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Static Output-feedback controller design for Two-Dimensional Roesser models*

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Abstract This paper focuses on the stabilization of two-dimensional discrete-time systems described by Roesser models via static output-feedback control. The existence of a static output-feedback control law is given by a new stability condition. It is proved that the proposed condition is equivalent to classical Lyapunov condition for stability analysis. Then, it is shown that the proposed condition leads to a sufficient solution to build a stabilizing static output-feedback controller which is less conservative than the Lyapunov-based condition. A design examples illustrate the applicability of the proposed approach.

Keywords 01 *2-D systems, static output feedback, LMI.*

1 Introduction

Two-dimensional (2-D) systems have attracted the attention of many scholars in the past years. Due to their effectiveness in describing systems in several modern engineering fields (image data processing and transformation, water stream heating, thermal processes, etc). The 2-D state-space theory was introduced by Roesser [3, 4]. Since then, several other works have appeared [5], [6], [7] and so far the use of 2-D systems do not cease to increase [12].

Based on these works, several properties concerning 2-D systems such as controllability, observability [4] and realization [9] have been investigated. This paper concentrates on stabilization of 2-D systems. In fact, the stability of 2-D systems using the 2-D Lyapunov equation has already been studied in [10, 11], while the state and output-feedback stabilization problem is treated in [13], by solving a set of 2-D polynomial equations. Further, most of the available works in the literature of 2-D systems consider only state-feedback stabilization [14, 15], or dynamic output-feedback control [16–18]. However, state-feedback controllers require the measurement of every state, some of which may be difficult to measure. On the other hand, dynamic output-feedback controllers (which include systems with state observers) result in high order controllers which may not be practical in real applications. Instead, the static output-feedback controllers are less expensive to implement and more reliable so they will be studied in this paper. In 2-D systems area, static output-feedback stabilization problem is not fully

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investigated and still not completely solved, because the major difficulty of designing stabilizing static output-feedback controllers is due to the non-convexity of the static output-feedback solution set [19, 20].

Thus, in this paper the stabilization for 2-D discrete-time systems is considered. The purpose is the design of static output-feedback controllers such that the resulting closed-loop system is asymptotically stable. In particular, two stability conditions are discussed, the second one is a new stability condition in which a slack variable is introduced to add more degrees of freedom. It is shown that for analysis only both stability conditions are equivalent. However, the second one leads to a sufficient solution in terms of LMI to find a stabilizing controllers gain which is less conservative than the solution based on first stability condition.

This paper is organized as follows: Section 2 presents a short description of discrete-time 2-D system described by Roesser model, and the problem formulation. Section 3 is dedicated to the stability analysis. Section 4 is interested in finding static output-feedback controllers using an LMI-based approach. Section 5 presents two examples to show the applicability of the proposed approach.

2 Problem formulation and Preliminaries

The following discrete-time 2-D system in Roesser models [4] are considered throughout the paper:

$$\begin{cases} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bu(i,j), \\ y(i,j) = C \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, \end{cases} \tag{1}$$

where $x^h(i,j) \in R^{n_1}$ is the horizontal state vector, $x^v(i,j) \in R^{n_2}$ is the vertical state vector, $u(i,j) \in R^m$ is the input vector, $y(i,j) \in R^l$ is the output vector; $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ and $C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$ are constant matrices of appropriate dimensions.

Consider the following unforced system:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, \tag{2}$$

this 2-D discrete-time system is asymptotically stable if and only if, its characteristic polynomial $\mathcal{C}(z_1, z_2)$ has no zeros inside the closed unit bi-disc $\overline{\mathcal{D}}^2 = \{(z_1, z_2) : |z_1| \leq 1, |z_2| \leq 1\}$.

Hence, the stability of 2-D discrete-time systems can be determined by checking the stability of the 2-D characteristic polynomial in the variables z_1 and z_2 [8, 23]. This appears to be difficult to use for control synthesis. In the literature, it has been shown that the stability analysis using Lyapunov functions is still efficient to derive sufficient conditions guaranteeing the asymptotic stability for 2-D discrete systems [7]. The well-known Lyapunov inequality to test the stability of the 2-D discrete system in (2) is given in the following condition [7].

