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State feedback control design for a class of nonlinear systems

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Abstract : This paper deals with a state feedback controller for a class of controllable and observable nonlinear systems with an admissible tracking capability. Two fundamental features are worth to be mentioned. The first one consists in the high gain nature of the underlying state feedback control and observer designs. More specifically, a unified high gain control design framework is proposed thanks to the duality between control and observation. The second feature consists in incorporating a filtered integral action into the control design. The filtering is mainly motivated by measurement noise sensitivity reduction while the integral action allows to achieve a robust offset free performance in the presence of step like disturbances. An academic servo problem, involving a nonlinear double integrator, is addressed to show the effectiveness of the proposed control design method.

Keywords : Nonlinear system, Output feedback control, Admissible tracking capability, High gain control, Sliding mode control, High gain observer, Filtered integral action.

1 Introduction

The problems of observation and control of nonlinear systems have received a particular attention throughout the last four decades (Agrawal and Sira-Ramirez (2004), Gauthier and Kupka (2001), Isidori (1995), Nijmeijer and der Schaft (1991), Krstič et al. (1995), Sepulchre et al. (1997)). Considerable efforts were dedicated to the analysis of the structural properties to understand better the concepts of controllability and of observability of nonlinear systems (Hammouri and Farza (2003), Gauthier and Kupka (2001), Rajamani (1998),

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Isidori (1995), Gauthier and Kupka (1994), Fliess and Kupka (1983), Nijmeijer (1981), Gauthier and Bornard (1981) Fliess and Kupka (1983)). Several control and observer design methods were developed thanks to the available techniques, namely feedback linearisation, flatness, high gain, variable structure, sliding modes and backstepping (Farza et al. (2005), Agrawal and Sira-Ramirez (2004), Boukhobza et al. (2003), Gauthier and Kupka (2001), Fliess et al. (1999), Sepulchre et al. (1997), Fliess et al. (1995), Isidori (1995), Krstič et al. (1995)). The main difference between these contributions lies in the design model, and henceforth the considered class of systems, and the nature of stability and performance results. A particular attention has been devoted to the design of state feedback control laws incorporating an observer satisfying the separation principle requirements as in the case of linear systems (Mahmoud and Khalil (1996)). Furthermore, various control design features have been used to enhance the performances, namely the robust compensation of step like disturbances by incorporating an integral action in the control design (Seshagiri and Khalil (1996)) and the filtering to reduce the control system sensitivity in the presence of noise measurements.

In this paper, one proposes a state feedback controller for a class of nonlinear controllable and observable systems. More specifically, one will address an admissible tracking problem for systems without zero dynamics, allowing thereby a comprehensive presentation of the proposed control design framework. The state feedback controller is obtained by simply combining an appropriate high gain state feedback control with a standard high gain observer (Gauthier and Kupka (2001), Farza et al. (2005)). The state feedback control design was particularly suggested from the the high gain observer design bearing in mind the control and observation duality. Of particular interest, the controller gain involves a well defined design function which provides a unified framework for the high gain control design, namely several versions of sliding mode controllers are obtained by considering particular expressions of the design function. Furthermore, it is shown that a filtered integral action can be simply incorporated into the control design to carry out a robust compensation of step like disturbances while reducing appropriately the noise control system sensitivity .

This paper is organized as follows. The problem formulation is presented in the next section. Section 3 is devoted to the state feedback control design with a full convergence analysis of the tracking error in a free disturbances case. Section 4 emphasizes the high gain unifying feature of the proposed control design. The possibility to incorporate a filtered integral action into the control design is shown in section 5. Simulation results are given in section 6 to highlight the performances of the proposed controller.

2 Problem formulation

One seeks to an admissible tracking problem for MIMO systems which dynamical behavior can be described by the following state representation

$$\begin{cases} \dot{x} = Ax + Bb(x)u + \varphi(x) \\ y = Cx = x^1 \end{cases} \tag{1}$$

with

$$x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix}, \ \varphi(x) = \begin{pmatrix} \varphi^1(x^1) \\ \varphi^2(x^1, x^2) \\ \vdots \\ \varphi^{q-1}(x^1, \ldots, x^{q-1}) \\ \varphi^q(x) \end{pmatrix} \tag{2}$$

$$A = \begin{pmatrix} 0 & I_{n-p} \\ 0 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ I_p \end{pmatrix}, \ C = \begin{pmatrix} I_p & 0_p & \ldots & 0_p \end{pmatrix} \tag{3}$$

where the state $x \in \vartheta$ an open subset $\mathbb{R}^n$ avec $x^k \in \mathbb{R}^p$, the input $u \in U$ a compact set of $\mathbb{R}^m$ with $m \geq p$, $b(x)$ is a rectangular matrix of dimension $p \times m$. This system is uniformly controllable and observable provided that the following assumptions holds.

