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Stabilization of a Genesio-Tesi Chaotic System Using a Fractional Order $PI^{\lambda}D^{\mu}$ Regulator

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Abstract—Based on the bifurcation diagram technique, this paper analyses the stabilization conditions of the nonlinear chaotic Genesio-Tesi system with the use of a fractional order control scheme. Stability analysis is performed for the closed-loop nonlinear chaotic system behavior, for the fractional order integral and derivative actions by mean of the bifurcation diagram. Numerical simulation examples illustrate the efficiency of the proposed state feedback controller with a fractional order $PI^{\lambda}D^{\mu}$ control structure, allowin the stabilization of the problem of unstable fixed points with good closed loop system performance.

Keywords- fractional order system; Genesio-Tesi chaotic system; chaos suppression, periodic orbit; fractional $PI^{\lambda}D^{\mu}$ controller; unstable fixed point.

I. Introduction

Fractional order control solutions are gathering more and more research efforts nowadays mainly because of the emergence of numerical tools to deal with the fractional order differential and integral equations [1,2]. In order to improve the performance of linear feedback systems, Podlubny [3] generalized the classical PID controller to the $PI^{\lambda}D^{\mu}$ form called fractional order PID, which has recently become very popular because it offers supplementary flexibility to deal with more complicated design specifications. Since, fractional order PID (FOPID) controllers have counted a wide range of practical applications in several power systems as cited in the following. In [4, authors deal with the design of fractional order PID controller applied to integer as well as fractional plants. In [5], a FOPID is designed for the hydraulic turbine regulating system (HTRS) system for concurrent performance exigencies. Besides that, a comparative study concerning PID and FOPID controllers illustrates the superiority of the fractional ones in performance and robustness. In [6], a Fractional-Order PID is designed for ship roll motion control with the use of chaos embedded PSO algorithm. In [7], optimum tuning of fractional order PID controller for AVR system using chaotic and swarm optimization technique. Authors in [8] consider the fractional order high gain adaptive control strategy with augmented tuning parameters for the performance enhancement of the closed loop control system. Whereas in [9] a fractional differentiator-based controller is proposed to suppress chaos in a 3D single input chaotic system by stabilizing some of the fixed points.

Chaotic systems are also affected by the fractional order, either modeling or control or both. Many famous fractional order systems, such as Rossler system, Lorenz system, Chua's circuit, Duffing system and so on, have been studied [10-12]. In view of the fact that fractional calculus provides another good way to describe, predict and control physical systems accurately, it has been applied to control system, physics and system modeling [13,14]. We cite for example in [15], the control of chaos in the fractional nonlinear model of Chen-type by mean of a state space linear feedback control.

In this work the problem of chaos stabilization is considered for the case of Genesio-Tesi-type system which was introduced by Genesio and Tesi since more than two decades [16]. Stability analysis of the nonlinear chaotic system is studied for the fractional order integral and derivative actions using the bifurcation diagram. The fractional $PI^{\lambda}D^{\mu}$ controller is implemented by mean of the Adams-Bashforth-Moulton method and results show the ability of the proposed simple control strategy to perform the desired stability, based on the fractional order integral action.

The remaining sections are organized as follows: Section II presents basic definitions of fractional order integrals and the Fundamental Predictor-Corrector Algorithm for numerical integration of fractional order differential equations. The nonlinear chaotic Genesio-Tesi system is introduced in section III. In section IV, the fractional $PI^{\lambda}D^{\mu}$ controller is presented, and in section V the stabilization problem is studied for different control actions. Concluding remarks are given in section VI.

II. Approximation of Fractional Operator

Research in fractional calculus goes three centuries back in history. The principle theory was initiated mainly around definitions of fractional derivative and integrals. Many reference books [17-18] retrace these mathematical developments. Applications of such concepts in automatic

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control were promoted more recently attracted by the special properties available in fractional order models [1,19].

A. *Basic definitions*

One of the commonly used definitions of the fractional order operators is the Riemann-Liouville (R-L) [17].

