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Design of the time-varying parametric estimation method using the Kalman filter

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ABSTRACT. *In this paper, we develop a recursive parametric estimation algorithm of time-varying systems, based on the Kalman filter. In order to be effective, this algorithm requires an exact knowledge of the parameters noise variance-covariance matrix and the measurement noise variance. Here, we consider that the last variable is known and constant, and the parameters noise variance-covariance matrix is unknown and/or time-varying. For estimating this matrix, we develop an estimation new method based on the fuzzy logic theory. A numeric simulation example is treated in order to validate the performances of the proposed method.*

KEYWORDS. *Time-varying systems; Input-output mathematical models; Kalman Filter; Estimation of the variance-covariance matrix; Fuzzy logic.*

1. Introduction

The control of a dynamic system requires the determination of the inputs and the outputs of this system, the development of a mathematical model describing the system and finally the structural and parametric identification of this model.

An input-output mathematical model is the most often an approximation of the relations that exists between the characteristic parameters of the system. The stage of identification includes two parts: structural estimation and parametric estimation. The structural estimation permits to specify the structure of the model to adopt. Whereas the parametric estimation permits to determine the value of the different parameters intervening in the mathematical model. In this paper, the structure of the model is supposed known and we are interested only on the parametric estimation.

Notice that in several types of industrial applications, the real physical systems are with time-varying parameters or are submitted to externs and/or interns

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disturbances which can be assimilated to variations of the model parameters. In general, the hypothesis of constant parameters is made only for the simplicity purpose of the estimation problem.

The estimation of the systems with time-varying parameters is a theme of actuality research. Recently, several researchers were interested in the analysis and the synthesis of the parametric estimation schemes that can be applied to this kind of systems. In the same way, various papers have been developed which are based on the introduction of weight terms in the computing procedure of the adaptation gain ([5], [10], [12] and [13]). Among the parametric estimation methods of the time-varying systems proposed in the literature, we are interested, thereafter, to the method using the Kalman filter that is developed extensively ([7], [14] and [22]). The advantage of this method consists in determining estimated values of the system variables when the environment presents random disturbances. But, the major problem of the practical implantation of the Kalman filter is the selection procedure for the filter parameters. It has been shown that insufficiently known a priori filter parameters can reduce the precision of the estimated filter states or introduces biases to their estimates. In addition, incorrect *a priori* information can lead to practical divergence of the filter [8]. Among the Kalman filter parameters, we can distinguish the parameters noise variance-covariance matrix $Q(k)$ and the measurement noise variance σ^2. Here, we consider that the variance σ^2 is known and constant, and the parameters of the matrix $Q(k)$ are unknown and/or time-varying.

In this paper, we are interested in the development of an estimation method of this matrix, which is used in the calculation of the adaptation gain for the estimation algorithm. The similar problematic has been treated by Abid and Kamoun [2].

This paper represents, in a first time, a recursive parametric estimation algorithm of no-stationary systems, based on the Kalman filter. In a second time, we give the earlier methods for estimating $Q(k)$ then the new method using the fuzzy logic. In order to validate the performances of the proposed method, a numeric simulation example will be treated in the last part.

2. Parametric estimation

The parametric estimation constitutes an important phase in the definition of the mathematical model. Indeed, it permits to determine, from the input-outputs information of the system, a mathematical model of an *a priori* fixed class, that submitted to the same solicitations of the initial system, must give equivalent results considering the aimed objectives and the expected precision.

When, we develop a mathematical model, Two categories are generally distinguished: the stationary models, where the parameters and the structure of these models are considered constant, and the no stationary models, where the no stationarity can be originated from variation of the parameters, the delay or the order of the system. In this paper, we will only consider the case of no stationarity with respect to a variation of the parameters.