$\mathcal{H}1$. The function $b(x)$ is Lipschitz in x over ϑ and there exists two positive scalars α and β such that for any $x \in \vartheta$, one has $\alpha^2 I_p \leq b(x)\,(b(x))^T \leq \beta^2 I_p$.

$\mathcal{H}2$. The function $\varphi(x)$ is Lipschitz in its arguments over the domain of interest ϑ.

The control problem to be addressed consists in an asymptotic tracking of an output reference trajectory that will be noted $\{y_r(t)\} \in \mathbb{R}^p$ and assumed to be enough derived, i.e.

$$\lim_{t \to \infty} (y(t) - y_r(t)) = 0 \tag{4}$$

Taking into account the class of systems, it is possible to determine the system state trajectory $\{x_r(t)\} \in \mathbb{R}^n$ and the system input sequence $\{u_r(t)\}$ corresponding to the output trajectory $\{y_r(t)\} \in \mathbb{R}^p$. This allows to define an admissible reference model as follows

$$\begin{cases} \dot{x_r} = Ax_r + Bb(x_r)u_r + \varphi(x_r) \\ y_r = Cx_r \end{cases} \tag{5}$$

The reference model state $x_r \in \mathbb{R}^n$ and its input $u_r \in \mathbb{R}^m$ can be determined as follows

$$\left\{\begin{array}{l} x_r^1 = y_r \\ x_r^k = \dot{x_r}^{k-1} - \varphi^{k-1}(x_r^1, \ldots, x_r^{k-1}) \ for \ k \in [2, q] \\ u_r = (b(x_r))^+ (\dot{x_r}^q - \varphi^q(x_r)) \end{array}\right. \tag{6}$$

By assuming that the reference trajectory is smooth enough, one can recursively determine the reference model state and input from the reference trajectory and its first derivatives, i.e. $y_r^{(i)} = \dfrac{d^i y_r}{dt^i}$ $for\ i \in [1, q-1]$, as follows

$$x_r^k = g^k\left(y_r, y_r^{(1)}, \ldots, y_r^{(k-1)}\right) \ for \ k \in [1, q]$$

where the functions g^k are given by

$$\left\{\begin{array}{l} g^1(y_r) = y_r \\ g^k\left(y_r, y_r^{(1)}, \ldots, y_r^{(k-1)}\right) = \\ \quad \sum_{j=0}^{k-2} \dfrac{\partial g^{k-1}}{\partial y_r^{(j)}}\left(y_r, \ldots, y_r^{(k-2)}\right) y_r^{(j+1)} \\ -\varphi^{k-1}\left(g^1(y_r), \ldots, g^{k-1}\left(y_r, y_r^{(1)}, \ldots, y_r^{(k-2)}\right)\right) \\ \ for \ k \in [2, q] \end{array}\right. \tag{7}$$

The output tracking problem (4) can be hence turned to a state trajectory tracking problem defined by

$$\lim_{t\to\infty} (x(t) - x_r(t)) = 0 \tag{8}$$

Such problem can be interpreted as a regulation problem for the tracking error system obtained from the system and model reference state representations (1) and (5), respectively.

$$\left\{\begin{array}{l} \dot{e} = Ae + B\left(b(x)u(x) - b(x_r)u_r\right) + \varphi(x) - \varphi(x_r) \\ e_m = y - y_r \end{array}\right. \tag{9}$$

