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A Fuzzy Compensator of Interactions for a Multivariable Generalised Predictive Control

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Abstract. A method for the design multivariable generalised predictive controllers based on the compensation of interactions is introduced. The design proceeds in two steps. In the first step, the interactions are ignored and single input single output generalised predictive controllers are designed for the resulting subsystems. In a second step, a fuzzy compensator of interactions acting feedforward produces compensation signal for each subsystem. A multiobjective genetic algorithm is used to find the optimal parameters of the fuzzy compensator. The method is applied in simulation to the multivariable control of a binary distillation column.

Keywords. Multivariable systems, Feedforward controllers, Fuzzy inference system, Multi Objective Genetic Algorithm

1. Introduction

The problem of the design of the controllers for a multivariable linear systems is strongly related to the presence of the interactions between the single-variable subsystems that constitute the overall system. Several approaches have been developed for the resolution of this problem, [1]. We can distinguish two classes of methods. The first proceed in two steps. In the first step, we seek a decoupling or pseudo decoupling between subsystems, in the second step, we build an independent controller for each subsystem thus obtained. These methods have shown a lack of ro-

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bustness [1]. The second class of methods considers the multivariable system as a whole and then tries to find a multivariable controller. Some of these methods have guaranteed properties of robustness.
In this work, we propose an approach which follows the methodology of the first class. In a first stage, each subsystem is controlled independently of the other by generalized predictive controller with parameters calculated without taking account of the interactions. In a second stage, a fuzzy compensator of interactions, based on Takagi Sugeno inference system [2] is designed with the objective of compensating the interactions. It is based on a feedforward control strategy after measurements of the signals from the single-variable controllers. The quasi-optimal parameters of the fuzzy compensator are obtained using a multiobjective genetic algorithm. This paper is organized as follows: in the second section, we introduce the background of the method: the feedforward control, the fuzzy inference systems and the multiobjective genetic algorithms, in the third section we present the algorithm and in the fourth section we describe an application to a distillation column.

2. Background:

2.1 The feedforward control

The feedforward control [4], is based on the anticipation of the effect of the measurable disturbance which acts on the variable to be controlled. It will then be possible to act on the control signal in order to keep constant the controlled variable. This type of control is only possible if the transfer between the disturbance and the variable to be controlled is known.

2.2 The fuzzy inferences systems

The fuzzy inferences systems operate on linguistic variables instead of the numeric variables. The fuzzy system is characterized by only one fuzzy relation which is determined by the combination of all the fuzzy rules, as follows:

$$R = also\,(R_1, R_2, ..., R_n) \tag{1}$$

A fuzzy inferences system is made up of three parts [5]:

- Fuzzification interface twhich transforms the numerical values into linguistics values
- A knowledge base defined by a data base providing the necessary information used for the exploration of the fuzzy rules and handling of the data in the fuzzy system of inference and a base of rules. A logic of decision able to simulate the human decisions based on fuzzy concept and to infer the control actions.
- The defuzzification which transforms the linguistic values of the output into numerical value.

2.3 Multi-Objective Genetic Algorithms, MOGA

A Simple Genetic Algorithm, SGA, is a heuristic algorithm of search for optimum based on the process of natural evolution [6]. The SGA simulates this process through coding and special operators and works with a population of individuals or chromosomes. A chromosome is composed of subchromosomes each representing a coding of one variable of the problem and represents a feasible solution to the problem with an associated value of the cost function, here the fitness, to be optimised. A population is then a set of admissible solutions. By applying genetic operators to the current population a new population is created with the goal of improving the fitness. The three classical genetic operators are reproduction, crossover and mutation [6]. SGA deals with problem of optimisation with only one criterion. This criterion (represented by the function to optimise) was transformed in the form of a function of adaptation. This approach works well for many problems. However, in some practical problems several criteria are used simultaneously and it is no more possible (nor desirable) to combine them in only one function. In this situation, we are confronted with a problem with multiple objectives. The problem of multiobjective optimisation consists in simultaneously optimising several objectives, it is formulated as follows. Let us consider the case of maximization of two objectives:

maximize $f(x)=(f_1(x),f_1(x))$

Such as $x\in X$

x_1 and x_1 are two solutions to be compared. It is said that x_1 dominates x_2 if and only if:

$$f_1(x_1) > f_1(x_2) \text{ and } f_2(x_1) \geq f_2(x_2) \text{ or } f_1(x_1) \geq f_1(x_2) \text{ and } f_2(x_1) > f_2(x_2)$$

The non-dominated individuals are defined as the Pareto front (criterion of non-predominance).

