International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 10, N°1, Special Issue AND, April 2016, pp 2091–2096.

Synthesis of a Sliding mode Multi-Observers for Nonlinear Perturbed Output Systems:Application to a denitrification process

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***Abstract*—In this paper we present an approach to design a multi-observers for a nonlinear perturbed output system. This approach is based on the multiple model and the sliding mode theories. the multiple model formulation is obtained by applying the method of the sector nonlinearity transformation to represent nonlinear systems into an equivalent multiple model. The sliding surface is chosen as the error between the estimated and the real system outputs. The convergence of the state estimation error is proved using an appropriate Lyapunov function and the stability conditions are given in terms of Linear Matrix Inequalities (LMIs). The gains of the multi-observer are given by resolving LMIs. An application to a denitrification process is considered and the effectiveness of the proposed multi-observers are illustrated through simulation results.**

Keywords*—Nonlinear systems, uncertain output systems, Multiple models, sliding mode observer (SMO), robust observer.

I. Introduction

Modeling systems is an important step that precedes the design of estimators. The methods of describing nonlinear systems are various and diverse. One of them is the multiple models approach, presented by[1], which is a representation of nonlinear system by an interpolation of linear local models through the activation functions with appropriate index variables.It is an interested tool by its use in many areas like control, estimation and diagnosis[2,3,4]. In fact, it provides a good representation of the dynamic behavior of nonlinear systems and an ease of elaborating estimation algorithm of state variables. In the literature, the obtention of multiple models' structures is presented in different ways [1,5].
On the other side, the sliding mode theory is considered as one of the most important approach in developing estimator schemes, since it shows good performance as the robustness against perturbations. The idea of such theory is based on the concepts of sliding surface and equivalent control. The design of sliding mode observers is well motivated for nonlinear systems [6-12] and linear ones [13-20]. The work [13] treated the problem of estimation through the sliding mode observer based on equivalent control. An other approach, in [14], based on Lyapunov method is established in the estimation of state variables, under matching condition, with the presence of unknown inputs and uncertainties.
The linear tools using sliding mode approach in state estimation was extended in the multiple models form [21-24].
This paper aims to design a robust observer based on multiple models structure for nonlinear systems with an output vector affected by uncertainties or disturbances. This paper is organized as follows.Section II presents the problem formulation. Section III presents the structure of the proposed observer and the convergence analysis. Section IV the multiple sliding mode observer is applied for the estimation of states in denitrification process in the presence of perturbations affecting the output of the system.

II. Problem statement

Consider a nonlinear system described by:

$$\begin{cases} \dot{x} = f(x,u) \\ y = Cx + d \end{cases} \quad (1)$$

Where $x \in R^n$,$u \in R^m$ and $y \in R^p$ are ,respectively, the state vector , the input vector and the output vector of the system.d is the perturbations vector which affects the output and verifying the following property:$\|d\| \leq \bar{d}$.
the system (1) can rewritten using the following multiple models form given by:

$$\begin{cases} \dot{x} = \sum_{1}^{M} \mu_i(x)(A_i x + B_i u) \\ y = Cx + d \end{cases} \quad (2)$$

A_i and B_i are matrices with appropriate dimensions.
$C = [I_{p\times p} 0_{n-p}]$ and verifying $C^{\mathrm{T}}C = I_{p\times p}$
The activation functions $\mu_i(x)$ verify the convex property:

$$\begin{cases} \sum_{1}^{M} \mu_i(x) = 1 \\ 0 \leq \mu_i(x) \leq 1 \end{cases} \quad (3)$$

This paper was recommended for publication in revised form by the Editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

The objective of this paper is to develop a sliding mode observer for the reconstruction of unmeasured variables of a nonlinear system with perturbed output.

III. Design of a multiple sliding mode observer

A. Structure of the observer

The structure of the proposed observer is given by:

$$\begin{cases} \dot{\hat{x}} = \sum_{1}^{M} \mu_i(\hat{x})(A_i\hat{x} + B_i u + L_i(y-\hat{y}) + L_i\alpha_i) \\ \hat{y} = C\hat{x} \end{cases} \quad (4)$$

Where $\hat{x} \in R^n$ is the estimate state vector, $L_i \in R^{n\times p}$ are the estimation gains. α_i represent the sliding mode gains.