3 State feedback control

As it was early mentioned, the proposed state feedback control design is particularly suggested by the duality from the high gain observer design proposed in Farza et al. (2005). The underlying state feedback control law is then given by

$$\left\{\begin{array}{l} \nu(e) = -B^T K_c\left(\lambda^q \bar{S} \Delta_\lambda e\right) \\ \\ u(x) = (b(x))^+\left(\dot{x}_r^q - \varphi^q(x_r) + \nu(e)\right) \end{array}\right. \tag{10}$$

where $(b(x))^+$ denotes the right inverse of the matrix $b(x)$, which exists according to $\mathcal{H}1$, Δ_λ is the block diagonal matrix defined by

$$\Delta_\lambda = diag\left(I_p, \frac{1}{\lambda}\ I_p, \ldots, \frac{1}{\lambda^{q-1}}\ I_p\right) \tag{11}$$

where $\lambda > 0$ is a positive scalar, $\bar{S}$ is the unique solution of the the following algebraic Lyapunov equation

$$\bar{S} + A^T\bar{S} + \bar{S}A = \bar{S}BB^T\bar{S} \tag{12}$$

and $K_c : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a bounded design function satisfying the following property

$$\forall \xi \in \Omega \ on\ a\ \xi^T BB^T K_c(\xi)\ \geq\ \frac{1}{2}\ \xi^T BB^T \xi \tag{13}$$

where Ω is any compact subset of $\mathbb{R}^n$.

Remark 3.1 *Taking into account the structure of the matrices B et C and the fact that the following algebraic Lyapunov equation*

$$S + A^T S + SA = C^T C \tag{14}$$

has a unique symmetric positive definite solution S (Gauthier et al. (1992)), one can deduce that equation (12) has a unique symmetric positive definite solution $\bar{S}$ which can be expressed as follows

$$\bar{S} = TS^{-1}T \quad avec \quad T = \begin{pmatrix} 0_p & \ldots & 0_p & I_p \\ \vdots & 0_p & I_p & 0_p \\ 0_p & I_p & 0_p & \vdots \\ I_p & 0_p & \ldots & 0_p \end{pmatrix} \tag{15}$$

Using some useful algebraic manipulations as in Farza et al. (2004) yields

$$B^T\bar{S} = CS^{-1}T = [C_q^q\ C_q^{q-1}\ \ldots\ C_q^1] \tag{16}$$

The above state feedback control law satisfies the tracking objective (8) as pointed out by the following fundamental result

Theorem 3.1 *The state and output trajectories of the (1)-(3) subject to the assumptions $\mathcal{H}1$ et $\mathcal{H}2$ generated from the input sequence given by (10)-(13) converge globally exponentially to those of the reference model (5) for relatively high values of λ.*

Proof. The state feedback control system can be written as follows

$$\begin{aligned}\dot{e} &= Ae + B\nu(e) + \varphi(x) - \varphi(x_r) \\ &= Ae - BB^T K_c(\lambda^q \bar{S}\Delta_\lambda e) + \varphi(x) - \varphi(x_r)\end{aligned}$$

The result will be established from a Lyapunov function using the state vector $\bar{e} = \lambda^q \Delta_\lambda e$ and henceforth governed by the equation

$$\dot{\bar{e}} = \lambda A\bar{e} - \lambda^q \Delta_\lambda BB^T K_c(\bar{S}\bar{e}) + \lambda^q \Delta_\lambda \left(\varphi(x) - \varphi(x_r)\right)$$

as $\Delta_\lambda A \Delta_\lambda^{-1} = \lambda A$ and $\Delta_\lambda B = \frac{1}{\lambda^{q-1}} B$, one can easily prove that $V : \bar{e} \mapsto V(\bar{e}) = \bar{e}^T \bar{S}\bar{e}$ is a Lyapunov function for the state feedback control system. Equation (12) yields

$$\begin{aligned}\dot{V} &= 2\bar{e}^T \bar{S}\dot{\bar{e}} \\ &= -\lambda V + \lambda \bar{e}^T \bar{S}BB^T \bar{S}\bar{e} - 2\lambda \bar{e}^T \bar{S}BB^T K_c(\bar{S}\bar{e}) + 2\lambda^q \bar{e}^T \bar{S}\Delta_\lambda(\varphi(x) - \varphi(x_r)) \\ &= -\lambda V - 2\lambda \left(\xi^T BB^T K_c(\xi) - \frac{1}{2}\xi^T B^T B^T \xi\right) + 2\lambda^q \bar{e}^T \bar{S}\Delta_\lambda(\varphi(x) - \varphi(x_r))\end{aligned}$$

where $\xi = \bar{S}\bar{e}$. Using inequality (13), one obtains

$$\dot{V} \leq -\lambda V + 2\lambda^q \bar{e}^T \bar{S}\Delta_\lambda(\varphi(x) - \varphi(x_r)) \tag{17}$$