The input-output mathematical model, allowing the description of a physical system with time-varying parameters operating in a slightly noisy environment, can be represented by the following equation:

$$A(q^{-1},k)y(k) = B(q^{-1},k)u(k) + e(k) \tag{1}$$

where $\{e(k)\}$ is a sequence of independent random variables with zero mean and finite variance σ^2, representing the whole disturbance acting on the system. $u(k)$ and $y(k)$ represent, respectively, the input and the output of the system at each discrete time *k*. $A(q^{-1},k)$ and $B(q^{-1},k)$ are time-varying polynomials which are defined by:

$$A(q^{-1},k) = 1 + a_1(k)q^{-1} + ... + a_n(k)q^{-n} \tag{2}$$

and

$$B(q^{-1},k) = b_1(k)q^{-1} + \cdots + b_n(k)q^{-n} \tag{3}$$

The polynomials $A(q^{-1},k)$ and $B(q^{-1},k)$ are assumed to have the same degree n. This assumption is made for the purpose of simplicity.

The equation (1) corresponds to an input-output, linear, monovariable mathematical model of noisy *DARMA* type (Deterministic Auto-Regressive Moving Average). The proposed problem consists of finding the parameters $a_i(k)$ and $b_i(k)$, $i = 1,\dots,n$ intervening in this mathematical model at each discrete time k.

For the parametric estimation of the systems described by input-output deterministic mathematical models, we can use the Recursive Least Squares *RLS* algorithm. The goal of the formulation of this algorithm is to find the new parameters estimated at discrete time k from those at time $k-1$.

It is important to underline that the different types of recursive estimation algorithms formulated for the estimation of the systems with constant parameters cannot be used to the systems with time-varying parameters.

Note that the main element in the structure of a recursive estimation algorithm corresponds to the adaptation gain. In general, the computing procedure of the adaptation gain implies a decreasing value of this gain. Thereafter, at the time of a parametric variation, this gain doesn't allow the estimation algorithm to make some corrections in due time in order to assure a better quality of estimation.

Besides, it is discriminating to reconfigure the recursive estimation algorithms in order to prevent the adaptation gain matrix from tending to zero and be come susceptible to follow the parametric variations. We can mention two types of algorithms which can attain this objective:

- recursive least squares estimation algorithm with forgetting factor,
- recursive estimation algorithm based on the Kalman filtering.

The modified version of the *RLS* algorithm consists of introducing a forgetting factor λ in the recursive calculation of the adaptation gain matrix. However, the value of λ must be chosen while respecting the following condition: $0 < \lambda < 1$. The choice of the value of the forgetting factor λ constitutes a major problem since there are not any universal analytic methods to follow. Indeed, a bad choice can lead estimation to a failure.

The *RLS* algorithm lacks directional possibilities in the update of the parameters, due to a scalar λ serving as the single design variable for this algorithm. This disadvantage has been clearly illustrated by several authors (e.g. [15]). However, since it seems possible that different parameters exhibit different time-variant characteristics, the estimation method based on the Kalman filter is more interesting.

3. Parametric estimation algorithm based on the Kalman filter

The Kalman filter is an estimator of the state variables of dynamic systems capable to be described by linear state models operating in a stochastic environment. The use of the Kalman filter for the parametric estimation requires the representation of the parameter vector with a state model.

Indeed, the output of the mathematical model given by the equation (1) can be expressed in the following compact form:

$$y(k) = \theta^T(k)\psi(k) + e(k) \tag{4}$$

where $\theta(k)$ is the vector of time-varying parameters and $\psi(k)$ is an observation vector, which are given respectively by:

$$\begin{aligned}\theta^T(k) &= [a_1(k)\cdots a_n(k)b_1(k)\cdots b_n(k)]\\ &= [\theta_1(k)\cdots\theta_{2n}(k)]\end{aligned} \tag{5}$$

$$\psi^T(k) = \left[-y(k-1)\cdots - y(k-n)\ u(k-1)\cdots u(k-n)\right] \tag{6}$$

Generally, the default description for the parameter variation when no specific information is at hand, is to model it as a process of Gauss-Markov, satisfying the differential stochastic equation:

$$\theta(k+1) = \Phi\theta(k) + w(k) \tag{7}$$

where Φ is a transition probability matrix supposed known and $w(k)$ is a disturbance vector. The elements of $w(k)$ are sequences of independent random variables with zero mean and with time-varying variance-covariance matrix $Q(k)$ given by:

$$Q(k) = \mathcal{E}\left[w(k) w^T(k) \right] \tag{8}$$

where $\mathcal{E}$ denotes the expectation symbol.

