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Disturbance Attenuation for Continuous-Time Linear Systems with State and Control Constraints

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Abstract The paper is devoted to the stabilization of linear systems having restricted states and controls. The determination of a large region of attraction for these systems is addressed. This region is described by an ellipsoid for which the volume is maximized. Necessary and sufficient LMI condition is given for the derivation of state feedback controllers driving the system asymptotically to the origin without violating the constraints for all states in the maximal set. This condition is then extended to determine the invariant sets for systems with persistent disturbances. LMI based methods are developed for constructing feedback law that achieve disturbance rejection with guaranteed stability requirements. The approach is illustrated by an example for continuous time cases.
keywords: Maximal domain of attraction; Constrained states; Constrained control; Disturbance; Linear Matrix Inequalities.

1 Introduction

In the last two decades, the problem of constraints had attracted a greater attention from workers in the field of control. In fact it was evident that constraints are inherent to practically all physical and/or industrial systems. These constraints may come out first from physical limitations as maximum flow for a valve, as constrained voltage and current for electrical processes and so on. Second another kind of constraints come out from approximations to obtain the desired model for the system as linearization approximations for naturally non linear systems. Many approaches have been then proposed to deal with such systems i.e., constrained systems. The positive invariance concept is one of that had emerged as appropriate to propose solutions to such problems. From the early results of [14], many work have been done with the use of positively invariant sets to deal with constrained systems one can see the overview of Blanchini [7] and the reference therein. It is worth to note here that this approach is based

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on control saturation avoidance and lead to limit the working area to the linear domain of behavior of the system [15]. On the other hand, new trends have emerged based on writing the saturation function as linear convex combination of some constrained variables [18]. Hence, a lot of work had been developed using this approach, without being exhaustive one can see [17], [16] and references therein. Other approaches can be found in the literature as the small and high gain concept [20] and l_1 concept [12] and so on. In the approaches above many problems have been studied and the common goal of almost all of these works is to find larger sets for initial conditions.

Hence, the problem of enlarging the set of attraction from the origin for constrained control and state systems is of interest in this work. A set of the state space including the origin in its interior is a controlled domain of attraction to the origin if there exists a control feedback law such that for any initial state in this set the states are driven asymptotically to the origin without constraints violation. In fact, many work had been done in order to characterize the maximal set of attraction for linear systems. From the work of Gilbert and Tan [13] and several studies have dealt with the construction of such domains. In the last years, many research work have been reported about the presence of limitation of certain variables inputs, states and/or, outputs within given sets. Without been exhaustive we can cite [5], [6], [8], [9], [19], [25], and the work of Blanchini [7] for an overview in this field. Mainly, the problem of constrained system variables is of continuing interest because of the wild sprite of possible applications. Almost all practical systems are subject to external disturbances that can in some situations degrade system performance if their effects are not considered during the design phase. In the current literature there are many ways to eliminate the effects of the external disturbances. One of them is the H_∞ control technique [3], [21]-[26]. It consists of designing a suboptimal control that minimizes the effects of the external disturbance on the output. The other approach that we will adopt here consists of assuming that the disturbance belongs to $\mathcal{L}2[0, \infty]$. This approach is referred to as H_∞ theory

In connection with the positive invariance concept, this paper deals with the problem of satisfying input and/or state constraints problems with a large domain of attraction. We are interested in rejecting the effect of the exogenous disturbance. Stability with respect to the constraints on the system and eliminate the effect of disturbance on system performance is guaranteed by synthesizing a Lyapunov quadratic function and by using the underlying positive invariance of its associated ellipsoids. The determination of a large region of attraction is addressed by an algorithmic procedure. This region of attraction is described by an ellipsoid for which the volume is enlarged by using an LMI-based algorithm. Also, Necessary and sufficient LMI conditions are given for the derivation of state-feedback controllers driving the system asymptotically to the origin without violating the constraints.