In other respects, according to the Mean value theorem, one has

$$\varphi(x) - \varphi(x_r) = \frac{\partial \varphi}{\partial x}(\zeta)(x - x_r) \tag{18}$$

for some $\zeta \in \vartheta$. one obtains

$$\begin{aligned}\|\Delta_\lambda \left(\varphi(x) - \varphi(x_r)\right)\| &= \|\Delta_\lambda \frac{\partial \varphi}{\partial x}(\zeta) e\| \\ &= \|\frac{1}{\lambda^q}\Delta_\lambda \frac{\partial \varphi}{\partial x}(\zeta)\Delta_\lambda^{-1}\bar{e}\| \\ &\leq \|\frac{1}{\lambda^q}\Delta_\lambda \frac{\partial \varphi}{\partial x}(\zeta)\Delta_\lambda^{-1}\|\|\bar{e}\|\end{aligned}$$

Since φ is Lipschitz over ϑ, the matrix $\frac{\partial \varphi}{\partial x}(\zeta)$ is bounded over ϑ. Taking into account the triangular structure of $\varphi(x)$, such a matrix is lower triangular and as a result the matrix $\Delta_\lambda \frac{\partial \varphi}{\partial x}(\zeta)\Delta_\lambda^{-1}$ depends only on terms which are in $1/\lambda$ and hence its norm is bounded by a constant which does not depend on λ for $\lambda \geq 1$. This leads to

$$2\ \lambda^q \|\bar{S}\bar{e}\|\|\Delta_\lambda \left(\varphi(x) - \varphi(x_r)\right)\| \leq \gamma V \tag{19}$$

where $\gamma > 0$ is a constant which does not depend on λ. Combining (17) and (19), one obtains

$$V(\bar{e}) \leq e^{-(\lambda-\gamma)t} V(\bar{e}(0))$$

Remark 3.2 *Consider the case where the state matrix structure is as follows*

$$A = \begin{pmatrix} 0 & A_1 & 0 & \dots & 0 \\ 0 & 0 & A_2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & A_{q-1} \\ 0 & \dots & \dots & 0 & 0 \end{pmatrix}$$

where $A_i \in \mathcal{R}^{p\times p}$ for $i \in [1, q-1]$ are invertible constant matrices. One can easily show that the corresponding control law $\nu(e)$ in the expression of the control law (10) is then given by

$$\nu(e) = -\lambda \left(\prod_{i=1}^{q-1} A_i \right)^{-1} B^T \Delta_\lambda^{-1} K_c \left(\bar{S} \Delta_\lambda \Lambda e \right) \tag{20}$$

with

$$\Lambda = \begin{pmatrix} I_p & 0 & \dots & \dots & 0 \\ 0 & A_1 & 0 & \dots & 0 \\ \vdots & 0 & A_1 A_2 & 0 & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \dots & \dots & 0 & \prod_{i=1}^{q-1} A_i \end{pmatrix} \tag{21}$$

To this end, let us consider the change of coordinates $z = \Lambda x$, the system can be rewritten as follows

$$\begin{cases} \dot{z} = \Lambda A \Lambda^{-1} z + \Lambda B b(x) u + \Lambda \varphi(x) \\ y = C \Lambda^{-1} z = z^1 \end{cases} \tag{22}$$

Taking into account the structure of the the system state realization as well as the transformation matrix, one gets

$$\Lambda A \Lambda^{-1} = \begin{pmatrix} 0 & I_{n-p} \\ 0 & 0 \end{pmatrix}$$

$$\Lambda B = B \left(\prod_{i=1}^{q-1} A_i \right) \text{ and } C \Lambda^{-1} = C \tag{23}$$

One hence recovers the structure of the considered class of systems, i.e. equations (1) to (3), and naturally deduces the expression of the state feedback control law (20).

4 Particular design functions

The control law involves a gain depending on the bounded design function K_c which is completely characterized by the fundamental property (13). Some useful design functions are given below to emphasize the unifying feature of the proposed high gain concept.

- The usual high gain design function given by

$$K_c(\xi) = k_c B B^T \xi \tag{24}$$

  where k_c is a positive scalar satisfying $k_c \geq \frac{1}{2}$. Notice that the required property is fulfilled over R^n.