The multiobjective Genetic Algorithm, MOGA, used here is given by the following steps [7]:

1. *Initialisation*

An initial population of individuals is generated randomly,

2. *Ranking:*

The operation of ranking consists in creating ranks of individuals based on the so-called dummy fitness:

a. For each population find the non-dominated individuals according to the definition above. Give the same pseudo-fitness (Dummy Fitness (DF)), f_0 for these individuals (generally f_0=1)

b. To maintain a diversity in the population, the DF of each individual in the front is multiplied by a quantity called niche and proportional to the number of individuals in the front, to obtain the new dummy fitness: $f_0.m_i$, where:

$$m_i = \sum_{j=1}^{M} Sh(d(i,j)) \tag{1}$$

i : index of the individual, M a number of individuals in the front,

Sh : the function of division, and it is defined by:

$$Sh(d(i,j)) = 1 - \frac{d(i,j)}{\sigma_{share}} \quad ; \quad \text{if } d<\sigma \tag{2}$$
$$Sh(d(i,j))=0 \quad ; \quad \text{if } d>0$$

with $Sh(d(0))=1$

σ : is the phenotypical distance between 2 individuals.

d(i, j) : is the distance between 2 individuals, it can be calculated by the Euclidean distance.

The individuals of this front are extracted from the population and the ranking operation is repeated with the remaining individuals to form the next front with initial dummy fitness equal to the smallest fitness of the preceding front.

***3**. Application of the Simple Genetic Algorithm*

Once the whole population is ranked, The Simple Genetic Algorithm (AGS) is applied to the ranked population obtained in step 2 to obtain a new population.

4. If the stopping criterion is verified *STOP* if not one returns to stage 2.

3. The Fuzzy Compensator of the Interactions, FCI

3.1 Basic structure

In the algorithm proposed, a Feedback/Feedforward control is introduced. The feedback consists in controlling each subsystems independently from the other by a single input single output, *SISO*, controller, the interactions considered as measurable disturbances, will be compensated by signals outputs from the fuzzy compensator of the interactions, FCI, based on Takagi Sugeno model. The strategy of control is done in two stages. In the first stage, single input single output, SISO, predictive controllers of the Generalised Predictive Controllers type, GPC, [8] are designed for each subsystem ignoring the interactions. In the second step, the interactions are introduced with the SISO GPC controllers in the loops. The objective is to find a fuzzy compensator for the compensation of the interactions. Figure 3 represents the diagram of such an approach for a 2x2 system.

a) The SISO GPC controllers:

In predictive control, the problem is to find at each sampling period, the sequence control signals that minimises the following objective function :

$$J(t) = \sum_{j=1}^{N} \left[\hat{y}(t+j) - r(t+j)\right]^2 + \sum_{j=N_1}^{N_u} \lambda\left[\Delta u(t+j-1)\right]^2 \tag{3}$$

where $\hat{y}(t+j)$ is the prediction of the output computed with a prediction model over an ouput prediction horizon *N*, *r(t+j)* is the reference signal, $\Delta u(t+j-1)$ the future control increment with $\Delta u(t) = u(t) - u(t-1)$, N_u the control horizon such that: $\Delta u(t+j-1)=0$ and λ a weighting factor. In the case of generalized predictive control GPC, the prediction model is a CARIMA model:

$$A(z^{-1})y(t)=B(z^{-1})\Delta)u(t-1)+C(z^{-1})e(t) \qquad (4)$$

$A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$ are polynomials in the unit delay operator z^{-1}. The role of the $\Delta=1-z^{-1}$ is to ensure integral action of the controller. The first element u(t) of the control signal is applied to the system.

b) The Fuzzy compensator of interactions

The technique of compensation is based on a feedforward control. The control signals output from the controllers are injected into the compensator to produce the two compensating signals, the final control is equal to a weighted sum of the two signals: *SISO* controllers and signal from the fuzzy compensator. The optimisation of the parameters of the compensator and weightings w_1, w_2 is realized by the multiobjective genetic algorithms. For a two dimensions system, two inputs and two outputs, the structure of the compensator is given by the following rules:

$$if \quad U_{gpc1} \text{ is } A_{i1} \text{ then } Uc_1=P_{i1}U_{gpc1}$$

$$if \quad U_{gpc2} \text{ is } A_{i2} \text{ then } Uc_2=P'_{i1}U_{gpc2}$$

U_{GPC1} and U_{GPC2} are the outputs of the *SISO* controllers GPC [8], and thus the inputs to the compensator, U_{C1} and U_{C2} are the outputs of the fuzzy compensator, A_{i1} and A_{i2}, $i=1 \ldots M$ are the fuzzy sets associated linguistic variables U_{GPC1} and U_{GPC2}. These sets will be isosceles triangles uniformly distributed on the universes of discourse.