The R-L integral of order $\lambda>0$ is defined as:

$$I_{RL}^{\lambda} f(t) = D^{-\lambda} f(t) = \frac{1}{\Gamma(\lambda)} \int_0^t (t-\tau)^{\lambda-1} f(\tau) d\tau \quad (1)$$

and the expression of the R-L fractional order derivative of order $\mu>0$ is:

$$D_{RL}^{\mu} f(t) = \frac{1}{\Gamma(n-\mu)} \frac{d^n}{dt^n} \int_0^t (t-\tau)^{n-\mu-1} f(\tau) d\tau \quad (2)$$

with $\Gamma(.)$ is the Euler's gamma function and the integer n is such that $(n-1) < \mu < n$. This fractional order derivative of equation (2) can also be defined from equation (1) as:

$$D_{RL}^{\mu} f(t) = \frac{d^n}{dt^n} \left\{ I_{RL}^{(n-\mu)} f(t) \right\} \quad (3)$$

B. *The Fundamental Predictor-Corrector Algorithm*

Now a definition of the fractional Adams-Bashforth-Moulton method introduced in [20] is given; as we shall later use it to approximate the fractional order integral operator. In fact it is more practical to use a numerical fractional integration method to compute fractional order integration or derivation as the approximating transfer functions are of relatively high orders.

Consider the differential equation

$$D^{\alpha} y(x) = f\left(x, y(x)\right) \quad (4)$$

with initial conditions:

$$y^{(k)}(0) = y_0^{(k)}, \qquad k = 0,1,\ldots,m-1, \quad (5)$$

where $m=[\alpha]$ and the real numbers $y^{(k)}(0) = y_0^{(k)}$, $k = 0,1,\ldots,m-1$, are assumed to be given.

The basics of this technique take profit of an interesting analytical property: the initial value problem (4), (5) is equivalent to the Volterra integral equation

$$y(x) = \sum_{k=0}^{[\alpha]-1} y^{(k)}(0) \frac{x^k}{k!} + \frac{1}{\Gamma(\alpha)} \int_0^{\infty} (x-t)^{\alpha-1} f\left(t, y(t)\right) dt \quad (6)$$

Introducing the equispaced nodes $t_j = jh$ with some given $h>0$ and by applying the trapezoidal integral technique to compute (6), the corrector formula becomes

$$y_h(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{t_{n+1}}{k!} y^{(k)}(0) + \frac{h^{\alpha}}{\Gamma(\alpha+2)} f\left(t_{n+1}, y_h^P(t_{n+1})\right) + \frac{h^{\alpha}}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, y_h(t_j)\right) \quad (7)$$

where

$$\begin{aligned} a_{0,n+1} &= n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha} \\ a_{j,n+1} &= (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}, \\ &(1 \le j \le n) \end{aligned} \quad (8)$$

and $y_h^P(t_{n+1})$ is given by,

$$y_h^P(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{t_{n+1}}{k!} y^{(k)}(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f\left(t_n, y_h(t_j)\right) \quad (9)$$

where now

$$b_{j,n+1} = \frac{h^{\alpha}}{\alpha} \left((n+1-j)^{\alpha} - (n-j)^{\alpha} \right) \quad (10)$$

III. CHAOTIC GENESIO-TESI SYSTEM

Genesio-Tesi system is defined by the following mathematical model

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -cx - by - az + x^2 \end{cases} \quad (11)$$

When $(a,b,c) = (1.2, 2.992, 6)$ [21], the Genesio-Tesi system presents a chaotic behavior as shown in *Fig. 1.* Initial conditions are the same in [21]: $x(0) = -1.0032$, $y(0) = 2.3445$ and $z(0) = -0.087$.

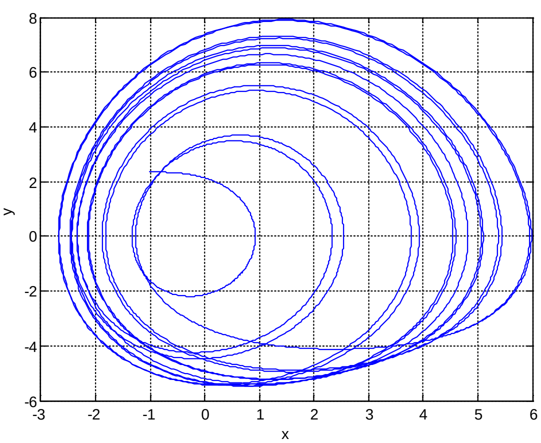

Fig. 1. Phase plane of chaotic Genesio-Tesi system

Many researchers have been attracted by the problem of control and synchronization of Genesio-Tesi nonlinear system [22]. They proposed different strategies for that aim such as adaptive control [23], LMI optimization approach [24], sliding mode control [25], single variable feedback control [26] …etc.