The remainder of this paper is organized as follows. In section 2, we provide the basic definitions together with the statement of the problem. Section 3 is devoted to preliminary results. The main results of the paper which consists of new statement of invariance conditions and their transformation into LMI's are stated in Section 4. The LMI's given enable us to derive stabilizing state feedback controllers driving the state asymptotically to the origin without of violation constraints. Sufficient conditions are established to design a controller that rejects the disturbance acting of the system. Further, the problem of the volume maximization for the invariant set leading to the largest set of invariance in cases of constraints in both control and state is studied. Examples are studied in section 5 to illustrate the application of our method. Section 6 is reserved to concluding remarks.

2 Problem formulation

Consider the following continuous time system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t) \quad (1)$$

where the matrices A, E and B are real constant of appropriate size. The vector disturbance $w(t)$ belongs to the set

$$\mathcal{W} = \{w \in \Re^l \quad / \quad w(t)^T w(t) \leq 1,\ \forall t \geq 0\}, \quad (2)$$

The sate vector $x(t)$ is constrained to belong to the following set

$$\mathcal{L}(F, v) = \{x \in \Re^n \quad / \quad Fx \leq v,\ F \in \Re^{q \times n}\}, \quad (3)$$

and the control law $u(t) \in \Re^m$ is constrained to evolve in the following set

$$\Omega = \{u \in \Re^m \quad / \quad |u_i| \leq 1, i = 1, \ldots, m\}. \quad (4)$$

The problem we are addressing below is to find a stabilizing state-feedback law $u(t) = Kx(t)$ and a set of initial conditions for which $u(t) \in \Omega$ and $x(t) \in \mathcal{L}(F, v)$ for every time $t > 0$ and $x(t) \to 0$ for $t \to \infty$.
Further, denote the Lyapunov level set as

$$\Omega(P,\ \gamma) = \left\{x \in R^n \quad / \ x^T P x \leq \gamma\right\}, \quad (5)$$

consequently $\Omega(P, \gamma)$ is an invariant set of the system (1) in the sense of the following definition:

Definition 1 *A set $S \subset \Re^n$ is said to be invariant with respect to motion of the system (1), if for all initial state $x(0) \in S$ the motion of the system $x(t)$ remains in S for all $t > 0$.*

Let a matrix K_i denote the i^{th} row of K and define

$$\mathcal{L}(K) = \{x \in \Re^n \quad / \quad |K_i x| \leq 1, i = 1, \ldots, m\} \quad (6)$$

Lemma 2 *For a given ellipsoid $\Omega(P,\gamma)$, if there exist an $K \in I\!R^{m\times n}$ and a scalar $\epsilon > 0$ such that*

$$(A+BK)^T P + P(A+BK) + \frac{1}{\epsilon} PEE^T P + \epsilon \frac{P}{\gamma} < 0, \tag{7}$$

then $\Omega(P,\gamma)$ is a invariant set for system 1.

Proof. For $V(x) = x^T Px$, we will show that

$$\dot{V}(x,w) = 2(x^T P(A+BK)x + Ew) < 0,$$
$$\forall \in \partial(P,\gamma), w^T w \leq 1.$$

Recall that for any number ϵ,

$$2a^T b \leq \frac{1}{\epsilon} a^T a + \epsilon b^T b, \qquad \forall a, b \in I\!R^n.$$

It follows that

$$2x^T PEw \leq \frac{1}{\epsilon} x^T PEE^T Px + \epsilon w^T w \leq \frac{1}{\epsilon} x^T PEE^T Px + \epsilon.$$

Hence,

$$\dot{V}(x,w) \leq 2(x^T P(A+BK)x) + \frac{1}{\epsilon} x^T PEE^T Px + \epsilon.$$

It follows from (7) that for all $x \in \Omega(P,\gamma), w^T w \leq 1$,

$$\dot{V}(x,w) < -\frac{\epsilon}{\gamma} x^T Px + \epsilon.$$

Observing that on the boundary of $\Omega(P,\gamma)$, $x^T Px = \gamma$, hence $\dot{V}(x,w) < 0$
This shows that $\Omega(P,\gamma)$ is invariant set

If P satisfies condition (7) and $\Omega(P,\gamma) \subset \mathcal{L}(K) \bigcap \mathcal{L}(F,v)$ then taking any initial condition in the ellipsoid $\Omega(P,\gamma)$ we have $u(t) \in \Omega$ and $x(t) \in \mathcal{L}(F,v)$ for every time $t > 0$. In the sequel we show how to compute such K, P and γ which determine the largest region of attraction $\Omega(P,\gamma)$ to the origin of the system (1) subject to state and control constraints.