While associating the equations (7) and (4), we get a state model permitting to describe the system with time-varying parameters. In this case, $\theta(k)$ represent a state vector and can be estimated using the Kalman filter approach. Such estimation algorithm requires the knowledge of the system input signals $u(k)$ and output signals $y(k)$ for several values of k, $k = 0, 1, \ldots, M$. So, we must have an information sequence I_M defined as: $I_M = \{u(0), u(1), ..., u(M); y(0), y(1), ..., y(M)\}$. From equations (5) and (7), it can be seen that the conditional distribution of $\theta(k+1)$ given I_M is Gaussian with mean $\hat{\theta}(k+1)$ and covariance $P(k+1)$ .Then the estimated parameters are given with the following recursive shape:

$$\hat{\theta}(k+1) = \Phi\hat{\theta}(k) + L(k)\varepsilon(k) \tag{9}$$

with $\varepsilon(k)$ is the prediction error given by:

$$\varepsilon(k) = y(k) - \hat{\theta}^T(k)\psi(k) \tag{10}$$

and $L(k)$ is the Kalman gain defined as follows:

$$L(k) = \frac{P(k-1)\psi(k)}{\sigma^2 + \psi^T(k)P(k-1)\psi(k)} \tag{11}$$

$$P(k) = \Phi\left[P(k-1) - \frac{P(k-1)\psi(k)\psi^T(k)P(k-1)}{\sigma^2 + \psi^T(k)P(k-1)\psi(k)} \right]\Phi^{\mathrm{T}} + Q(k) \tag{12}$$

The introduction of the variance-covariance matrix $Q(k)$ in the equation of the adaptation gain matrix $P(k)$ permits to determine a prediction error gain $L(k)$ allowing the update of each respective parameter by each diagonal element of $Q(k)$. Due to $Q(k)$ describes the variations in the parameters.

It is obvious that the practical implantation of this algorithm requires, in addition to the knowledge of the initial conditions of the parameters vector and the adaptation gain matrix, the knowledge of the values of $Q(k)$ and the value of σ^2. Let us consider that σ^2 is known and constant, so we will be interested for estimating the parameters of the matrix $Q(k)$, which are unknown and/or time-varying.

4. Earlier approaches for estimating the matrix $Q(k)$

Several methods are used in order to estimate the variance-covariance matrix Q of the noise sequence acting on the state of the system. We can distinguish:

1- Bayesian estimation methods, which essentially assume that the values of the unknown variances form a finite set with known probabilities of each value in the set occurring [11];

2- correlation methods, which use the fact that the sequence of the prediction error for a Kalman filter is a Gaussian white noise with zero mean [18];

3- covariance-matching methods, which strive to make the residuals consistent with their theoretical values. The sequence of the prediction error is given by [17]:

$$E\left[\varepsilon^2(k)\right] = \psi^T(k)P(k-1)\psi(k) + \sigma^2 \tag{13}$$

4- maximum likelihood methods, which are based on the maximization of some density functions, for example ([17], [19],[20]):

$$F(\sigma^2, Q) = -\frac{1}{2}\sum_{t=1}^{N}\frac{\varepsilon^2(t)}{E\left[\varepsilon^2(t)\right]} \tag{14}$$

The first approach changes merely the type of conception variables, i.e., the set and the probabilities, for the filter. The second and the third approach rely on the fact that the description of the variations in the parameters, by a state model, being correct. This description supposes that the real system can be modelled as a parametric combination of the chosen observation vector. However, for a real system, the mathematical model can be valid only for the modelling goals, and cannot be considered like an exact description of the system. For these reasons, the ad hoc methods are generally adopted in parametric estimation.

Other works ([6], [9], [16], [21], [23] and [24]) are interested to develop an adaptive filter, in order to adjust on line the variance-covariance matrix $Q(k)$ so called adaptive matrix. Among the estimation methods for $Q(k)$, we can mention the one that augments the state vector. In addition to representative variables of the system, the state vector can be augmented with other variables which are estimated by the filter. Indeed, in [3] the noise statistics are added to the state vector to estimate the noise and to make an adaptive compensator.

Thereafter, we propose an ad hoc method for the estimation of the matrix $Q(k)$ on line, using the fuzzy logic.