Theorem : The error estimation between the system (2) and (4) converges asymptotically to zero if there exist matrix positive definite symmetric $P = P^{\mathrm{T}} > 0$, and F_i satisfying the following constraints

$$\begin{cases} \begin{bmatrix} -A_i^{\mathrm{T}}P - PA_i + C^{\mathrm{T}}W_i + W_i{}^{\mathrm{T}}C - \lambda & P \\ P & I \end{bmatrix} > 0 \\ C^{\mathrm{T}}F_i^{\mathrm{T}} = PL_i \end{cases} \quad (5)$$

And the following conditions are fulfilled :

$$\begin{cases} if r \neq 0, then\ \alpha_i = \bar{d}\dfrac{F_i r}{\| F_i r \|} \\ if r = 0, then\ \alpha_i = 0 \end{cases} \quad (6)$$

where$\lambda = \mu^2$, $L_i = P^{-1}W_i$ and $r = Ce$

B. Stability of the observer

In the proof of theorem, the following lemma is used :

Lemma : for every two matrices X and Y with appropriate dimensions, the following property holds :

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq X^{\mathrm{T}}X + Y^{\mathrm{T}}Y \quad (7)$$

In order to prove the stability of the observer, let's define the state error:

$$e = x - \hat{x} \quad (8)$$

The dynamic of estimation error is given by :

$$\dot{e} = \dot{x} - \dot{\hat{x}} \quad (9)$$

$$\dot{e} = \sum_{1}^{M} \mu_i(\hat{x})((A_i - L_iC)e - L_i d - L_i\alpha_i) + \Delta \quad (10)$$

where:

$$\Delta = \sum_{1}^{M}(\mu_i(x) - \mu_i(\hat{x}))(A_i x + B_i u)$$

it is assumed that Δ verifies the Lipchitz condition:

$$\| \Delta \| \leq \mu \| e \| \quad (11)$$

Considering the following candidate Lyapunov function:

$$V = e^{\mathrm{T}}Pe \quad (12)$$

Using(10), its time derivative is given by :

$$\dot{V} = \dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e}$$

$$\dot{V} = \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}(A_i - L_iC)^{\mathrm{T}}Pe - d^{\mathrm{T}}L_i{}^{\mathrm{T}}Pe - \alpha_i^{\mathrm{T}}L_i{}^{\mathrm{T}}Pe + e^{\mathrm{T}}P(A_i - L_iC)e - e^{\mathrm{T}}PL_id - e^{\mathrm{T}}PL_i\alpha_i + \Delta^{\mathrm{T}}Pe + e^{\mathrm{T}}P\Delta) \quad (13)$$

using the lemma: $\Delta^{\mathrm{T}}Pe + e^{\mathrm{T}}P\Delta \leq e^{\mathrm{T}}PPe + \Delta^{\mathrm{T}}\Delta$

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta - 2e^{\mathrm{T}}PL_id - 2e^{\mathrm{T}}PL_i\alpha_i) \quad (14)$$

Using the constraint:$C^{\mathrm{T}}F_i^{\mathrm{T}} = PL_i$ a new inequality can be obtained:

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta - 2e^{\mathrm{T}}C^{\mathrm{T}}F_i{}^{\mathrm{T}}d - 2e^{\mathrm{T}}C^{\mathrm{T}}F_i{}^{\mathrm{T}}\alpha_i) \quad (15)$$

$r = Ce$

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta - 2r^{\mathrm{T}}F_i{}^{\mathrm{T}}d - 2r^{\mathrm{T}}F_i{}^{\mathrm{T}}\alpha_i) \quad (16)$$

Remembering that d is bounded($\|d\| \leq \bar{d}$). This leads to obtain:

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta + 2\bar{d}\|F_ir\| - 2r^{\mathrm{T}}F_i{}^{\mathrm{T}}\alpha_i) \quad (17)$$

Two cases can be investigated:

1^{rst} **case** :if $r \neq 0$ then:

$$2r^{\mathrm{T}}F_i{}^{\mathrm{T}}\alpha_i - 2\bar{d}\|F_ir\| = 0$$

$$\alpha_i = \bar{d}\frac{F_i r}{\| F_i r \|}$$

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta) \quad (18)$$

2^{nd} **case** :if $r = 0$ then$\alpha_i = 0$

finally, we can obtain the following inequality:

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \Delta^{\mathrm{T}}\Delta) \quad (19)$$

using(11),$\dot{V}$ becomes:

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2)e + \mu^2\|e\|^2)$$

$$\dot{V} \leq \sum_{1}^{M} \mu_i(\hat{x})(e^{\mathrm{T}}((A_i - L_iC)^{\mathrm{T}}P + P(A_i - L_iC) + P^2 + \mu^2 I)e) \tag{20}$$

Using the Schur complement with supposing $\lambda = \mu^2$ and $W_i = PL_i$.The following inequality must be verified to ensure the stability of the observer ($\dot{V} < 0$):

$$\begin{bmatrix} -A_i^{\mathrm{T}}P - PA_i + C^{\mathrm{T}}W_i + W_i{}^{\mathrm{T}}C - \lambda & P \\ P & I \end{bmatrix} > 0 \tag{21}$$

In summary, $V > 0$ and $\dot{V} < 0$, the error estimation converges asymptotically to zero.

IV. Numerical example

A. Description of denitrification process

In this section, the proposed observer approach is applied to a model of denitrification process. The process is described by the following model[25]:

$$\begin{cases} \dot{S}_1 = -y_{11}\mu_1 X + D(S_{1in} - S_1) \\ \dot{S}_2 = (y_{12}\mu_1 - y_{22}\mu_2)X + D(S_{2in} - S_2) \\ \dot{S}_3 = -(y_{13}\mu_1 + y_{23}\mu_2)X + D(S_{3in} - S_3) \\ \dot{X} = (\mu_1 + \mu_2)X - k_d X - DX \end{cases} \tag{22}$$

Where S_1, S_2, S_3 and X are respectively the concentrations of the respective species. S_{1in}, S_{2in} and S_{3in} are the respective supply of S_1, S_2 and S_3 concentrations. k_d is the mortality rate of the microorganisms. D is the dilution rate, and the y_{ij} denote yield coefficients and finally μ_1 and μ_2 are specific growth rates of the biomass respectively acetic acid and nitrite and have the following expressions :

$$\mu_1 = \mu_{1max}\frac{S_3}{(S_3 + k_{S_3})}\frac{S_1}{(S_1 + k_{S_1})}$$
$$\mu_2 = \mu_{2max}\frac{S_3}{(S_3 + k_{S_3})}\frac{S_2}{(S_2 + k_{S_2})}$$

Let's define the state vector x, the input vector u and the output vector y of the system : $x = [S_1 \quad S_2 \quad S_3 \quad X]^{\mathrm{T}}$, $u = [S_{1in} \quad S_{2in} \quad S_{3in}]^{\mathrm{T}}$ and $y = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} + d$

B. Multiple Model form

The multiple model is adopted as an approach in order to design the observer for state estimation. We employ the method of the sector nonlinearity transformation. Considering the process, we define the following nonlinearities as the premise variables :

$$z_1 = D \tag{23}$$

$$z_2 = \frac{S_3}{(S_3 + k_{S_3})}\frac{S_1}{(S_1 + k_{S_1})} \tag{24}$$

$$z_3 = \frac{S_3}{(S_3 + k_{S_3})}\frac{S_2}{(S_2 + k_{S_2})} \tag{25}$$

The nonlinear model can be written in the following quasi-Linear Parameter Variant (LPV) form :

$$\dot{x} = A(z)x + B(z)u \tag{26}$$

With matrix $A(z)$ and $B(z)$ are expressed by using the premise variables:

$$A(z) = \begin{bmatrix} -z_1 & 0 & 0 & a_{14} \\ 0 & -z_1 & 0 & a_{24} \\ 0 & 0 & -z_1 & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \tag{27}$$

$$B(z) = \begin{bmatrix} -z_1 & 0 & 0 \\ 0 & z_1 & 0 \\ 0 & 0 & -z_1 \\ 0 & 0 & 0 \end{bmatrix} \tag{28}$$

where

$$a_{14} = -y_{11}\mu_{1max}z_2$$
$$a_{24} = y_{12}\mu_{1max}z_2 - y_{22}\mu_{2max}z_3$$
$$a_{34} = -y_{13}\mu_{1max}z_2 - y_{23}\mu_{2max}z_3 \tag{29}$$
$$a_{44} = -z_1 + \mu_{1max}z_2 + \mu_{2max}z_3 - k_d$$

Each one of the premise variables can be expressed as :

$$z_j = F_{j1}z_{j1} + F_{j2}z_{j2}, \ for, \ j = 1, 2, 3 \tag{30}$$

where

$$z_{j1} = max\{z_j\}$$
$$z_{j2} = min\{z_j\}$$
$$F_{j1} = \frac{z_j - z_{j2}}{z_{j1} - z_{j2}} \tag{31}$$
$$F_{j2} = \frac{z_{j1} - z_j}{z_{j1} - z_{j2}}$$