3 Preliminaries

This section provides some useful key lemmas. We derive two key lemmas that give necessary and sufficient conditions for the inclusion of an ellipsoidal set in respectively two kind of polyedral sets.

Lemma 3 *[1] The inclusion $\Omega(P,\gamma) \subset \mathcal{L}(K)$ is equivalent to*

$$\begin{bmatrix} r & Y_i \\ Y_i^T & Q \end{bmatrix} \geq 0, \quad i = 1, \ldots, m \tag{8}$$

where $Q = P^{-1}$, $r = \gamma^{-1}$ and $Y_i = K_i Q$.

Proof. Rewrite $x \in \Omega(Q^{-1}, r^{-1})$ as $Q_1(x) = x^T Q^{-1} x - r^{-1} \leq 0$ and $x \in \mathcal{L}(K)$ as $Q_2(x) = x^T K_i^T K_i x - 1 \leq 0 \quad i = 1, \ldots, m$. Since the condition $\Omega(P, \gamma) \subset \mathcal{L}(K)$ is nothing than the implication $Q_1(x) \leq 0 \Rightarrow Q_2(x) \leq 0$, then by using S-procedure Lemma , this condition is equivalent to the existence of $\alpha_i > 0$ such that $x^T K_i^T K_i x - 1 \leq \alpha_i (x^T \; Q^{-1} \; x - r^{-1})$ for $i = 1, \ldots, m$.
Now taking any arbitrary scalar β we have
$\beta x^T K_i^T K_i x \beta - \beta^2 \leq \alpha_i \beta x^T \; P \; x \beta - \alpha_i \beta^2 r^{-1}$
and by making the change of variable $\tilde{x} = x\beta$, we obtain $\tilde{x}^T K_i^T K_i \tilde{x} - \beta^2 \leq \alpha_i \tilde{x}^T \; P \; \tilde{x} - \beta^2 \alpha_i r^{-1}$. Or equivalently for $i = 1, \ldots, m$,

$$\begin{bmatrix} \tilde{x} \\ \beta \end{bmatrix}^T \begin{bmatrix} K_i^T K_i - \alpha_i Q^{-1} & 0 \\ 0 & -1 + \alpha_i r^{-1} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \beta \end{bmatrix} \leq 0,$$

the above inequality reduces to:

$$\begin{aligned} K_i^T \, K_i \; &\leq \; \alpha_i \, Q^{-1}, \\ \alpha_i r^{-1} \; &\leq \; 1, \end{aligned}$$

this leads equivalently to

$$K_i^T K_i \; \leq r Q^{-1}.$$

Let $Y_i \; = \; K_i \, Q$ and by using the Schur lemma, we have that

$$\begin{bmatrix} r & Y_i \\ Y_i^T & Q \end{bmatrix} \; \geq \; 0,$$

is equivalent to $\Omega(P, \gamma) \subset \mathcal{L}(K)$

It is worth to note that lemma 3above is well known as a sufficient condition given by [10]. Here we proove that this condition is also necessary.

Lemma 4 *[2] The inclusion $\Omega(P, \gamma) \subset \mathcal{L}(F, v)$ is equivalent to the existence of $\alpha_1 > 0, \ldots, \alpha_q > 0$ such that:*

$$\begin{bmatrix} 4 \, \alpha_i \, v_i \; - \; F_i \; Q \; F_i^T & 2 \, \alpha_i \\ 2 \, \alpha_i & r \end{bmatrix} \geq 0, \tag{9}$$

where $Q = P^{-1}$ and $r = \gamma^{-1}$ and F_i is the i^{th} row of F.