Lemma 1. *[7] The 2-D discrete-time system* (2) *is asymptotically stable if there exists matrices* $P_h > 0$ *and* $P_v > 0$ *satisfying the following LMI:*

$$A^T PA - P < 0, \tag{3}$$

where $P = diag(P_h, P_v)$.

Now, let us consider the 2-D discrete-time system (1) with the following static-output controller

$$u(i, j) = Ky(i, j), \tag{4}$$

where K is the controller gain to be determined, $K \in R^{m \times l}$.
By applying this control law to the Roesser model described by (1), the following closed-loop system is obtained:

$$(\Sigma_c) : \begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = (A + BKC) \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}. \tag{5}$$

Then, the static output-feedback stabilization problem to be addressed in this paper can be formulated as follows: given a 2-D discrete time system described by (1), determine a static output-feedback controller in the form of (4) such that the resulting closed-loop system (5) is asymptotically stable.

3 Stability analysis

In this section, the stability of the 2-D discrete time systems described by Roesser models is investigated using the Lyapunov theory. We recall the following Lemma which is useful in this purpose.

Lemma 2. *[22] Let* Ψ, M *and* R *be matrices of appropriate dimensions. Let* $\mathcal{N}_M$ *and* $\mathcal{N}_R$ *be the orthogonal complements of* M *and* R*, respectively. Then, the following propositions are equivalent:*

i) $\Psi + MXR^T + RX^T M^T < 0$,
ii) $\mathcal{N}_M^T \Psi \mathcal{N}_M < 0, \quad \mathcal{N}_R^T \Psi \mathcal{N}_R < 0$.

Now, consider the closed-loop system (Σ_c).

$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = A_c \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}, \tag{6}$$

where $A_c \equiv A + BKC$.
By applying the stability condition for the unforced system (Lemma 1) to (6), the stability condition guaranteeing the asymptotic stability of the closed-loop system (6) can be derived as follows.

Theorem 1. *If there exist matrices* $P = diag(P_h, P_v) > 0$ *and* K *such that:*

$$A_c^T PA_c - P < 0, \tag{7}$$

where $A_c \equiv A + BKC$*, then closed-loop system (6) is asymptotically stable.*

In the sequel a new sufficient stability condition which is equivalent to the one given by Theorem 1 is provided using direct Lyapunov method and Lemma 2.

Theorem 2. *Consider the 2-D discrete-time system (6). If there exist a block-diagonal matrix $Q = diag(Q_h, Q_v) > 0$, $Q_h \in R^{n_h \times n_h}$, $Q_v \in R^{n_v \times n_v}$ and matrices K, V such that*

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^T A_c^T & \frac{1}{2}V + V^T - Q \\ A_c V & -Q & -A_c V \\ \frac{1}{2}V^T + V - Q & -V^T A_c^T & -V - V^T \end{bmatrix} < 0, \qquad (8)$$

where $A_c \equiv A + BKC$, then the closed-loop system (6) is asymptotically stable.

Proof. Regarding the previous results, the asymptotic stability of closed-loop system (6) is guaranteed by the condition (7) of Theorem 1. Then, it is sufficient to proof that the conditions of Theorem 1 and Theorem 2 are equivalent. Thus, multiplying the right and the left of condition (7) by P^{-1} and taking $Q = P^{-1}$ in the resulting inequality, we have to show the following equivalence:

$$QA_c^T Q^{-1} A_c Q - Q < 0 \Leftrightarrow \begin{bmatrix} -\frac{1}{2}(V+V^T) & V^T A_c^T & \frac{1}{2}V + V^T - Q \\ A_c V & -Q & -A_c V \\ \frac{1}{2}V^T + V - Q & -V^T A_c^T & -V - V^T \end{bmatrix} < 0. \qquad (9)$$