The constant matrices A_i and $B_i$defining the 8 submodels are determined by using the matrices $A(z)$ and $B(z)$ and $z_(j,i)$ $i = 1, 2$ and $j = 1, 2, 3$.

$$\begin{aligned} A_1 = A(z_{11}, z_{21}, z_{31}), A_2 = A(z_{11}, z_{21}, z_{32}) \\ A_3 = A(z_{11}, z_{22}, z_{31}), A_4 = A(z_{11}, z_{22}, z_{32}) \\ A_5 = A(z_{12}, z_{21}, z_{31}), A_6 = A(z_{12}, z_{21}, z_{32}) \\ A_7 = A(z_{12}, z_{22}, z_{31}), A_8 = A(z_{12}, z_{22}, z_{32}) \\ B_1 = B(z_{11}), B_2 = B_3 = B_4 = B_1 \\ B_5 = B(z_{12}), B_6 = B_7 = B_8 = B_5 \end{aligned} \tag{32}$$

The activation functions have the following expressions :

$$\begin{aligned} \mu_1(z) = F_{11}F_{21}F_{31}, \mu_2(z) = F_{11}F_{21}F_{32} \\ \mu_3(z) = F_{11}F_{22}F_{31}, \mu_4(z) = F_{11}F_{22}F_{32} \\ \mu_5(z) = F_{12}F_{21}F_{31}, \mu_6(z) = F_{12}F_{21}F_{32} \\ \mu_7(z) = F_{12}F_{22}F_{31}, \mu_8(z) = F_{12}F_{22}F_{32} \end{aligned} \tag{33}$$

Finally the nonlinear model can be written in multiple model form:

$$\begin{cases} \dot{x} = \sum_{1}^{8} \mu_i(x)(A_i x + B_i u) \\ y = Cx + d \end{cases} \tag{34}$$

Table I. INITIAL CONDITIONS

Variables	Values
$S_1(0)$	0.6 g/l
$S_2(0)$	0 g/l
$S_3(0)$	2.77 g/l
$X(0)$	0.15 g/l
$\hat{S}_1(0)$	0.8 g/l
$\hat{S}_2(0)$	0.1 g/l
$\hat{S}_3(0)$	3 g/l
$\hat{X}(0)$	0.2 g/l

Table II. PARAMETERS VALUES

Variables	Values
y_{11}	6.2
y_{12}	3.3
y_{22}	1.2
y_{13}	1.1
y_{23}	1.6
μ_{1max0}	0.17 h^{-1}
μ_{2max0}	0.085 h^{-1}
k_{S_1}	0.05 g/l
k_{S_2}	0.07 g/l
k_{S_3}	0.1 g/l
k_d	0.025 h^{-1}

C. Simulation Results

The initial conditions and parameters values are given, respectively, by tables 1 and 2.

The evolution of the known inputs of the denitrification process are shown by Figure 1:

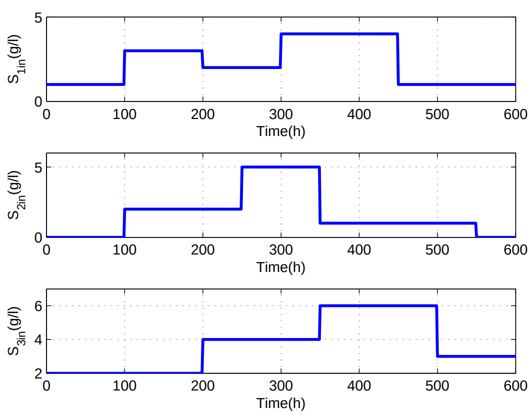

Fig. 1 Evolution of the known inputs

the perturbations vector d has the following expression:

$$d = \begin{bmatrix} 0.02\sin(\frac{2\pi t}{15}) \\ 0.02\sin(\frac{2\pi t}{15}) \\ 0.02\sin(\frac{2\pi t}{15}) \end{bmatrix}$$