IV. FRACTIONAL PID CONTROLLER

In this section, the proposed fractional calculus-based control strategy that is able to suppress chaotic oscillations in a Genesio-Tesi single input three state chaotic system is presented. The considered control strategy is a fractional order PID-like controller (FOPID) as a state feedback mechanism [3]. A FOPID controller calculates an error value and tries to minimize it by adjusting the process using a manipulated variable. In owner study we have selected each action separately.

The Genesio-Tesi system model is given by:

$$\begin{cases} \dot{x} = y + u_x \\ \dot{y} = z + u_y \\ \dot{z} = -cx - by - az + x^2 + u_z \end{cases} \tag{12}$$

A. Fractional proportional-integral controller

The control law is expressed by the formula,

$$u(t) = k_p X + k_I I^{\lambda} X \tag{13}$$

where λ is the fractional order.

For a suitable choice of k_p and k_I gains we use the bifurcation diagrams presented in *fig. 2* and *fig. 3*. First, we fixe k_I to 2, and traces the evolution of the x variable according to k_p which varied between 0.5 and 0.85. Then, from *fig. 2* we take one value from the interval that ensures stability of the system, and we seek to know the behavior of the system by varying k_I between 1 and 3. In our case we took $k_p = 0.65$

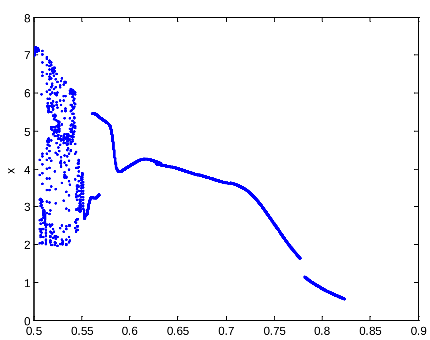

Fig. 2. Bifurcation diagram x=f(kp) Bifurcation diagram x=f(ki)

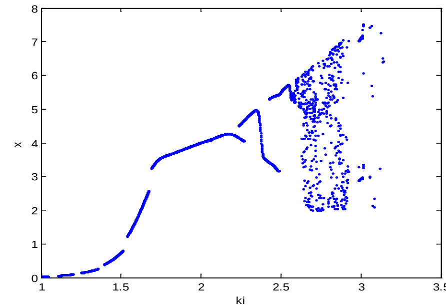

Fig. 3. Bifurcation diagram x=f(ki)

B. Fractional proportional-derivative controller

The control law for the case of fractional proportional-derivative control is given by the expression.

$$u(t) = k_p X + k_d D^{\mu} X \tag{14}$$

Where μ is the fractional order.

Here, we use also bifucation diagram presented in *fig. 4* to determine the gain k_d. Always keeping $k_p = 0.65$.

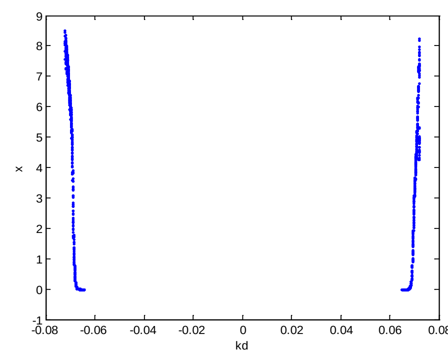

Fig. 4. Bifurcation diagram x=f(kd)

C. Fractional proportional-integral-derivative controller

The fractional proportional-integral-derivative controller applied on Genesio-Tesi system can be schematized as follow.