5. New approach for estimating the variance-covariance matrix

Under the assumption that the elements of $w(k)$ are sequences of independent random variables with zero mean, we can conclude that the corresponding variance-covariance matrix is diagonal. Let us define the vector $q(k)$ as the vector of the elements of the diagonal matrix $Q(k)$. Letting $q_i(k)$ be the i^{th} element in $q(k)$, we can write :

$$Q(k) = diag\ [q_1(k)\ q_2(k)\ ...\ q_{2n}(k)] \tag{15}$$

Therefore, the objective is to elaborate a new approach, which estimates the appropriate coefficients of the vector $q(k)$ at each discrete time *k*.

The idea repose on the fact that each diagonal element of the matrix $Q(k)$ influences on the adjustment of each corresponding model parameter. For the choice of these matrix elements, we can notice the influence of the prediction error. Indeed, a high value of the prediction error indicates the bad quality of the parametric estimation, therefore, we must increase the Kalman gain by increasing the matrix $Q(k)$. In addition, in a regulating strategy, it is required to derive some parameters (parameter to adjust or an internal parameter) in order to represent their variations. In fact, the variation of each estimated parameter is defined by:

$$\Delta\hat{\theta}_i(k) = \hat{\theta}_i(k) - \hat{\theta}_i(k-1) \tag{16}$$

with $i = 1, 2, \ldots, 2n$, indicates the system variation dynamic. If the variation of the parameter $\hat{\theta}_i(k)$ is large, the corresponding coefficient $q_i(k)$ must be chosen big otherwise a small value of $q_i(k)$ suffice.

For these reasons, we will use an estimation strategy for each coefficient $q_i(k)$ based on the evaluation of the two parameters: the prediction error and the variation of the corresponding estimated parameter.

In our case, we have accurate information on the variables error and variation of the estimated parameter at the discrete time *k*-1, but the facts to evaluate are vague. To specify this empiric concept, we must use the formalisms permitting the representation of the error and the variation of the estimated parameter, and elaborate the methods permitting to manipulate them, for the estimation. In order to avoid the problems of discontinuity in the systems with classic threshold, the theory of the fuzzy sets is recommended since it takes in account and solves this type of problem. So the fuzzy logic theory will be used to manipulate these fuzzy sets [4].

Therefore, the objective is to elaborate a fuzzy system, which provides in output, in each discrete time *k*, the appropriate coefficients of the vector $q(k)$ from the error (in absolute value) and the variation of the estimated parameters at the discrete time *k*-1. The functional diagram of this fuzzy system is given by Figure 1.

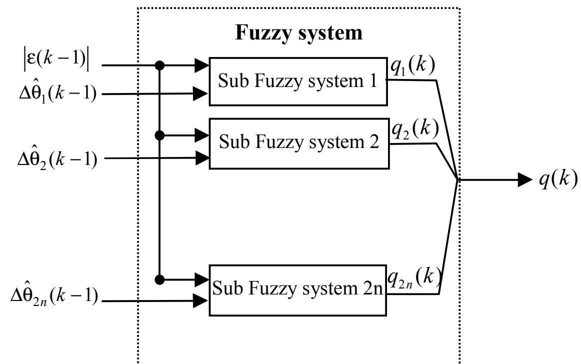


Fig. 1. Functional diagram of the fuzzy system.

We obtain $2n$ sub fyzzy systems. Each sub system leans on fuzzy rules of inputs-outputs type. Indeed, the output of this sub system $q_i(k)$ according to the two inputs $|\varepsilon(k-1)|$ and $\Delta\hat{\theta}_i(k-1)$ is gotten with the rules manipulating the linguistic variables of $|\varepsilon(k-1)|$, $\Delta\hat{\theta}_i(k-1)$ and $q_i(k)$. It is required to begin with describing the input and output signals by the fuzzy language. This operation requires the dividing of the variation domains for these signals in fuzzy sets. The choice of these last has a considerable effect on the contribution of the fuzzy estimation and only the experimental studies and in simulation can lead to appropriate choice.