- The design function involved in the actual sliding mode framework

$$K_c(\xi) = k_c B B^T sign(\xi) \tag{25}$$

  where k_c is a positive scalar and 'sign' is the usual signum function. It is worth mentioning that the required property (13) holds in the case of bounded input bounded state systems. However, this design function induces a chaterring phenomena which is by no means suitable in practical situations.

- The design functions that are commonly used in the sliding mode practice, namely

$$K_c(\xi) = k_c B B^T Tanh(k_0 \xi) \tag{26}$$

  where $tanh$ denotes the hyperbolic tangent function and k_c and k_o are positive scalars. One can easily shows that the design function (26) satisfies the property (13)for relatively great values of k_o. More particularly, recall that one has $\lim_{k_0 \longrightarrow +\infty} Tanh(k_0 \tilde{z}) = sign(\tilde{z})$.

It is worth noticing that the choice of the design function K_c is not limited to the functions presented above. Other expressions could be considered, namely the inverse tangent function or any suitable combination of a usual high gain design function with a sliding mode design function.

5 Incorporation d'une action intégrale filtrée

One can easily incorporate a filtered integral action into the proposed state feedback control design, for performance enhancement considerations, by simply introducing suitable state variables as follows

$$\begin{cases} \dot{\sigma}^f = e^f \\ \dot{e}^f = -\Gamma e^f + \Gamma e^1 \end{cases} \tag{27}$$

where $\Gamma = Diag\{\gamma_i\}$ is a design matrix that has to be specified according to the desired filtering action. The state feedback gain is then determined from the control design model

$$\begin{cases} \dot{e}_a = A_a e_a + \psi(e_a + x_{ra}) - \psi(x_{ra}) \\ \qquad +B_a\left(b(e + x_r)u(e_a + x_{ra}) - b(x_{ra})u_{ra}\right) \\ y_a = \sigma^f \end{cases} \tag{28}$$

avec

$$\dot{y}_r^f = -\Gamma y_r^f + y_r,\ \dot{\sigma}_r^f = y_r^f$$

$$e_a = \begin{pmatrix} \sigma^f \\ e^f \\ e \end{pmatrix},\ x_{ra} = \begin{pmatrix} \sigma_r^f \\ y_r^f \\ x_r \end{pmatrix}$$

$$A_a = \begin{pmatrix} 0 & I_p & 0 \\ 0 & 0 & \Gamma \\ 0 & 0 & A \end{pmatrix},\ B_a = \begin{pmatrix} 0_p \\ 0_p \\ B \end{pmatrix}$$

$$\psi(e_a) = \begin{pmatrix} 0_p \\ -\Gamma e_f \\ \varphi(e) \end{pmatrix}$$

Indeed, the control design model structure (28) is similar to that of the error system (9) and hence the underlying state feedback control design is the same. The output feedback control law incorporating a filtered integral action is then given by

$$\begin{cases} u(e_a) = \left(b(\hat{e}_a + x_{ra})\right)^+ \left(\dot{x}_r^q - \varphi^q(x_r) + \nu(e_a)\right) \\ \nu(e_a) = -\Gamma^{-1} B_a^T K_{ac}(\lambda^q \bar{S}_a \Delta_{a\lambda} \Lambda e_a) \end{cases} \tag{29}$$

avec

$$e_a = \begin{pmatrix} \sigma^f \\ e^f \\ e \end{pmatrix} \tag{30}$$

$$\Delta_{a\lambda} = diag\left(I_p, \frac{1}{\lambda}\ I_p, \ldots, \frac{1}{\lambda^q}\ I_p, \frac{1}{\lambda^{q+1}}\ I_p\right) \tag{31}$$

$$\Lambda = \begin{pmatrix} I_p & 0 & \ldots & \ldots & 0 \\ 0 & I_p & 0 & \ldots & 0 \\ \vdots & 0 & \Gamma & 0 & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \ldots & \ldots & 0 & \Gamma \end{pmatrix} \tag{32}$$

where $\bar{S}_a$ is the unique symmetric positive definite matrix solution of the following Lyapunov algebraic equation

$$\bar{S}_a + \bar{S}_a \bar{A}_a + \bar{A}_a^T \bar{S}_a = \bar{S}_a \bar{B}_a \bar{B}_a^T \bar{S}_a \tag{33}$$

and $K_{ac} : \mathbb{R}^{n+2p} \to \mathbb{R}^{n+2p}$ is a bounded design function satisfying a similar inequality as (13), namely

$$\forall\ \xi_a \in \Omega \quad on\ a \quad \xi_a^T B_a B_a^T K_{ac}(\xi_a)\ \geq\ \frac{1}{2}\ \xi_a^T B_a B_a^T \xi_a \tag{34}$$

where Ω is any compact subset of $\mathbb{R}^{n+2p}$.