The resolution of the inequalities in (5) leads to obtain the scalar $\mu = 0.051$ and the matrices P and L_i:

$$P = \begin{bmatrix} 12.9785 & -0.0045 & 0.0197 & 0.0212 \\ -0.0045 & 12.9569 & 0.0058 & 0.0215 \\ 0.0197 & 0.0058 & 12.9747 & 0.0475 \\ 0.0212 & 0.0215 & 0.0475 & 0.0772 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 112.3997 & -0.0105 & 0.0188 \\ 0.2790 & 112.2738 & 0.1277 \\ 0.4038 & -0.1284 & 112.5503 \\ -151.6356 & 22.9951 & -103.8554 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 112.3981 & -151.6278 & -0.0110 \\ 0.2797 & 112.2504 & 0.0976 \\ 0.4035 & -0.1785 & 112.4834 \\ -151.6278 & 36.7194 & -85.5495 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 112.1589 & 0.1245 & -0.0151 \\ 0.0547 & 112.3908 & 0.0850 \\ -0.1087 & 0.1473 & 112.4632 \\ -9.4321 & -52.6683 & -78.5916 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 112.1679 & 0.0976 & -0.0433 \\ 0.0502 & 112.3700 & 0.0533 \\ -0.1063 & 0.0957 & 112.3963 \\ -9.4971 & -38.9164 & -60.3130 \end{bmatrix}$$

$$L_5 = \begin{bmatrix} 112.4695 & -0.0105 & 0.0189 \\ 0.2795 & 112.3435 & 0.1276 \\ 0.4040 & -0.1285 & 112.6203 \\ -151.6084 & 23.0185 & -103.7999 \end{bmatrix}, \quad L_6 = \begin{bmatrix} 112.4681 & -0.0315 & -0.0111 \\ 0.2798 & 112.3202 & 0.0972 \\ 0.4040 & -0.1785 & 112.5534 \\ -151.5966 & 36.7434 & -85.4895 \end{bmatrix}$$

$$L_7 = \begin{bmatrix} 112.2453 & 0.1169 & -0.0110 \\ 0.0465 & 112.4640 & 0.0823 \\ -0.1047 & 0.1445 & 112.5332 \\ -9.4527 & -52.5975 & -78.5156 \end{bmatrix}, \quad L_8 = \begin{bmatrix} 112.2496 & 0.0916 & -0.0409 \\ 0.0436 & 112.4428 & 0.0514 \\ -0.1047 & 0.0939 & 112.4656 \\ -9.3883 & -38.8725 & -60.1583 \end{bmatrix}$$

The values of F_i solutions of the constraint : $C^T F_i^T = PL_i$

$$F_1 = 10^3 \begin{bmatrix} 1.4556 & -0.0001 & 0.0003 \\ -0.0002 & 1.4552 & 0.0001 \\ 0.0003 & 0.0001 & 1.4554 \end{bmatrix}, \quad F_2 = 10^3 \begin{bmatrix} 1.4556 & -0.0001 & 0.0002 \\ -0.0001 & 1.4552 & 0.0001 \\ 0.0003 & 0.0001 & 1.4554 \end{bmatrix}$$

$$F_3 = 10^3 \begin{bmatrix} 1.4556 & 0.0000 & 0.0004 \\ -0.0000 & 1.4551 & 0.0001 \\ 0.0004 & 0.0001 & 1.4554 \end{bmatrix}, \quad F_4 = 10^3 \begin{bmatrix} 1.4556 & -0.0001 & 0.0004 \\ -0.0001 & 1.4551 & -1.8677 \\ 0.0004 & 1.8677 & 1.4554 \end{bmatrix}$$

$$F_5 = 10^3 \begin{bmatrix} 1.4565 & -0.0001 & 0.0003 \\ -0.0000 & 1.4561 & 0.0001 \\ 0.0003 & 0.0001 & 1.4563 \end{bmatrix}, \quad F_6 = 10^3 \begin{bmatrix} 1.4565 & -0.0001 & 0.0003 \\ -0.0001 & 1.4561 & 0.0001 \\ 0.0003 & 0.0001 & 1.4563 \end{bmatrix}$$

$$F_7 = 10^3 \begin{bmatrix} 1.4565 & -0.0001 & 0.0003 \\ -0.0001 & 1.4561 & 0.0001 \\ 0.0003 & 0.0001 & 1.4564 \end{bmatrix}, \quad F_8 = 10^3 \begin{bmatrix} 1.4566 & -0.0001 & 0.0004 \\ -0.0001 & 1.4561 & 0.0000 \\ 0.0004 & 0.0000 & 1.4563 \end{bmatrix}$$

The simulations are carried out with variations of some parameters given by the following figure:

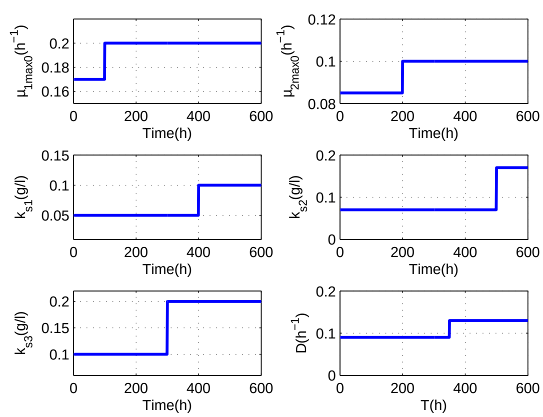

Fig. 2 Parameters variations

First, the observer is applied without the robust term, and the estimation performance is shown in Fig.1-4. It is shown that the estimation is not good specially for the unmeasured

state$X$. Then, the proposed robust observer is applied, and the improved estimation performance is shown in Fig.5-8.
Each sliding mode gain α_i is modified as follows :

$$\begin{cases} if r \neq 0, then\ \alpha_i = \bar{d}\dfrac{F_i r}{\| F_i r \| + \delta} \\ if r = 0, then\ \alpha_i = 0 \end{cases} \tag{35}$$

The parameter δ is a small scalar and it is used in order to smooth out the discontinuity.