For the input signal $|\varepsilon(k-1)|$, it is necessary to determine membership functions that value the relevance of the following facts: absolute value of the error is large (L) and absolute value of the error is small (S), considering the situation described by the numeric value for this signal. In the same way, we define the fuzzy distribution of the variable, variation of the estimated parameter $\Delta\hat{\theta}_i(k-1)$ that associates its compatibility with the vague term that can be: positive (P), approximate of zero (AZ) and negative (N). To accomplish the stage of fuzzyfication, we define fuzzy sets for the output variable. We must take in account that the matrix $Q(k)$ must be chosen non-negative definite. Indeed, we choose a subdivision for discourse domain of $q_i(k)$ in three fuzzy sets: small (S), medium (M) and large (L). The membership functions for these fuzzy sets relative to

$|\varepsilon(k-1)|$, $\Delta\hat{\theta}_i(k-1)$ and $q_i(k)$, which are designed using a heuristic approach, are shown in Figure 2.

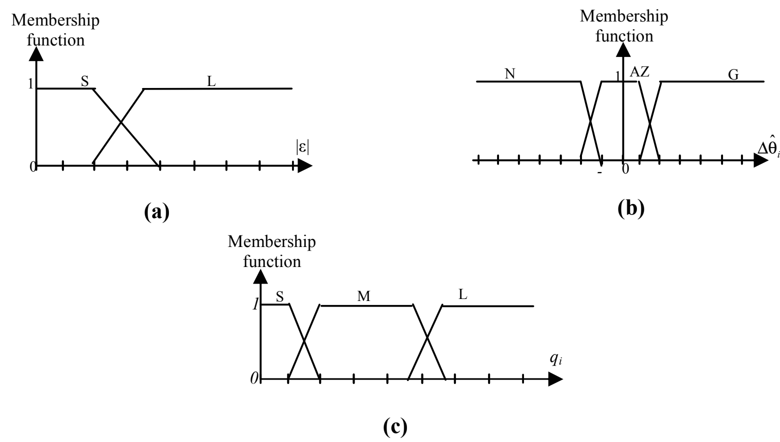


Fig. 2. Membership function of: **(a)** $|\varepsilon(k-1)|$; **(b)** $\Delta\hat{\theta}_i(k-1)$ and **(c)** $q_i(k)$.

Now, the step of inference consists in establishing the fuzzy rules of the kind:

IF <*antecedent 1*> and <*antecedent 2*> **THEN** <*consequent*>,

where antecedent 1 is of the form $|\varepsilon(k-1)|$ is {S ou L}, antecedent 2 is of the form $\Delta\hat{\theta}_i(k-1)$ is {N, AZ ou G} and consequence is of the form $q_i(k)$ is {S, M ou L}. The development of these rules leans, generally, on the experience and the ability acquired by a human operator and on the theories of the parametric estimation algorithms to use. Among which, the coefficient $q_i(k)$ should be chosen small when the system variations are feeble in order to prevent against complicated statistical properties estimation.

For each sub fuzzy system *i*, we elaborate six fuzzy rules that are represented by the following inference matrix.

Table 1. Inference matrix for the inputs $|\varepsilon(k-1)|$ and $\Delta\hat{\theta}_i(k-1)$ and the output $q_i(k)$.

$q_i(k)$		$|\varepsilon(k-1)|$	
		S	*L*
$\Delta\hat{\theta}_i(k-1)$	*N*	***M***	***L***
	AZ	***S***	***M***
	P	***M***	***L***

6. Illustrative example

In this paragraph, we treat a numeric simulation example in order to test the performances of the recursive parametric estimation algorithm based on the Kalman filter, using the estimated variance-covariance matrix determined by the fuzzy method.

Let us consider a linear system, with time-varying parameters, which can be described by the following mathematical model:

$$y(k) = -a_1(k)y(k-1) - a_2(k)y(k-2) + b_1(k)u(k-1) + b_2(k)u(k-2) + e(k) \quad (17)$$

where $y(k)$ and $u(k)$ designate respectively the output and the input of the system at the discrete time k, $e(k)$ represent the disturbance acting on the measured output, and $a_1(k), a_2(k), b_1(k)$ and $b_2(k)$ are unknown and slightly time-varying parameters defined as follow:

$$\begin{aligned} a_1(k) &= \Phi_{11}a_1(k-1) + w_1(k-1) \\ a_2(k) &= \Phi_{22}a_2(k-1) + w_2(k-1) \\ b_1(k) &= \Phi_{33}b_1(k-1) + w_3(k-1) \\ b_2(k) &= \Phi_{44}b_2(k-1) + w_4(k-1) \end{aligned} \quad (18)$$

We assume that $e(k)$ is a sequence of independent random variables with zero mean and variance σ^2.