Proof. Using Schur Lemma the LMI condition (9) is equivalent to:

$$-4 \, \alpha_i^2 \, r^{-1} + 4 \, \alpha_i \, v_i \; - \; F_i \; Q \; F_i^T \geq 0,$$

using again Schur Lemma , we obtain

$$\begin{bmatrix} -\alpha_i r^{-1} \; + \; v_i & -\frac{1}{2} F_i \\ -\frac{1}{2} F_i^T & \alpha_i Q^{-1} \end{bmatrix} \; \geq \; 0.$$

So that for any vector $[z\ \tilde{x}]^T$, we have

$$\begin{bmatrix} z \\ \tilde{x} \end{bmatrix}^T \begin{bmatrix} -\alpha_i r^{-1} + v_i & -\frac{1}{2}F_i \\ -\frac{1}{2}F_i^T & \alpha_i Q^{-1} \end{bmatrix} \begin{bmatrix} z \\ \tilde{x} \end{bmatrix} \geq 0,$$

by developing the above inequality, we obtain

$$\frac{z}{2}F_i\tilde{x} + \frac{z}{2}\tilde{x}^T F_i^T - v_i z^2 \leq \alpha_i \tilde{x}^T P \tilde{x} - \alpha_i r^{-1} z^2,$$

making the change of variable $\tilde{x} = z\, x$ we have

$$\frac{1}{2}F_i x + \frac{1}{2}x^T F_i^T - v_i \leq \alpha_i x^T Q^{-1} x - \alpha_i r^{-1},$$

or equivalently

$$F_i x - v_i \leq \alpha_i (x^T Q^{-1} x - r^{-1}).$$

By using the S-procedure Lemma the proof is complete

4 Main results

In this section we give necessary and sufficient conditions for an ellipsoid to be invariant with respect to motion of constrained control and state system. These conditions are formulated in terms of LMI. Further, the LMI formulation enables us to compute the maximal invariant set for these systems. The enlargement procedure is based on the determinant maximization.

Proposition 5 *There exists a stabilizing state feedback control law such that the system (1) satisfies the state and control constraints (3)-(4) if there exist matrices $Y \in R^{m\times n}$, $Q > 0$, a scalar $\epsilon > 0$ and scalars $\alpha_i > 0$ such that:*

$$(i)\ QA^T + AQ + BY + Y^T B^T + \tfrac{1}{\epsilon}EE^T + Q\tfrac{\epsilon}{\gamma} > 0,$$

$$(ii)\quad \begin{bmatrix} 4\alpha_i v_i - F_i Q F_i^T & 2\alpha_i \\ 2\alpha_i & r \end{bmatrix} \geq 0, \quad for\ \ i = 1\ldots, q$$

$$(iii)\quad \begin{bmatrix} r & Y_j \\ Y_j^T & Q \end{bmatrix} \geq 0, \quad for\, j = 1\ldots, m$$

where Y_i's are the rows of the matrix Y.
Moreover, letting $P = Q^{-1}$ and $\gamma = r^{-1}$ we have that the ellipsoid $\Omega(P,\gamma)$ is invariant for the system (1) with respect to the state and control constraints (3)-(4) under the control law $u = YQ^{-1}x$.

Proof. Let $u = Kx$ be a stabilizing state feedback then there exists $P > 0$ and a scalar $\epsilon > 0$ such that

$$(A + BK)^T P + P(A + BK) + \frac{1}{\epsilon}PEE^T P + \epsilon\frac{P}{\gamma} < 0, \tag{10}$$

multiplying by P^{-1} the left and the right of the above inequality and using Schur lemma we obtain condition (i) with $Q = P^{-1}$, $Y = KQ$. In addition, the ellipsoid $\Omega(P, \gamma)$ is positively invariant for the system (1) which satisfies the state and control constraints (3)-(4). So that we have $\Omega(P, \gamma) \subset \mathcal{L}(K) \bigcap \mathcal{L}(F, v)$ and then by using Lemma 3 and Lemma 4 we lead to conditions (ii) and (iii).