The two control signals will be then: $U_1=(U_{GPC1}+U_{C2})w_1$; $U2=(U_{GPC2}+U_{C1})w_2$;

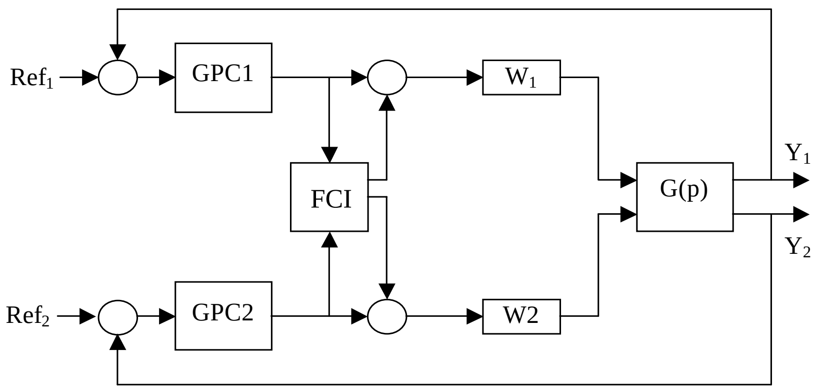

Figure 1. Compensation of interactions structure

3.2 Description of the parameters optimisation algorithm

The multi objective genetic algorithm, MOGA, is used to determine the parameters of the fuzzy compensator: the length of the universes of discourse of the input variables of the fuzzy compensator, the parameters of the consequences and to refine the parameters of single-variable controllers GPC. Note that since the distribution of the fuzzy sets is uniform, it is only necessary to determine the length of the universe of discourse. In the case of a two inputs two outputs system, the two objective functions to be minimised by the MOGA are:

$$f_1 = \sum_{i=1}^{T} |e_1(i)| \ , \ f_2 = \sum_{i=1}^{T} |e_2(i)| \tag{5}$$

T is the simulation time which must be sufficiently long so as the problem can be considered as an optimal control problem with infinite horizon and thus guarantees that the optimal solution if it exits is stable.

Optimisation is carried out off line and in closed loop with the controllers and the fuzzy compensator of interactions in the loop. The variables included in the optimisation are: the widths of the universes of discourse, the parameters of SUGENO, the weighting vector, the output prediction horizons of the SISOGPC and the weighting factors λ. This set of variables forms the chromosome.

The optimisation process goes along the following steps:

1- Initialisation : Generate randomly an initial population of chromosomes

2- For each chromosome (namely the set of variables included in the optimisation) compute the fitness as follows: construct the associated closed loop system with SISO GPC and Fuzzy compensator. Simulate this system from t=0 to t=T and compute the cost functions according to (5).

Apply ranking to the current population as described in section 2.3 above

3- Apply SGA to the ranked population

4- If the stopping criterion satisfied stop extract the best solution in last generation otherwise return to 2. The stopping criterion is usually a maximum number of generations.

4. Results of simulation

In this section, we present the simulations which were carried out in order to test the method introduced. The selected system is a binary distillation given as reference model in 1991 by Limebeer [3] with strong interactions. It is described by the following model:

$$\begin{bmatrix} Y_1(p) \\ Y_2(p) \end{bmatrix} = \frac{1}{75p+1} \begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \begin{bmatrix} k_1 e^{\theta_1 p} & 0 \\ 0 & k_2 e^{\theta_2 p} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{6}$$

$Y_{1(p)}$ and $Y_{2(p)}$ are the concentrations of the products, the inputs $U_{1(\mathrm{p})}$ and $U_{2((\mathrm{p})}$ are the reflux and vapour boilup. θ_i =1*mn*, k_1=k_2=1. The compositions are normalised between 0 and 1. The references *are* $r_1(t)$ for $y_1(t)$ and $r_2(t)$ for $y_2(t)$. The objective is to design a controller for the multivariable system that must satisfy the following robustness performances when the gains k_1 and k_2 vary between 0.8 and 1.2:

-For a reference $r_1\ (t)=1$ $(r_2(t)=1)$ we must have $0.9 \le y_1(t) \le 1.2$ ($0.9 \le y_2(t) \le 1.2$)

- For reference $r_1\ (t)=0$ $(r_2(t)=0)$ we must have $|y_1(t)| \le 0.4$ ($|y_2(t)| \le 0.4$)

These are to be satisfied for all $t \ge 0$

The design of controllers GPC is done for the two following subsystems:
The MOGA is used to find the following parameters:

For the SISO GPC, the objective is to refine the parameters N^1 and N^2 the prediction horizon for the two subsystems, λ_1 and λ_2 the weighting factors for the two subsystems. We set to one both control horizons $N_{\underline{u1}}$ and N_{u2}.

For the fuzzy compensator of interactions: $p_{11}, p_{12}, p_{13}, p_{21}, p_{22}, p_{23}$, g_1 and g_2 and weightings factor: w_1, w_2 .

The cost functions to be minimized by the MOGA are:

$$f_1=\sum_{i=1}^{100}|e_1(i)| \qquad\qquad f_2=\sum_{i=1}^{100}|e_2(i)|$$

$e_1(t)$ and $e_2(t)$ are the cumulated errors on outputs 1 and 2.

The results of the MOGA optimisation procedure are presented in table I.

Table I : Optimal parameters

Parameters	Values
Parameters of Sugeno	P_{11}= -0.80107 ; P_{12}= 0.7078 ; P_{13}= -0.5388
Universe of discourse	g_1 = 378.1355 ; g_2 = 484.8989
Parameters of GPC SISO1	N_{21} = 16; λ_1 = 0.6894 ; N_{u1} = 1
Parameters of GPC SISO2	N_{22} = 16 ; λ_2 = 0.1215 ; N_{u2} = 1
weighting Vector	W_1 = 2.367 ; w_2 = 2.550

The optimal parameters find by the MOGA are given in table I. Figures 2 (a) and (b) show the results of the simulation of SISO control GPC without compensation of the interactions. It is seen that the responses are very sluggish and the compositions does not reach their references even at time 100 minutes. Figures 3, 4, 5 and 6 show the results of the control GPC with the fuzzy compensator of interactions for the nominal case k_1=1 and k_2=1. It is noticed that the effect of the interactions is practically eliminated. In order to check if the robustness performances are satisfied, the GPC controllers with the fuzzy compensator are tested for the extreme gains namely a variation of ±10%. Figures7, 8, 9 and 10 show the result of these simulations. The stability is preserved but the performances are slightly degraded. This is shown in figure 7 for the reference $r_1(t)=1$ and $r_2(t)=0$ with gains $k_1=0.8$ and $k_2=1.2$ where the output $y_1(t)$ reaches 1.25. In figure 9, the degradation concerns also $y_1(t)$ which overshoots up to 0.8 for the case $r_1(t)=0$ *and* $r_2(t)=1$ with the same gains. In figure 10, the output $y_2(t)$ reaches 1.25. Remarkably however, the performances are maintained for the case $k_1=0.8$ and $k_2=1.2$.

5. Conclusion

A technique for the design of multivariable controllers has been introduced. It is based on the principle of decoupling by compensation of the interactions. In a first stage, the interactions are ignored and of the controllers are designed for the resulting SISO subsystems. In a second stage, a fuzzy compensator having as inputs the outputs of the SISO GPC controllers produces signals of compensations. These which be added to the GPC control signals, with effect to minimize the effect of the interactions while acting feedforward. The solution uses the genetic algorithms multiobjective to find one quasi optimal compensator. The approach is relatively simple to design. The method is applied in simulation to a binary distillation column. The results show a good compensation of interactions for the nominal case and a robustness withy respect to stability with a slight degradation of the performance.