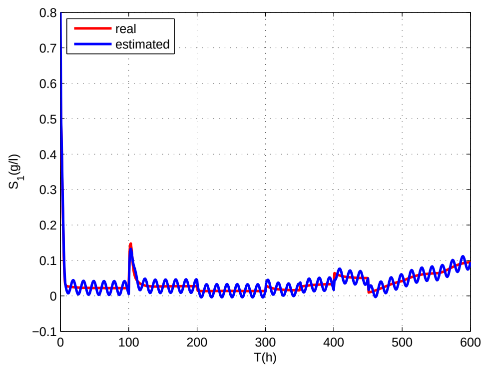

Fig. 3 Evolution of S_1 and $\hat{S}_1$ in the absence of robust term

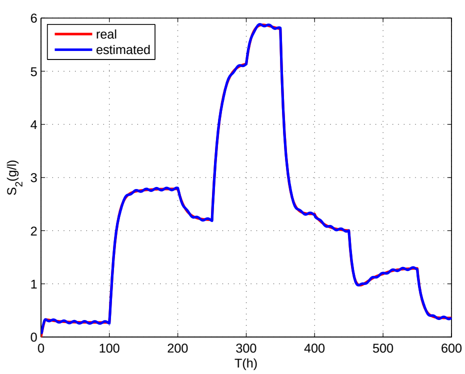

Fig. 4 Evolution of S_2 and $\hat{S}_2$ in the absence of robust term

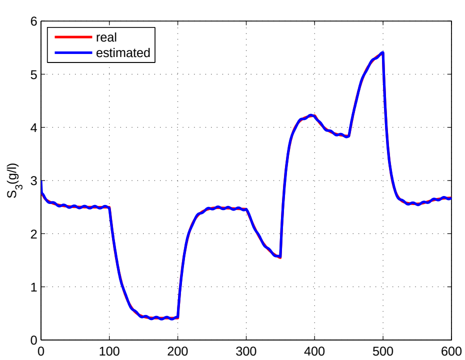

Fig. 5 Evolution of S_3 and $\hat{S}_3$ in the absence of robust term

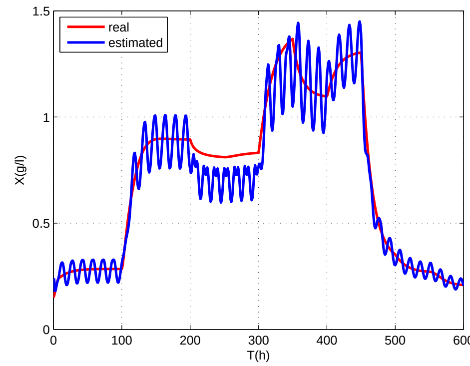

Fig. 6 Evolution of X and $\hat{X}$ in the absence of robust term

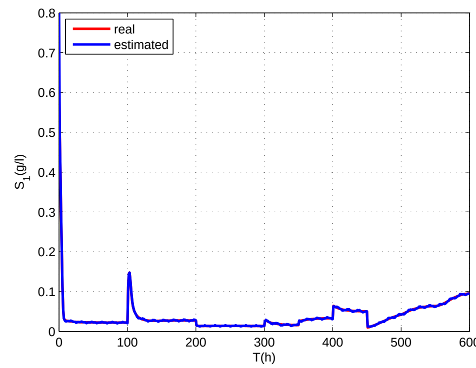

Fig. 7 Evolution of S_1 and $\hat{S}_1$ with robust term

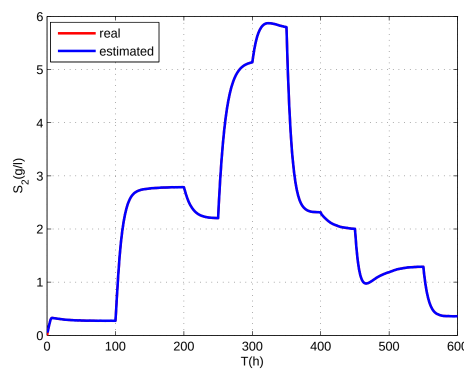

Fig. 8 Evolution of S_2 and $\hat{S}_2$ with robust term

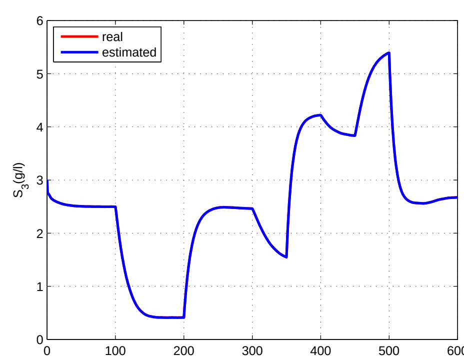

Fig. 9 Evolution of S_3 and $\hat{S}_3$ with robust term

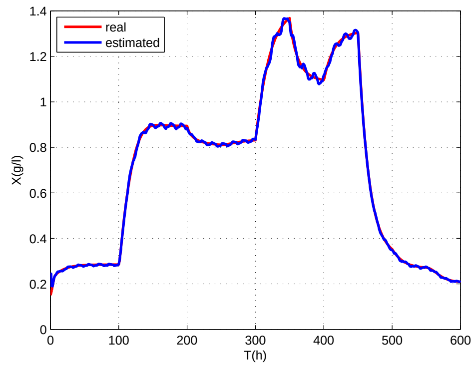

Fig. 10 Evolution of X and $\hat{X}$ with robust term

V. CONCLUSION

In this work, a sliding mode multi-observers for nonlinear output perturbed systems is developed. The nonlinear system is consistently represented by a multiple model. The stability is studied with the Lyapunov theory that allows to derive conditions ensuring the convergence of the state estimation error. The existence conditions are expressed in terms of LMIs that can be solved with LMIS Toolbox in Matlab in order to obtain observer gains. The synthesized multi-observers is applied to a denitrification process. From the simulation results, it is clear that the discontinuous term is able to eliminate an output perturbation.

REFERENCES

[1] R. Murray-Smith, T. A. Johansen, *Multiple model approaches to nonlinear modelling and Control*. Taylor and Francis, 1997.

[2] S. Ben Attia,A. Messeaoud, M. Ltaief,R. Ben Abdennour,*Synthesis of multi-observers for discrete-time nonlinear systems with delayed output*. International Journal of Sciences and Techniques of Automatic Control, vol. 8, no. 1, pp.1966–1981,2014.

[3] M. Besbes, E. Maherzi,M. Ellouze ,R. Mhiri,*Stabilization of uncertain polytopic system by switched command law:multiobserver based*. International Journal of Sciences and Techniques of Automatic Control, vol. 2, no. 1, pp.368–385,2008.

[4] T. Zouari,K. Laabidi,M. Ksouri,*Multimodel Approach Applied for Failure Diagnosis*. International Journal of Sciences and Techniques of Automatic Control, vol. 8, no. 1, pp.500–515,2008.

[5] A .M . Nagy, G. Mourot, B. Marx, G. Schutz, J. Ragot, *Model structure simplification of a biological reactor*, 15th IFAC Symposium on System Identification, Saint-Malo, France,vol. 15,no. 1, pp. 257–262, 2009.

[6] Y. Xiong and M. Saif,*Sliding mode observer for uncertain systems. II.Nonlinear case* IEEE.Conference on Decision and Control, Sedney, Australia, vol.1, pp. 322–327,December, 2000.

[7] Y. Xiong and M. Saif,*Sliding mode observer for nonlinear uncertain systems*, IEEE Trans. Automatic Control, vol.46, no.12, December 2001.