For this, let us start by rewriting inequality (8) as follows:

$$\begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}I \\ -A_c \\ -I \end{bmatrix} V \begin{bmatrix} -I & 0 & I \end{bmatrix} + \begin{bmatrix} -I \\ 0 \\ I \end{bmatrix} V^T \begin{bmatrix} \frac{1}{2}I & -A_c^T & -I \end{bmatrix} < 0. \qquad (10)$$

Next, respecting the notation in Lemma 2, let $M^T = \begin{bmatrix} \frac{1}{2}I & -A_c^T & -I \end{bmatrix}$, $R^T = \begin{bmatrix} -I & 0 & I \end{bmatrix}$ and explicit the orthogonal complements of M and R ($\mathcal{N}_M$ and $\mathcal{N}_R$ respectively):

$$\mathcal{N}_M = \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A_c^T \end{bmatrix}, \quad \mathcal{N}_R = \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}. \qquad (11)$$

Applying Lemma 2, we obtain:

$$\mathcal{N}_M^T \Psi \mathcal{N}_M = \begin{bmatrix} I & 0 & \frac{1}{2}I \\ 0 & I & -A_c \end{bmatrix} \begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A_c^T \end{bmatrix} = \begin{bmatrix} -Q & QA_c^T \\ A_c Q & -Q \end{bmatrix},$$

and

$$\mathcal{N}_R^T \Psi \mathcal{N}_R = \begin{bmatrix} I & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix} = \begin{bmatrix} -2Q & 0 \\ 0 & -Q \end{bmatrix}.$$

Now, assume that the condition in the left-hand of the equivalence (9) holds. Taking its Schur complement we obtain: $\begin{bmatrix} -Q & QA_c^T \\ A_c Q & -Q \end{bmatrix} < 0$. So, it is obvious to see that if the

stability condition (7) in Theorem 1 holds, then by using Lemma 2 the condition (8) in Theorem 2 holds.
Reciprocally, assume that (8) holds. Using again Lemma 2, and according to the above discussion, it can be seen that (7) holds, completing the proof.

Remark 1. Note that the condition (7) can be recovered by imposing $V = Q$ and letting $Q^{-1} = P$ in condition (8). However, it is shown that both the stability conditions provided by Theorems 1 and 2 are equivalent, so they present the same level of conservatism. As it will be seen in the next sections, the contribution of Theorem 2 in addition to prove the equivalence with the condition (7) in Theorem 1, is to propose a condition which is less conservative when the output-feedback control synthesis is considered.

4 LMI-based stabilization solution

This part is interested in finding static output-feedback controllers, such that the closed-loop of 2-D discrete-time system is asymptotically stable. For this goal, it is proposed to employ a direct procedure based on the change of variables technique, which has been shown to be very efficient for solving related control problems (such as state-feedback controllers).

Hence, the synthesis of static output-feedback controllers can be reduced, using Theorems 1 and 2, to find P and K such that:

$$\begin{bmatrix} P & (A+BKC)^T P \\ P(A+BKC) & P \end{bmatrix} > 0, \tag{12}$$

or equivalently, to find V, K and Q such that:

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^T(A+BKC)^T & \frac{1}{2}V+V^T-Q \\ (A+BKC)V & -Q & -(A+BKC)V \\ \frac{1}{2}V^T+V-Q & -V^T(A+BKC)^T & -V-V^T \end{bmatrix} < 0. \tag{13}$$

It should be noted that in general the problem of solving numerically (12) for (K, P) or (13) for (K, P, V) is non-convex in general. This makes the output-feedback control problem difficult to solve.
However, the introduction of a new slack variable G in (13), such that $KCV = KGC$ leads to a convex sufficient condition in terms of LMI as in the following Theorem:

Theorem 3. *Consider the 2-D system (5); if there exist a block-diagonal matrix $Q = diag(Q_h, Q_v) > 0$ with $Q_h \in R^{n_1 \times n_1}$ and $Q_v \in R^{n_2 \times n_2}$, V, G and Y such that*

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^TA^T+C^TY^TB^T & \frac{1}{2}V+V^T-Q \\ AV+BYC & -Q & -(AV+BYC) \\ \frac{1}{2}V^T+V-Q & -V^TA^T-C^TY^TB^T & -V-V^T \end{bmatrix} < 0, \tag{14}$$

$$G+G^T > 0, \tag{15}$$

$$CV = GC, \tag{16}$$

then the control law $u(i,j) = Ky(i,j)$, where $K = YG^{-1}$, stabilizes asymptotically the closed-system (6).

