P_{22}, P_{33}, K_i$ and L_i. So, we can't use software packages such as LMI optimization toolbox in MATLAB. Thus to solve this problem we propose the following sequential algorithm in two steps:

i.) In the first step, we can note that (29) implies that

$$M_{22} < 0 \tag{31}$$

where M_{22} is defined in (30).

Using Schur complement, and with the following variable change

$$W_{22} = P_{22}^{-1},\ Y_j = K_j W_{22}$$

Conditions (29) becomes

$$\begin{pmatrix} W_{22} A_i^T + Y_i^T B_i^T + A_i W_{22} + B_i Y_i & W_{22} & W_{22} \\ W_{22} & -N & 0 \\ W_{22} & 0 & -\left(q_{22}^{ii}\right)^{-1} \end{pmatrix} < 0 \tag{32a}$$

$$\begin{pmatrix} \Phi_{ij} & W_{22} & W_{22} \\ W_{22} & -\frac{N}{2} & 0 \\ W_{22} & 0 & -\left(q_{22}^{ij} + \left(q_{22}^{ij}\right)^T\right)^{-1} \end{pmatrix} < 0 \tag{32b}$$

where

$$\Phi_{ij} = W_{22} A_i^T + Y_j^T B_i^T + A_i W_{22} + B_i Y_j + W_{22} A_j^T + Y_i^T B_j^T + A_j W_{22} + B_j Y_i \tag{33a}$$

$$N = Q^{-1} \tag{33b}$$

ii.) In the second step, we substitute P_{22}, Q and K_j in (15a) and (15b). From these inequalities we obtain the following *LMIs* on variables P_{11}, P_{33}, $Z_i = P_{11}L_i$ and $\alpha_{ij} = \varepsilon_{ij}^{-1}$.

Finally, the observer gains are obtained as follows:

$$L_i = P_{11}^{-1} Z_i \tag{34}$$

3. Simulation example

Consider a T-S uncertain model with two local models as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i \left(z(t)\right)\left((A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)\right) + w(t) \\ y(t) = Cx(t) \end{cases} \tag{35a}$$

where

$$A_1 = \begin{pmatrix} 0 & 1.5 \\ 15 & 0.5 \end{pmatrix}, A_2 = \begin{pmatrix} 0.8 & 1 \\ 9 & 3 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0 \\ -0.2 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ -0.01 \end{pmatrix} \tag{35b}$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \tag{35c}$$

and $z(t) = y(t)$. The parametric uncertainties as defined as:

$$D_1 = D_2 = \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix} \tag{35d}$$

$$E_{11} = E_{12} = \begin{pmatrix} 0.8 & 1 \\ 9 & 3 \end{pmatrix} \quad E_{21} = E_{22} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{35e}$$

Considering the reference model

$$A_r = \begin{pmatrix} 0 & 1 \\ -4 & -3 \end{pmatrix}, r(t) = \begin{pmatrix} 0 \\ \sin(t) \end{pmatrix} \tag{36}$$

The resolution of conditions (15-a), (15-b) given in theorem 1, and using the algorithm described in section (II-C), feedback and observer gain matrices can be obtained as

$$Q=\begin{pmatrix}503.11 & -38.36\\ -38.36 & 560.94\end{pmatrix} \qquad P_{11}=\begin{pmatrix}6275.8 & -303.0\\ -303.0 & 26.7\end{pmatrix} \tag{37a}$$

$$P_{22}=\begin{pmatrix}0.55 & 0.26\\ 0.26 & 0.13\end{pmatrix} \qquad P_{33}=\begin{pmatrix}689.17 & 626.27\\ 626.27 & 889.86\end{pmatrix} \tag{37b}$$

$$K_1=\begin{pmatrix}849.68 & 382.11\end{pmatrix}, K_2=\begin{pmatrix}2693.3\ 1260.6\end{pmatrix} \tag{38a}$$

$$L_1=\begin{pmatrix}92.7 & 1219.6\end{pmatrix}^T, L_2=\begin{pmatrix}77.46\ 877.02\end{pmatrix}^T \tag{38b}$$

Figure 1 illustrates that the error of estimation converge towards zero, despite of the presence of norm-bounded parametric uncertainties

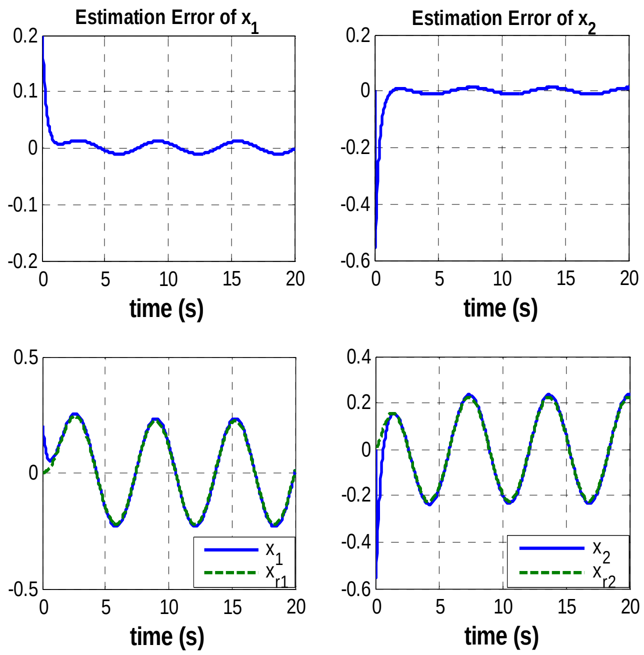

Fig 1 Errors of estimation of state plant and the trajectories of the state variables superposed with the reference state variables

Note that with the same model (35a), by using conditions proposed in [14], there is no feasibility for the problem, which proves the interest of the introduction of relaxations.

4. Conclusion

In this paper, we have developed a robust tracking observer-based control design for uncertain T-S fuzzy model. The proposed conditions are formulated in the LMIs terms and relaxed by the introduction of new variables. Sequential algorithms on two steps have been proposed to design observer and controller gains. Finally a numerical example is proposed to demonstrate the effectiveness of the proposed algorithm method.

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