The noise vector $w(k)$ is given by:

$$w^T(k) = [w_1(k)\ w_2(k)\ w_3(k)\ w_4(k)] \quad (19)$$

We suppose that the vector $w(k)$ is constituted by a sequence of random variables with zero mean and variance-covariance matrix $Q(k)$.

For the estimation of the parameters $a_1(k), a_2(k), b_1(k)$ and $b_2(k)$, we apply the estimation algorithm based on the Kalman filter (equations: (9), (11) and (12)). To estimate the unknown and time-varying matrix $Q(k)$, we utilize the fuzzy method proposed in this paper. In order to evaluate the parametric estimation results obtained with this method, we suppose, in a first time, that the matrix $Q(k)$ is known and constant. We apply, in a second time, an estimation method of $Q(k)$ using an auxiliary filter proposed in ([1] and [16]), that is one among the covariance-matching methods.

For the practical implementation of this algorithm, we consider that the input $u(k)$ is a squared signal of level [+2,-2] and of period 20. This type of input signal has been selected so that it can continually excite the considered system in the whole region of its working domain. The initial conditions of the algorithm based on the Kalman filter are chosen as: $\hat{\theta}^T(0) = [0\ 0\ 0\ 0]$, $P(0) = 1000I$ and $\sigma^2 = 0.02$. Indeed the transition probability matrix is selected such as: $\Phi = [0.5\ 0\ 0\ 0;\ 0\ 0.3\ 0\ 0;\ 0\ 0\ 1\ 0;\ 0\ 0\ 0\ 0.09]$.

The objective of this example is to show the performances of the proposed method for searching the matrix $Q(k)$ and the capacities of the estimation algorithm with Kalman filter using this matrix $Q(k)$ in order to follow the variations of the parameters $a_1(k)$, $a_2(k)$, $b_1(k)$ and $b_2(k)$ during the time.

Figures 3 and 4 give the evolution curves, respectively, of the prediction error $\varepsilon(k)$ and, as an example, the real and the estimated parameter $b_1(k)$ for known $Q(k)$, estimated $Q(k)$ with the fuzzy method and estimated $Q(k)$ with the method using an auxiliary filter.

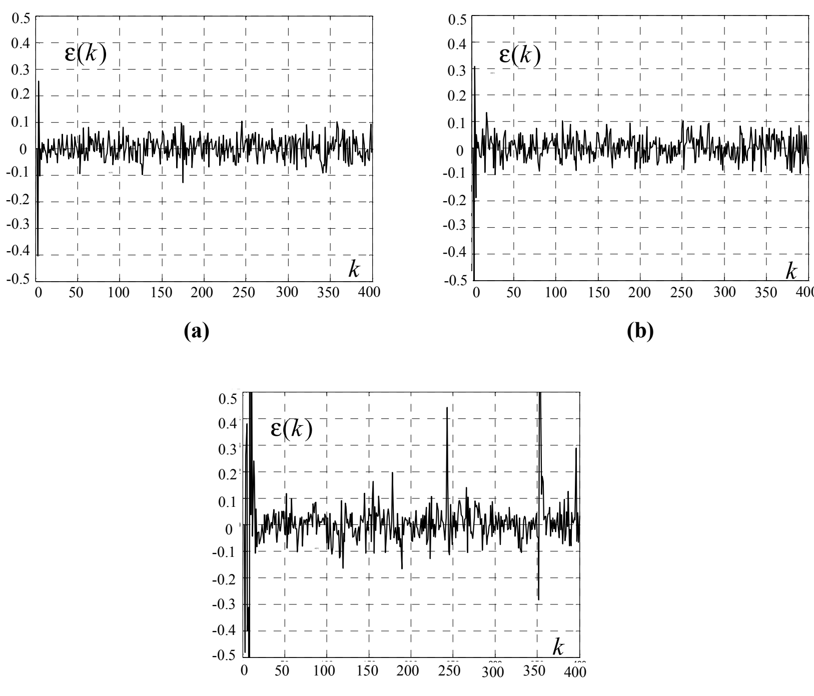
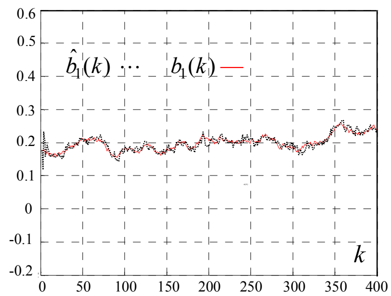
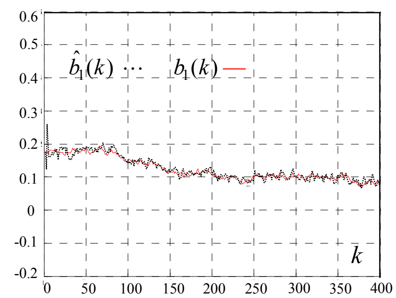