Remark 2. Note that in Theorem 3, the Lyapunov matrix $Q = P^{-1}$ has to satisfy only the stability condition, so there are more degree of freedoms. Moreover, the equality constraint (16) is satisfied by the slack variable G.

Remark 3. Note that the invertibility of matrix G is guaranteed by the condition $G + G^T > 0$.

5 Numerical Examples

Two examples are now considered to show the applicability of the proposed approach.

5.1 First example:

Consider the following 2D Roesser model of the form 1:

$$A = \left[\begin{array}{ccc|cc} 0.2156 & 0.4832 & 0.2301 & 0.5780 & 0.9791 \\ 0.4390 & 0.9834 & 0.0235 & 0.1234 & 0.4585 \\ 0.7395 & 0.7219 & 0.7538 & 0.9841 & 0.0737 \\ \hline 0.0299 & 0.6821 & 0.8434 & 0.8563 & 0.0523 \\ 0.2393 & 0.6449 & 0.1974 & 0.4616 & 0.5539 \end{array}\right], B = \left[\begin{array}{ccc} 0.2092 & 0.7572 & 0.7605 \\ 0.8062 & 0.0939 & 0.8473 \\ 0.5108 & 0.4564 & 0.5678 \\ \hline 0.3766 & 0.0206 & 0.2455 \\ 0.0441 & 0.1583 & 0.7634 \end{array}\right],$$

$$C = \left[\begin{array}{ccc|cc} 0.2534 & 0.2092 & 0.6996 & 0.5146 & 0.3776 \\ 0.8883 & 0.9392 & 0.65 & 0.7429 & 0.5017 \end{array}\right].$$

In this example, the open-loop system is unstable (matrix A_{11} contains an eigenvalue outside the unit circle given by 1.7). The purpose is to design a static output-feedback controller such that the closed-loop system is asymptotically stable.
Using the proposed Theorem 3, a feasible solution can be obtained, such as the following:

$$Q_1 = \begin{bmatrix} 2365.3 & 72.7 & -155 \\ 72.7 & 2264.4 & -144.8 \\ -155 & 144.8 & 2136.8 \end{bmatrix}, Q_2 = \begin{bmatrix} 2560.9 & 108.4 \\ 108.4 & 2147.4 \end{bmatrix},$$

$$V = \left[\begin{array}{ccc|cc} 1328.2 & 46.3 & 45.4 & 21.8 & 66.1 \\ 31.4 & 1.3554 & -46.0 & -35.7 & -36.9 \\ -161.7 & -116.3 & 1194.1 & -182.8 & -47.2 \\ \hline -22.2 & -85.8 & -126 & 1349.4 & 15.9 \\ 63.4 & 28.5 & 184.8 & 78.4 & 1207.5 \end{array}\right],$$

$$Y = \begin{bmatrix} 148.2 & -672 \\ -1778.6 & 0.4562 \\ 840.5 & -1016.9 \end{bmatrix}, G = \begin{bmatrix} 1292.9 & -95.8 \\ -91 & 1286.3 \end{bmatrix}.$$

Then, the gain of a stabilizing static output-feedback controller is:

$$K = YG^{-1} = \begin{bmatrix} 0.0782 & -0.5166 \\ -1.3578 & 0.2535 \\ 0.5976 & -0.7461 \end{bmatrix}.$$

5.2 Second example

This example deals with stabilization processes in a Darboux equation (Marszalek, 1984) and is considered by many works treating 2-D systems [2]. The Darboux equation corresponds to the following PDE:

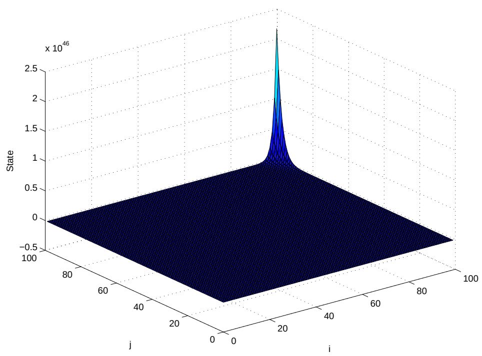
$$\frac{\partial^2 s(x,t)}{\partial x \partial t} = a_1 \frac{\partial s(x,t)}{\partial t} + a_2 \frac{\partial s(x,t)}{\partial x} + a_0 s(x,t) + bf(x,t), \tag{17}$$

where $s(x,t)$ is an unknown function at space $x \in [0, x_f]$ and time $t \in [0, \infty)$, a_0, a_1, a_2 and b are real coefficients, and $f(x,t)$ is the input function.
In [2] the partial differential equation (PDE) model (17) was converted into a 2-D Roesser model of the form (1), where:
$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \Delta x & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} a_1 & a_1 a_2 + (a_0 + 0.7\delta) \\ 1 & a_2 \end{bmatrix}$,
$B = \begin{bmatrix} \Delta x & 0 \\ 0 & \Delta t \end{bmatrix} \begin{bmatrix} b + 0.1\delta \\ 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 10 \end{bmatrix}$.
Following [2], let $a_0 = 0.2, a_1 = -3, a_2 = -1, b = 0.3, \delta = 0.03^{-1} \Delta x = 0.1$ and $\Delta t = 0.5$, which gives an unstable open-loop system, see (Fig. 1(a)).
A feasible solution of the LMI of Theorem 3 gives:
$Q_1 = 987.8379; Q_2 = 1019.4; V = \begin{bmatrix} 565.9338 & -76.3366 \\ 1.8807 & 592.3742 \end{bmatrix}$;
$Y = -424.8591; G = 584.7406$.

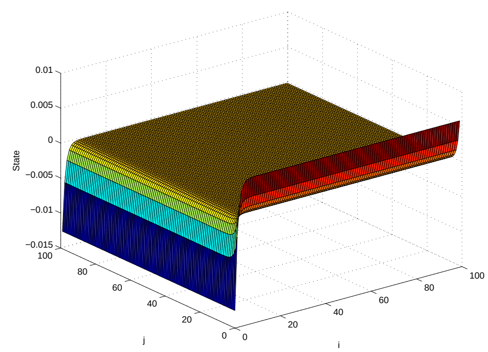
Thus, the gain of a stabilizing controller is: $K = YG^{-1} = -0.7266$.
Simulation result using this controller gain is shown in Fig. 1(b). It can be seen that the closed-loop system is stabilized. The state response x^v is similar, so it is omitted.

6 Conclusions

This paper has proposed a solution for the static output stabilization problem of 2-D systems described by Roesser models. It has been shown that this solution can be recast as a convex optimization under LMI form. The proposed approach is systematic for controller design of 2-D systems. Examples are given to demonstrate the applicability of the proposed methodology. This result can be extended to more complex systems (system with uncertainties, H_∞, etc) and can also open a new avenue in designing controllers for 2-D systems.



(a)



(b)

Figure1. (a) The open-loop response of x^h, (b)The closed-loop response of x^h.