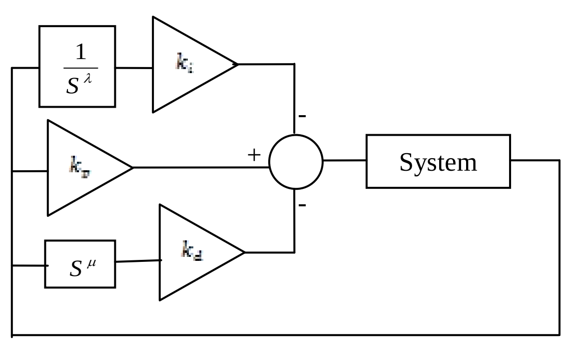

Fig. 5. Controlled system using FOPID

V. SIMULATION RESULTS

In this part, we provide the numerical results to illustrate the effectiveness of the regulator designed to deal with this class of nonlinear systems. Since direct implementation of the fractional order transfer functions is problematic, to implement these transfer functions, integer order approximations of the fractional transfer functions can be used in practical applications [27]. The fractional Adams-Bashforth-Moulton method is used for numerical approximation of the control system using Matlab/ Simulink.

A. Using PI^{λ} controller

The stabilization of the chaotic system on a fixed point or periodic orbit depends essentially on the choice of k_p and k_i gains. For (k_p , k_i) = (0.65, 2), the simulation result is presented in *fig.6*. The control is started at boot.

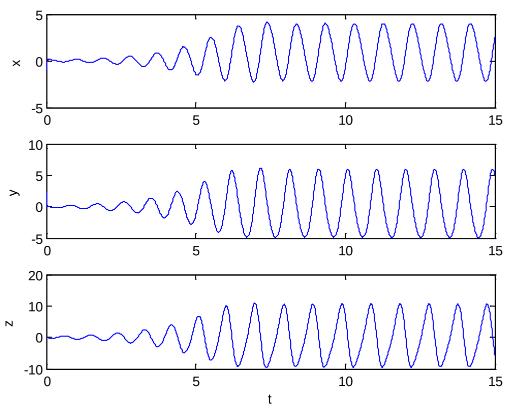

Fig. 6. State variables for (k_p , k_i) = (0.65, 2)

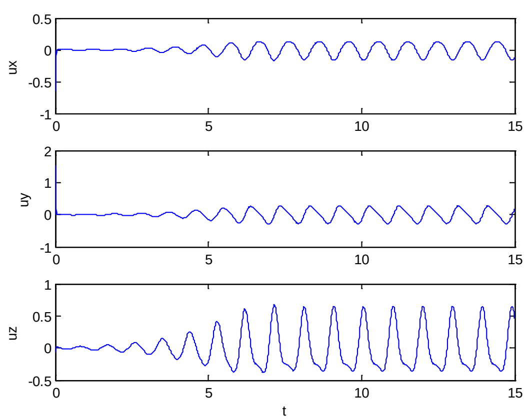

Fig. 7. PI controller signal

After more than 6 second, the controller succeeded to stabilize the system on a periodic orbit presented clearly in the phase plane by fig. 7.

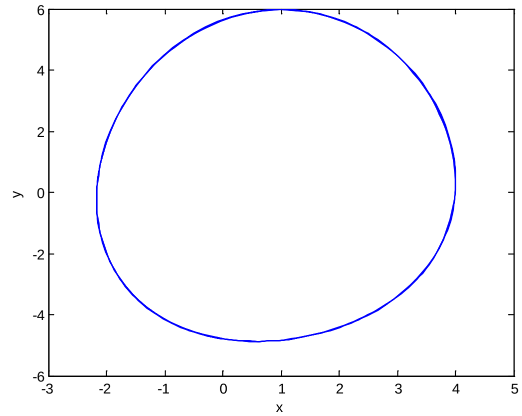

Fig. 8. Phase plane y(t) vs. x(t) from the 10th second

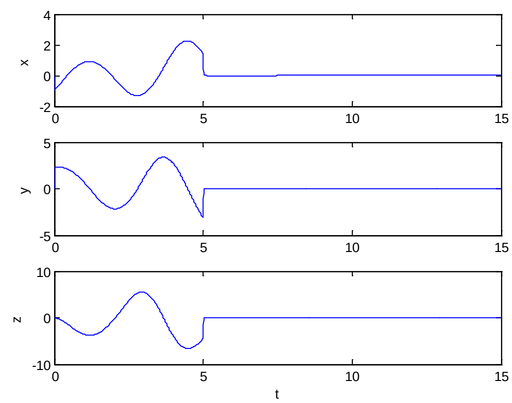

Fig. 9. State variables of controlled Genesio-Tesi system with (k_p , k_d) = (0.65, 0.05)