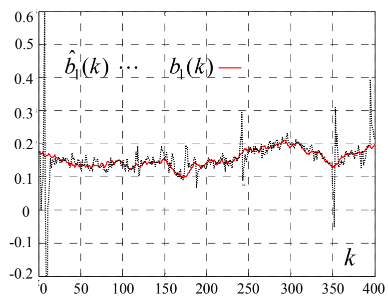
Fig. 3. Evolution curves of the prediction error $\varepsilon(k)$ for: **(a)** known $Q(k)$, **(b)** fuzzy $Q(k)$ and **(c)** estimated $Q(k)$ with the auxiliary filter.



(a)



(b)



(c)

Fig. 4. Evolution curves of the real and the estimated parameter $b_1(k)$ for: **(a)** known $Q(k)$, **(b)** fuzzy $Q(k)$ and **(c)** estimated $Q(k)$ with the auxiliary filter.

The performances of the recursive parametric estimation algorithm based on the Kalman filter with fuzzy estimation of the matrix $Q(k)$ can be valued while examining the obtained results of numeric simulations. We can notice that the plots of the Figure 3 relative to the evolution curves of the prediction error $\varepsilon(k)$ for the matrix $Q(k)$ known and the one estimated by the fuzzy method, indicate as well the good quality of the estimation. Besides, the evolution curves of the prediction error $\varepsilon(k)$ gotten for $Q(k)$ estimated with the fuzzy method and the one estimated with the auxiliary filter Figure 3, show as well that the quality of the estimation with the fuzzy method is better. In the same way, the evolution curves of the real and the estimated parameter $b_1(k)$ Figure 4, for $Q(k)$ known and the one estimated by the two methods, enforce the same observation.

Then these results prove the efficiency of the ad hoc methods for the variance-covariance matrix estimation with regard to the covariance-matching methods. In conclusion, we can affirm that the matrix $Q(k)$ is often unknown in engineering applications of the kalman filter. In order to assure the convergence of the parametric estimation algorithm, we can estimate $Q(k)$ by the method proposed using the fuzzy logic.

7. Conclusion

In this paper, a recursive parametric estimation algorithm for the no-stationary systems was developed, by using the Kalman filtering. We have studied the dynamic systems that can be described by input-output models, linear, monovariable, with time-varying parameters operating in a slightly noisy environment.

This parametric estimation algorithm is able to compensate the estimation errors resulting from insufficient knowledge and/or variation of the noise statistics by estimating, on line, the variance-covariance matrix of the noise. An estimation method of this matrix has been proposed. This method leans on the fuzzy logic theory for the evaluation of the prediction error and its variation at each time, in order to settle on the value of the variance-covariance matrix.

The efficiency of this method has been tested on a numeric simulation example. A comparison of the results has been given for a known variance-covariance matrix and the one valued by the fuzzy method and by the method using an auxiliary filter. The numeric simulation results obtained, indicate as well the efficiency of the method proposed.