References

1. M. Nachidi, A. Benzaouia, F. Tadeo and M. Ait Rami, "LMI-based approach for output-feedback stabilization for discrete time Takagi-Sugeno systems", *45th IEEE Conference on Decision and Control*, San Diego, pp. 6301-6306, 2006.
2. C. Du, L. Xie, C. Zhang, "H_∞ control and robust stabilization of two-dimensional systems in Roesser models," *Automatica,* vol. 37, pp. 205-211, 2001.
3. D. D. Givone and R. P. Rosser, "Multidimensional linear iterative circuits-general properties," *IEEE Transaction on Computers,* vol. 21, no. 10 pp. 1067-1073, 1972.
4. R. Roesser, "A discrete state-space model for linear image processing", *IEEE Transactions on Automatic Control,* vol. AC-20. pp.1-10, 1975.
5. S. Attasi, "Systmes linéaire homogènes à deux indices", *IRIA Rapport Labria,* no. 31, 1973.
6. E. Fornasini and G. Marchesini, "Doubly-indexed dynamical systems: state-space models and structural properties", *Mathematical Systems Theory,* vol. 1, no. 1, 1978.
7. T. Kaczorek, *Two-Dimensional linear systems,* Springer-Verlag, 1985.
8. E. Rogers, D. H. Owens, *Stability analysis for linear repetitive processes,* vol. 175. Lecture notes in control and information sciences series. Springer-Berlin, 1992.
9. R. Eising, "Realization and stabilization of 2-D systems," *IEEE Transactions on Automatic Control,* vol. 23, no. 5, pp.793-799, 1978.
10. P. Agathoklis, E. Jury, and M. Mansour, "The discrete-time strictly bounded-real lemma and the computation of positive definite solutions to the 2D Lyapunov equation," *IEEE Transactions on Circuits and Systems*, vol. 36, pp. 830–837, 1989.
11. B. D. O. Anderson, P. Agathoklis, E. I. Jury, and M. Mansour, "Stability and the matrix Lyapunov equation for discrete 2-dimensional systems," *IEEE Transactions on Circuits and Systems*, vol. 33, pp. 261-267, 1986.
12. N.E. Mastorakis and M. Swamy, "A new method for computing the stability margin of two-dimensional continuous systems", *IEEE Transactions on Circuits and Systems*, vol. 49, 869-872, 2002.
13. M. Bisiacco, "State and output feedback stabilizability of 2-D systems," *IEEE Transactions on Circuits and Systems*, vol. 32, 1246-1254, 1984.
14. S. Xu, J. Lam, Z. Lin, and K. Galkowski, "Positive Real Control for Uncertain Two-Dimensional Systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and application*, vol. 49, no. 11, 2002.

15. C. Du, and L. Xie, "Stability analysis and stabilization of uncertain two-dimensional discrete systems: An LMI approach," *IEEE Transactions Circuits and Systems I*, vol. 46, pp. 1371-1374, 1999.
16. L. Xie, "LMI approach to output feedback stabilization of 2-D discrete systems," *International Journal of control*, vol. 72, pp. 97-106, 1999.
17. S. Xu, J. lam, C. Yang and E. I. Verrest, "An LMI approach to guaranteed cost control from uncertain linear neutral delay systems," *International Journal of Robust and Nonlinear Control*, vol. 13, pp. 35-53, 2003.
18. J. Lam, S. Xu, Y. Zou, Z. Lin, K. Galkowski, "Robust output feedback stabilization for two-dimensional continuous systems in Rosser form," *Applied Mathematical Letters*, vol. 17, pp. 1331-1334, 2004.
19. R. E. Benton, Jr. and D. Smith, Static Output Feedback Stabilization with Prescribed Degree of Stability," *IEEE Transactions on Automatic Control*, vol.43, pp. 1493-1496, 1998.
20. J. C. Geromel, C. C. de Souza and R. E. Skelton, "Static Output Feedback Controllers: Stability and Convexity", *IEEE Transactions on Automatic Control*, vol. 43, pp. 120-125, 1998.
21. L. El Ghaoui, F. Oustry and M. Ait Rami, "A cone complementarity linearisation algorithm for static output-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 11-71, 1997.
22. S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear matrix inequalities in system and control theory*. SIAM, Philadelphia, PA: SIAM, 1994.
23. J. Kang and W. Tang, "The Method for Characteristic Polynomial Assignment of 2-D Singular Systems," *Proceedings of the 5" World Congress on Intelligent Control and Automation*, 2004, pp. 1027-1029.