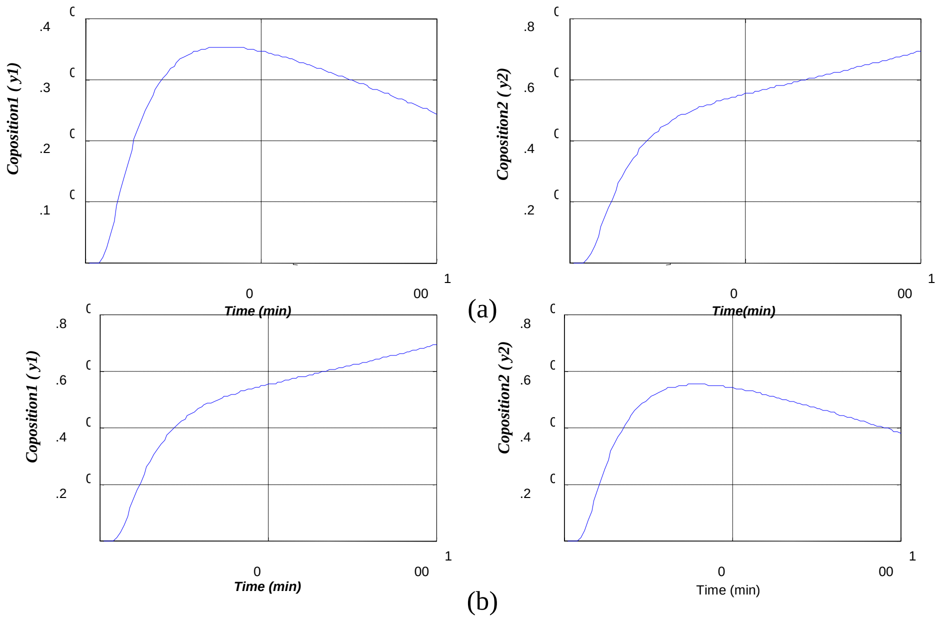

Figure 2: SISO controller without compensation (a) reference (0,1) and (b) reference (1, 0)

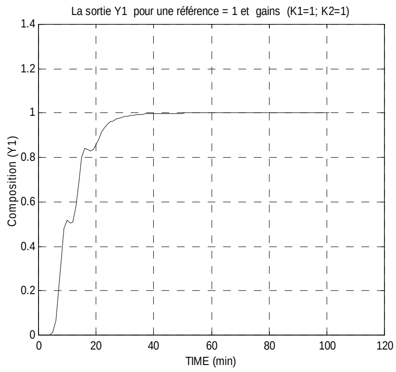

Figure 3 : The output y_1 for references (1,0) and gains ($k_1=1; k_2=1$)

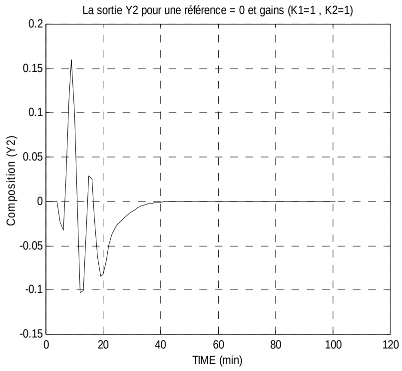

Figure 4: The output y_2 for references (1,0) and gains ($k_1=1; k_2=1$)

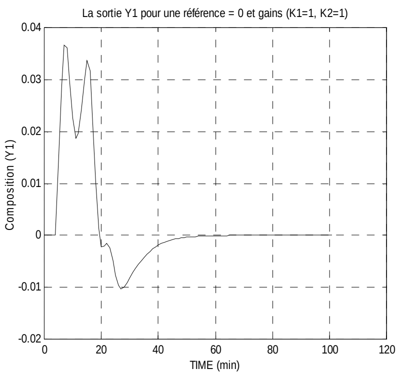

Figure 5: The output y_1 for references (0,1) and gains ($k_1=1; k_2=1$)

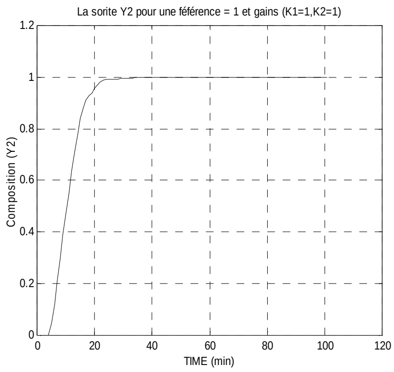

Figure 6 : The output y_2 for references (0,1) and gains ($k_1=1; k_2=1$)