References

1. Abid, A. (2006): Identification de systèmes à paramètres variables au cours du temps en utilisant le filtre de Kalman. CD-Rom de la septième conférence internationale des Sciences et des Techniques de l'Automatique STA'2006, 17-19 Décembre, Hammamet, Tunisie.
2. Abid, A. et M. Kamoun (2007): Estimation de la matrice de variance-covariance du bruit du Filtre de Kalman moyennant la Logique Floue. CD-Rom de la huitième conférence internationale des Sciences et des Techniques de l'Automatique STA'2007, 5-7 novembre, Monastir, Tunisie.
3. Bai, M.; H. D. Zhou and H. Schwarz (1998): Adaptive augmented state feedback control for an experimental planar two-link flexible manipulator. IEEE Transactions on Robotics and Automation, vol. 14, pp. 894-901.
4. Borne, P.; J. Rozinoer; J. Y. Dieulot et L. Dubois (1998): Introduction à la commande floue. Editions Technip, Collection Sciences et Technologies, Paris.
5. Campi, M. (1994): Performance of RLS identification algorithms with forgetting factor: A Φ–Mixing approach. Journal of Mathematical Systems, Estimation, and Control vol. 4, pp. 1-25.
6. Chaer, W. S. and R. H. Bishop (1997): A mixture-of-experts framework for adaptive Kalman filtering. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, vol. 27, pp. 452-464.

7. Favier, G. (1988): Numerically efficient adaptive identification algorithms. R.A.I.R.O. APII, vol. 22, pp. 27-52.
8. Fitzgerald, R. J. (1971): Divergence of the Kalman filter. IEEE Transactions on Automatic Control, vol. AC-16, pp. 736-747.
9. Guo, L. and L. Ljung (1995): Performance analysis of general tracking algorithms. IEEE Transactions on Automatic Control, vol. 40, pp. 1388-1402.
10. Hadrich, A.; A. Abid and A. Kamoun (2005): On the parametric identification of time-varying parameter systems. CD-Rom of the third International Conference on Signals Systems Decision and Information Technology SSD'2005, 22-24 March, Sousse.
11. Isaksson, A. (1988): On system identification in one and two dimensions with signal processing applications. Thèse de doctorat, département de Génie Electrique, université Linköping, Sued.
12. Kamoun, S. (2003): Contribution à l'identification et à la commande adaptative de systèmes complexes. Thèse de doctorat, Ecole Nationale d'Ingénieurs de Sfax, Tunisie.
13. Li, Z. and R. J. Evans (2002): Generalised minimum variance control of linear time-varying systems. IEE Proc.-Control theory Appl., vol. 149, January.
14. Ljung L. (2001): Recursive least-squares and accelerated convergence in stochastic approximation schemes. Int. J. Adap. Control Signal Process, vol. 15, pp. 169-178.
15. Ljung, L. and T. Söderström (1983): Theory and practice of recursive identification. London: The MIT Pres.
16. Maitelli, A. L. and T. Yoneyama (1997): Adaptive control scheme using real time tuning of the parameter estimator. IEE Proc.-Control theory Appl., vol. 144, pp. 241-248.
17. Mehra, R. K. (1972): Approaches to adaptive filtering. IEEE Transactions on Automatic Control, vol. 17, pp. 693-698.
18. Mehra, R. K. (1970): On the identification of variances and adaptive Kalman filtering. IEEE Transactions on Automatic Control, vol. 15, pp. 175-184.
19. Enns, P. G.; J. A. Machak; W. A. Spivey and W. J. Wrobleski (1982): Forecasting Applications of an Adaptive Multiple Exponential Smoothing Model. Management Science, vol. 28, September, pp. 1035-1044.
20. Smith, P. L. (1971): Estimation of the covariance parameters of non-stationary time-discrete linear systems. In Proc. 2nd Symp. Nonlinear Estimation Theory and Its Applications, San Diego, CA, September, pp. 323-328.
21. Song, Q.; Z. Jiang and J. Han (2007): Noise covariance identification based adaptive UKF with application to mobile robot systems. IEEE International Conference on Robotic and Automation, 10-14 April, Roma, Italy.
22. Sorenson, H. W. (1995): Kalman filtering: theory and application. Academic Press, New York.
23. Waller, M. and H. Saxen (2000): Estimating the degree of time variance in a parametric model. Automatica, vol. 36, pp. 619-625.
24. Wira, P. and J. P. Urban (2000): A new adaptive Kalman filter applied to visual servoing tasks. Fourth International Conference on knowledge-based Intelligent Engineering Systems and Allied Technologies, 30 Aug-1 Sept, Brighton, UK.