Note that $\Omega(P, \gamma) = \Omega(\gamma^{-1}P, 1) = \Omega((r^{-1}Q)^{-1}, 1)$, and since condition (i), LMI's(ii) and (iii) are homogeneous we have that any invariant ellipsoid with respect to the state and control constraints can be parametrized as $\Omega(Q^{-1}, 1)$. Consequently, with this parametrization the LMI's (i), (ii), (iii) are expressed equivalently by

$$Q\,A^T \;+\; A\,Q + B\,Y \;+ Y^T\,B^T + \frac{1}{\epsilon}EE^T + Q\epsilon > 0, \tag{11}$$

$$\begin{bmatrix} 4\alpha_i v_i - F_i Q F_i^T & 2\alpha_i \\ 2\alpha_i & 1 \end{bmatrix} \geq 0, \quad 1 \leq i \leq q \tag{12}$$

$$\begin{bmatrix} 1 & Y_j \\ Y_j^T & Q \end{bmatrix} \;\geq\; 0, \quad 1 \leq j \leq m, \tag{13}$$

Theorem 6 *The largest invariant ellipsod of the system (1) with state and control constraints (3)-(4) is the ellipsoid $\Omega(P)$ where P is solution to the following optimization problem (PB1):*

Maximize $\;\mathbf{Log(Det}(P^{-1}))$

$P > 0, Q > 0, \epsilon > 0, Y, \alpha_i$

subject to:

$$\begin{bmatrix} P & I \\ I & Q \end{bmatrix} > 0, \tag{14}$$

$$QA^T + AQ + BY + Y^T B^T + \frac{1}{\epsilon}EE^T + Q\epsilon > 0 \tag{15}$$

$$\begin{bmatrix} 4\alpha_i v_i - F_i Q F_i^T & 2\alpha_i \\ 2\alpha_i & 1 \end{bmatrix} \geq 0, \quad i = 1, \ldots, q \tag{16}$$

$$\begin{bmatrix} 1 & Y_j \\ Y_j^T & Q \end{bmatrix} \;\geq\; 0, \quad j = 1, \ldots, m \tag{17}$$

Proof. The volume of $\Omega(P)$ is proportional to $Det(P^{-1}))$. Maximizing this volume is equivalent to maximize $log(Det(P^{-1})))$. By using the Schur Lemma, the condition (14) is equivalent to $Q > P^{-1}$. By maximizing we can prove that the optimum is reach for $Q = P^{-1}$. The rest of the proof is straightforward.

If we fix ϵ, then the constraints of the optimization problem (PB1) become LMIs. To obtain the global infinum, we may vary ϵ $(0 < \epsilon < \infty)$.

5 Example

In order to illustrate the proposed procedure we consider two examples:
Our first example is the following 2-dimensional problem adopted from [18]:

$$\dot{x} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix} x + \begin{bmatrix} 2 \\ 4 \end{bmatrix} u(t) + E\, sign(sin(0.3t)), \tag{18}$$

where the additive disturbance matrix is $E = [0.1\ 0.1]^T$, with the state constraint set as:

$$F = \begin{bmatrix} 6 & 3 \\ 7 & 1 \end{bmatrix} \quad , \quad v = \begin{bmatrix} 21 \\ -20 \end{bmatrix}. \tag{19}$$

We derive a state feedback $u = K\ x$ which stabilizes the closed loop system while guaranteeing that for every initial state $x(0) \in \Omega(P, \gamma)$ we have $|K\ x| \leq 1$ and $F\ x \leq v$ along the system trajectory.
By solving the corresponding set of linear matrix inequalities LMI (14), (15), (16), (17) and by sweeping throught ϵ we obtain $\epsilon^* = 0.0246$ which correspond to maximal volume (figure 1).