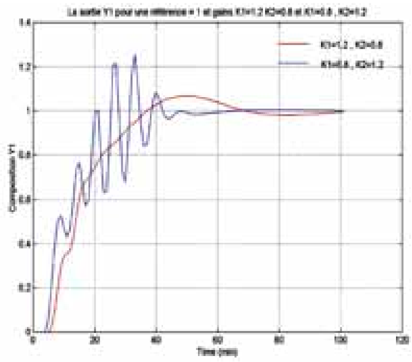

Figure 7: The output y_1 for references (1,0) $k_1=0.8$; $k_2=1.2$ and

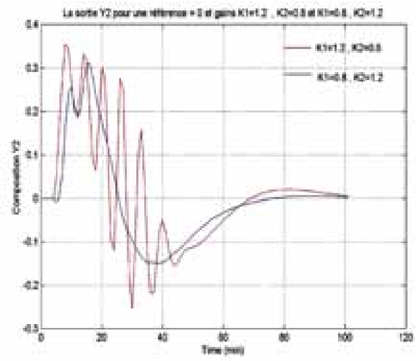

Figure 8: The output y_2 for references (1,0) and $k_1=0.8$; k_2

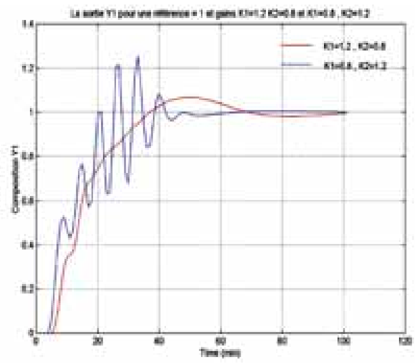

Figure 7: The output y_1 for references (1,0) $k_1=0.8$; $k_2=1.2$ and $k_1=1.2$, $k_2=0.8$

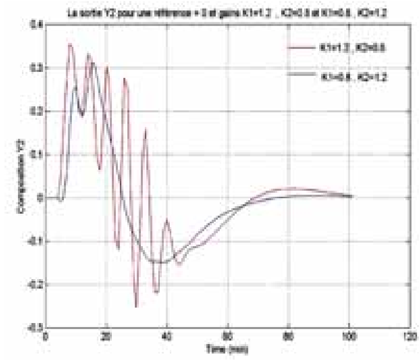

Figure 8: The output y_2 for references (1,0) $k_1=0.8$; $k_2=1.2$ and $k_1=1.2$, $k_2=0.8$

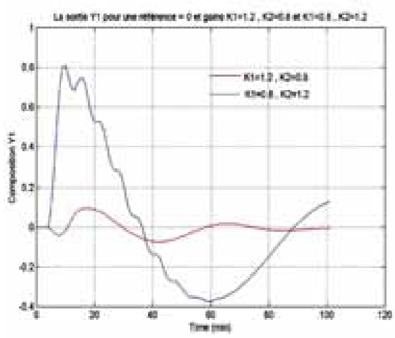

Figure 9: The output y_1 for references (0,1) $k_1=0.8$: $k_2=1.2$ and $k_1=1.2$. $k_2=0.8$

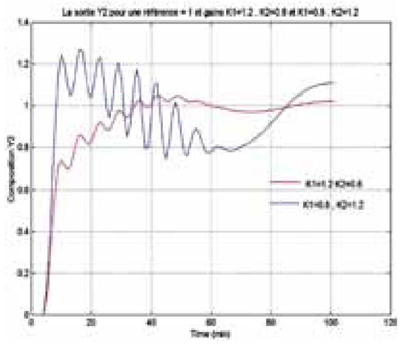

Figure 10: The output y_2 for references (0,1) $k_1=0.8$: $k_2=1.2$ and $k_1=1.2$. $k_2=0.8$

References

[1] Maciejowski,J.M., *Multivariable Feedback design,* Addison-Wesley Publishing Company,1994.

[2] Takagi, T, M. Sugeno, "Fuzzy identification of systems and its application to modeling and control ", *IEEE Trans. Syst. Man and Cybern*, vol SMC-15, N°1, pp.117-130, 1985.

[3] Limebeer D. J. N., "The specification and purpose of a controller design case study", in *Proc. IEEE Conf. Dec. Contr.,* Brighton, UK.,pp. 1581-1585, 1991.

[4] Luyben, W.L., *Process Modeling, Simulation and control for Chemical engineers*, McGraw Hill, 1973.

[5] Lee, C.C., "Fuzzy logic in control systems: Fuzzy logic controller Part I and II", *IEEE Trans. Syst.Man, Cybern*, SMC-20, pp.404-418, 1990.

[6] Goldberg D.E., *Genetic algorithms in search optimisation and machine learning*, Addison Wesley, 1989.

[7] C.A.Coello "A comprehensive survey of evolutionary-based multiobjective optimization techniques",Knowledge Inform. Syst., Vol.1, N°3, pp.269-308, 1999.

[8] D. W. Clarke, C. Mohtadi, P.S. Tuffs,"Generalised predictive control part I, the basic algorithm", Automatica, vol 23, N°2, pp.137-148, 1987.