It can be easily shown that the resulting output feedback control system is globally stable and performs an asymptotic rejection of state and/or output step like disturbances.

6 Illustrative example

Let consider an academic servo problem for the nonlinear double integrator described by

$$\begin{cases} \dot{x}_1 = x_1 + d_1 sin(x_1) + v(t) \\ \dot{x}_2 = -d_2 x_2^3 + (2 + tanh(x_2))u \\ y = x_1 \end{cases}$$

where $d_1 = d_2 = 1$ are constant parameters, $v(t)$ is a step like disturbance with unitary magnitude occurring over the time interval $[27,\ 60]$ and $\gamma(t)$ is a zero mean measurement noise of variance 0.001. The desired output reference trajectory is generated from a second order generator with unitary static gain and two equal poles $p_1 = p_2 = -0.8$ which input sequence is shown by the figure 1. The involved servo system is based on the proposed output feedback control with a filtered integral action as follows

$$\begin{cases} \dot{\sigma}^f = e^f \\ \dot{e}^f = -\gamma e^f + \gamma e^1 \\ u(e) = \dfrac{1}{1 + tanh(x_2)} \left(\dot{x}_r^2 + \left(x_r^2\right)^3 + \nu(e)\right) \\ \nu(e) = -\dfrac{k_c}{\gamma}\, tanh(k_o \left(\lambda^4 \sigma^f + 4\lambda^3 e^f + 6\gamma\lambda^2 e^1 + 4\gamma\lambda e^2\right) \\ \varepsilon_1 = x_1 - y \\ e^1 = y - y_r \\ e^2 = x_2 - x_r^2 \\ x_r^2 = \dot{y}_r - sin(y_r) \\ \dot{x}_r^2 = y_r^{(2)} - \dot{y}_r cos(y_r) \end{cases}$$

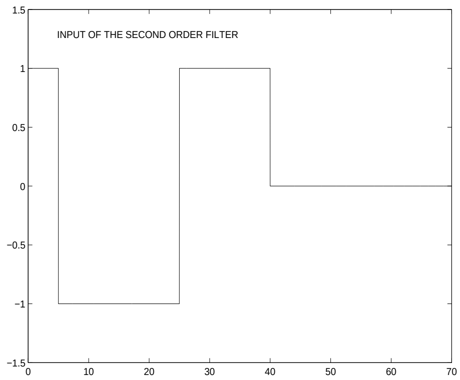

FIG. 1 – The reference generator input sequence

An intensive simulation study has been made using all the design functions that has been described above. As the performances were almost comparable, one will present only those obtained with the design function given by the expression (26). The design parameters have been specified as follows

$$k_c = 0.5;\ k_o = \lambda = 5;\ \gamma = 0.01;$$

Le premier ensemble de résultat est donné par la figure 2 et il a été obrenu avec les valeurs nominales des paramètres $d_1 = 1$ et $d_2 = 2$. Le deuxième ensemble est donné par la figure 3 et il a été obtenu avec des erreurs de 100 % qui ont été intentionnellement introduites sur les valeurs de chacun des paramètres d_1 et d_2, i.e les valeurs de $d_1 = d_2 = 2$ont utilisées dans le modèle sans changer l'expression de la commande. Sur chacune des figures 2 et 3, nous avons présenté l'évolution du comportement d'entrée-sortie du système ainsi que l' erreur de poursuite. Par ailleurs, la perturbation a été bien rejetée dans les deux cas et les comportements en sortie sont relativement bien filtrés avec la fonction de synthèse et l'action intégrale filtrée considérées. On notera aussi le bon comportement du système de commande vis-à-vis de fortes variations sur les paramètres du modèle.