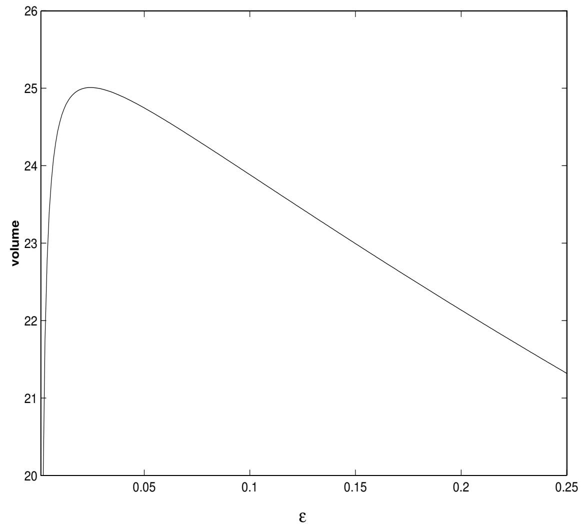

figure 1 : evolution of the volume of ellipsoid

and feedback

$$K = \begin{bmatrix} 0.2272 & -0.5757 \end{bmatrix}, \tag{20}$$

The ellipsoidal domain of attraction is

$$\Omega(P, \gamma) = \left\{ x \in R^n \mid x^T P x \leq 1 \right\}. \tag{21}$$

where,

$$P = \begin{bmatrix} 0.3655 & -0.2948 \\ -0.2948 & 0.4171 \end{bmatrix} \tag{22}$$

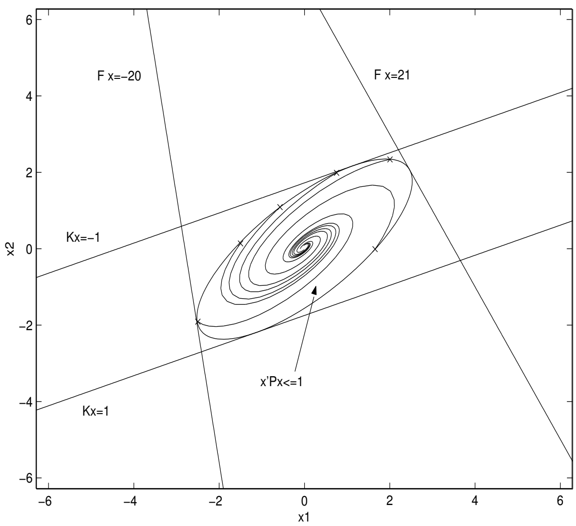

figure 2 : Domain of attraction

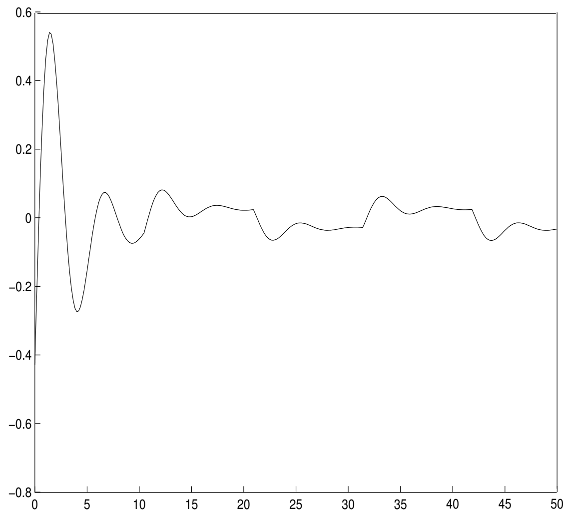

figure 3 : evolution of control action

The set $\Omega(P, \gamma)$ is the ellipsoid depicted in Figure 1 and is by construction contained in the region where $|K\ x| \leq 1$ and $F\ x \leq v$ (also reported in Figure 1). It is clear from the figure 2 that the feedback K is stabilizing and all constraints are respected. Figure 3 shows the control evolution for the state beginig at $xo = [-1.5\ \ 0.15]^T$.

Our second example is the following 3-dimensional system [24]:

$$\dot{x} = \begin{bmatrix} -0.82 & 17.76 & 90.24 \\ 0.17 & -0.75 & -11.10 \\ 0 & 0 & -250 \end{bmatrix} x + \begin{bmatrix} -91.24 \\ 0 \\ 250 \end{bmatrix} u(t) + E\,w(t), \tag{23}$$

where the additive disturbance matrix is

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

with the state constraint set as:

$$F = \begin{bmatrix} 8 & 20 & 15 \\ -4 & 10 & 5 \\ 0 & 0 & 1 \\ -2 & -3 & 1 \end{bmatrix} \quad , \quad v = \begin{bmatrix} 120 \\ 60 \\ -15 \\ 60 \end{bmatrix}. \tag{25}$$

We derive a state feedback $u = K\ x$ which stabilizes the closed loop system while guaranteeing that for every initial state $x(0) \in \Omega(P, \gamma)$ we have $|K\ x| \leq 1$ and $F\ x \leq v$ along the system trajectory.