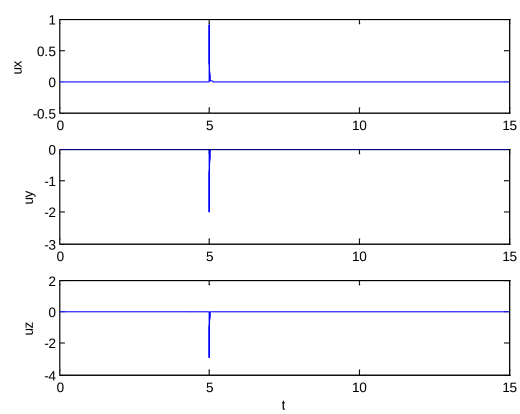

Fig. 10. PD controller signal

B. Using PI^{μ} controller

From the bifurcation diagram shown in the *fig. 4*, is clearly seen that despite the smallness of k_d gain interval, system behavior changes between chaotic and stable states.

From $k_d = -0.066$ to $k_d = 0.067$ the system is stable. Applying the fractional proportional-derivative controller on Genesio-Tesi system with $k_d = 0.05$, the results shown in *fig.9* are obtained.

Here, the control is triggered at instant $t = 5$ sec. The system converges abruptly towards the unstable fixed point$(0,0,0)$.

C. Using $PI^{\lambda}D^{\mu}$ controller

Obtained result using $PI^{\lambda}D^{\mu}$ controller is presented in *fig.11*.

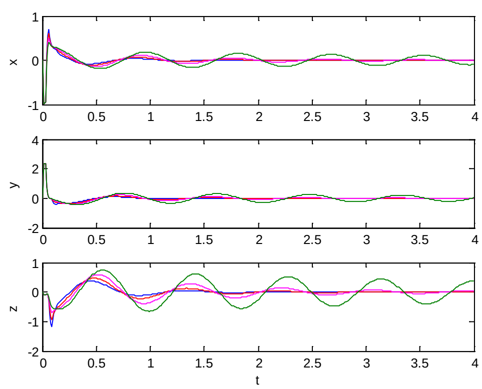

Fig. 11. State variables with different k_d values

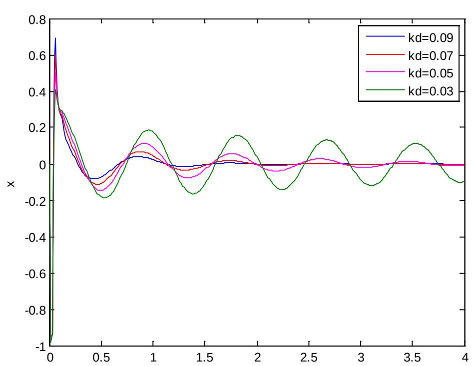

Fig. 12. Zoom of x variable

Here, the value of k_d that garantees the best performance of controller with respect to the overshoot and the response time is $k_d = 0.07$.

VI. CONCLUSION

In this paper, we propose a dynamic system, nonlinear complex Genesio-Tesi system, and study its dynamic properties. We consider the control and chaos suppression in this nonlinear chaotic system using of a fractional order control strategy. A linear fractional order state space feedback controller is designed for the chaotic system stabilization. By means of the bifurcation diagram and the phase portrait, the dynamic behaviors of system are depicted. In particular, the coefficients of the fractional order derivative and integral actions are considered as a bifurcation parameter. The proposed controller has a simple structure and can stabilize some fixed points in the single input 3D chaotic systems.

Stability analysis of the nonlinear chaotic system is studied for the fractional order integral and derivative actions using the bifurcation diagram. We show by numerical simulations that the fractional $PI^{\lambda}D^{\mu}$ controller provides a good closed loop system performance for stabilizing the problem of unstable fixed point.

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