7 Conclusion

In this paper, a unified high gain state feedback control design framework has been developed to address an admissible tracking problem for a class of controllable and observable nonlinear systems. Thanks the duality from the high gain system observation, a framework has been particularly suggested. The unifying feature is provided through a suitable design function that allows to rediscover all those well known high gain control methods, namely the sliding modes control. A Lyapunov approach has been adopted to show that the required tracking performances are actually handled.

Of practical purpose, a filtered integral action has been incorporated into the proposed control design to deal with step like disturbances while ensuring an adequate insensitivity to measurement noise. The effectiveness of the proposed state feedback control method has been emphasized throughout simulation results involving a nonlinear double integrator subject to state step like disturbances.

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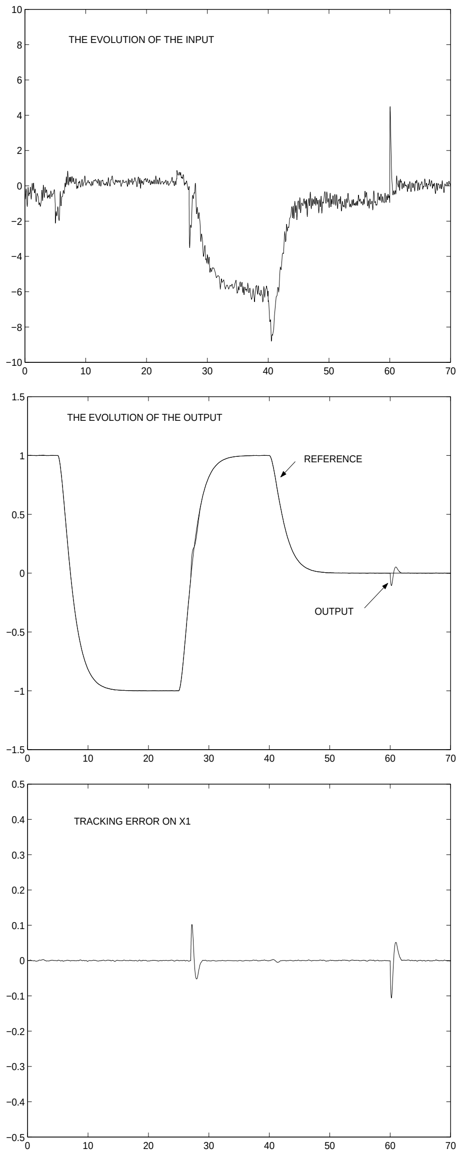

FIG. 2 – Nominal performances

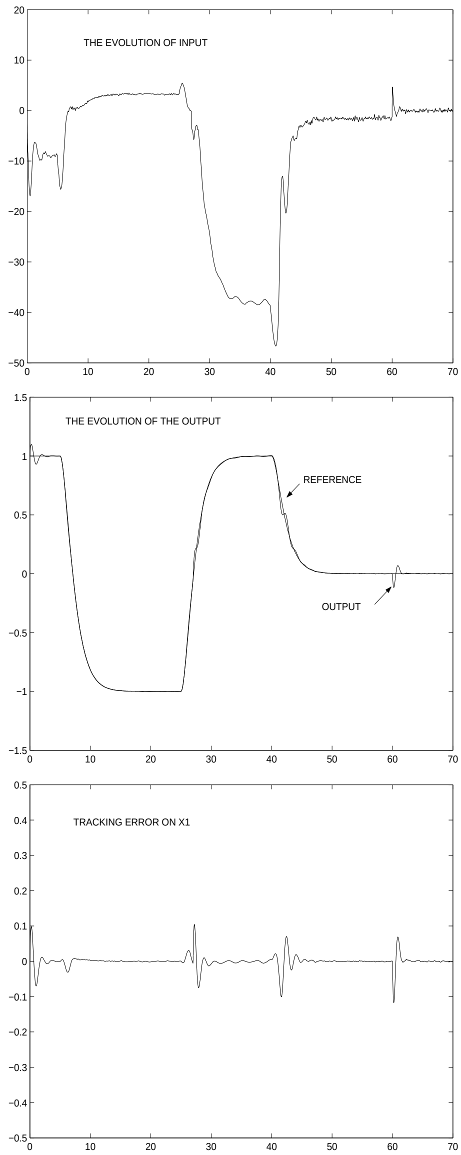

FIG. 3 – Robust performances

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