By solving the corresponding set of linear matrix inequalities LMI (14), (15), (16), (17) and by sweeping throught ϵ we obtain $\epsilon^* = 0.31$ which correspond to maximal volume (figure 4).

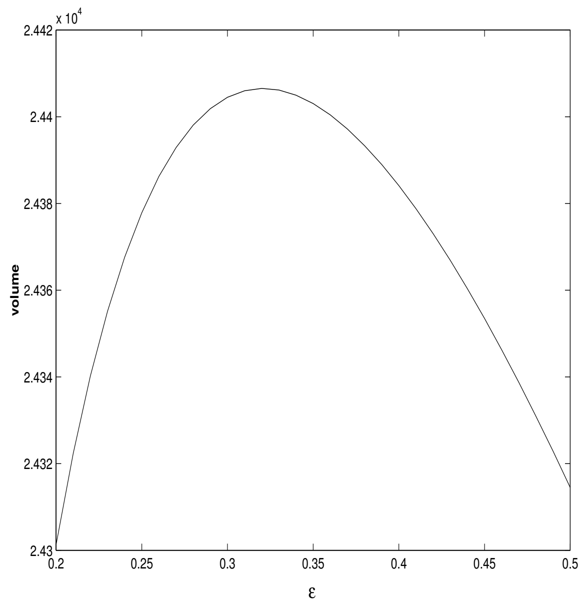

figure 4 : evolution of ellipsoid volume

and feedback

$$K = \begin{bmatrix} 0.0666 \; 0.3059 \; 0.02 \end{bmatrix} \tag{26}$$

The ellipsoidal domain of attraction is

$$\Omega(P, \gamma) = \left\{ x \in R^n \mid x^T P x \leq 1 \right\}. \tag{27}$$

where,

$$P = \begin{bmatrix} 0.0132 & 0.0097 & -0.0005 \\ 0.0097 & 0.1090 & 0.0003 \\ -0.0005 & 0.0003 & 0.0305 \end{bmatrix} \tag{28}$$

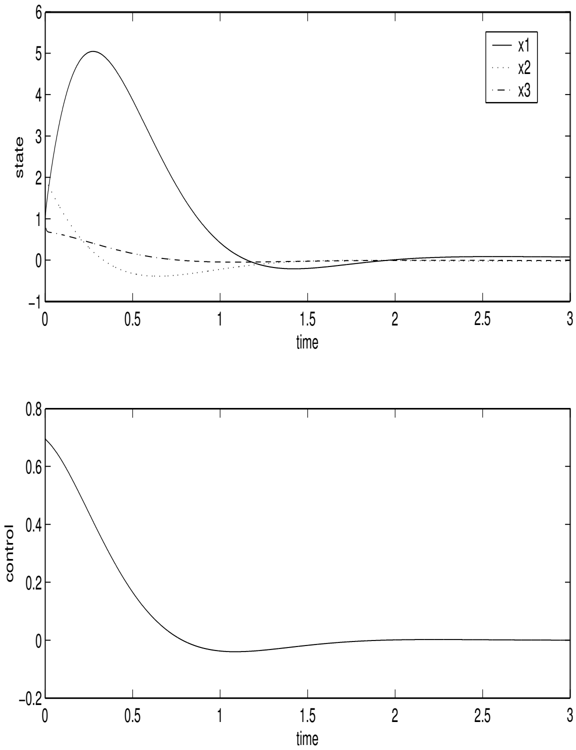

figure 5 : Time-evolution of state and control action

Figure 5 shows the state trajectory from initial condition $xo = [1 \;\; 2 \;\; 1]^T$. The trajectory state are all driven to the origin by the provided robust state-feedback control, with respected constraints on the state and the control.

6 conclusion

Necessary and sufficient conditions for an ellipsoid to be invariant with respect to motion of constrained control and state system are derived. We have proposed an LMI based approach to the enlarging of the linear invariant ellipsoid for linear system subject to state and control constraints. The proposed controller robustly drives the state into the target set. Examples are worked out to demonstrate the effectiveness of the proposed design techniques.

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