[8] W. Chen and M. Saif,*Robust fault detection in uncertain nonlinear systems via a second order mode observer*, ÒIEEE. Conference on Descision and Control, Orlando, Florida, vol.1, pp. 573–578,December, 2001.

[9] K.C. Veluvolu, Y.C. Soh and W. Cao, *Robust observer with sliding mode estimation for nonlinear uncertain systems*,IET Control Theory Appl., vol.1, no.5, pp. 1533–1540, September 2007.

[10] N. Patel, C. Edwards and S.K. Spurgeon, *An analysis of two nonlinear observers in the presence of noise*, Proceeding of the American Control Conference, Washington, pp. 242–247,June 2008.

[11] K.C. Veluvolu, Y.C. Soh,*Multiple sliding mode observers and unknown input estimations for Lipchitz nonlinear systems*, International Journal of Robust and Nonlinear Control, vol.21, n11, pp. 1322–1340, 2011.

[12] S. Drakunov and V. Utkin, *Sliding Mode Observers. Tutorial*. IEEE. Decision and Control, New Orleans, vol.4, pp. 3376–3378, December 1995.

[13] Y. Xiong and M. Saif, *Sliding mode observer for uncertain systems. I. Linear systems case*, IEEE.Conference on Decision and Control, Sedney, Australia, pp. 317–321, December 2000.

[14] S. Hui and S.H. Zak, *Observer design for systems with unkown inputs*, International J. Appl. Math.Comput.Sci, vol.15, no.4, pp. 431–446, 2005.

[15] K.C. Veluvolu, Y.C. Soh, W. Cao and Z..Y. Liu, *Observers with multiple sliding modes foe uncertain linear MIMO systems*, IEEE. Conference on Industrial Electronics and Applications, Singapore, pp. 1–6, May 2006.

[16] T. Floquet, C. Edwards and S.K. Spurgeon, *On sliding mode observers for systems with unknown inputs*, International Journal of Adaptive Control and Signal Processing, vol. 21, Italy, pp. 638–656, 2007.

[17] K. Kalsi, S. Hui and S.H. Zak, *unknown input and sensor fault estimation using sliding mode observers*, Proceeding of the American Control Conference, San Fransisco, pp. 1364–1369, June 2011.

[18] S. Dhahri, F. Ben Hmida, A. Sellami, M. Gossa, *LMI-based sliding-mode observer design method for reconstruction of actuator and sensor faults*.hskip 1em plus 0.5em minus 0.4emInternational Journal of Sciences and Techniques of Automatic Control, vol. 1, no. 1, pp. 91-107,2007.

[19] W.L. Liu, *Adaptive sliding mode observer design for a class of uncertain systems*,International Journal of Adaptive Control and Signal Processing, vol. 28, pp. 1341–1356, 2014.

[20] A. Akhenak, M. Chadli, D. Maquin and J. ragot, *Sliding mode multiple observer for fault detection and isolation*. IEEE. Decision and Control, Maui, Hawaii vol.1, pp. 953–958, December 2003.

[21] A. Akhenak, M. Chadli, D. Maquin and J. ragot, *Design of robust observer for uncertain Takagi-Sugeno models*. IEEE. International Conference on Fuzzy System, Budapest, Hungary vol.3, pp. 1327–1330 , July 2004.

[22] A. Akhenak, M. Chadli, D. Maquin and J. ragot, *State estimation of uncertain multiple model with unkown inputs*. IEEE. Decision and Control, Atlantis, Bahamas vol.4, pp. 3563–3568, December 2004.

[23] A. Akhenak, M. Chadli, D. Maquin and J. ragot, *Design of sliding mode unknown input observer for uncertain Takagi-Sugeno model*. IEEE. Decision and Control, Athens, Greece, pp. 1–6, June 2007.

[24] A. Akhenak, M. Chadli, D. Maquin and J. ragot, *fault detection and isolation using sliding mode observer for uncertain Takagi-Sugeno fuzzy model*. IEEE. 16th Mediterranean Conference on Control and Automation, Ajaccio, France, pp. 286–291, June 2008.

[25] K. Dahech, T. Damak, A. Toumi, *Joint Estimation of State variables and Kinetics Parameters of denitrification Process*.hskip 1em plus 0.5em minus 0.4emInternational Journal of Information and Systems Sciences, vol. 3, no. 1, pp. 54–66,2007.