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Sliding mode control of nonlinear SISO systems with both matched and unmatched disturbances

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Abstract: *In this paper, the nonlinear sliding mode control (SMC) with mismatch disturbances is proposed. We treat the problem of control with this class of disturbances and the chattering phenomena. The proposed method attenuates the effect of both uncertainties, external disturbances and eliminates the chattering phenomenon introduced by classical sliding mode control. The model of a hydraulic system is used to test the suggested procedure.*

Keywords: *Chattering phenomena, second order Sliding Mode Control, Three-Tank system and robust control.*

1. Introduction

Variable structure control (VSC) results in high performance systems that are robust to parameter uncertainties and noise. Most of the early works in the area are in the Russian literature (see Utkin [17] and the references within). Subsequently, various VSC algorithms have been successfully used for trajectory tracking problems [1], [4], [8], [12] and [14]. Good results have been reported in eliminating external disturbances, addressing nonlinearities, and achieving acceptable control in the presence of modelling errors. A popular VSC approach for trajectory tracking problems is based on Lyapunov's method. This approach yields multivariable designs that produce sliding mode on the intersection of several switching surfaces.

The control laws are designed so that the system trajectory always reaches the sliding surface. This is known as the reaching phase. Once on the sliding surface, the control structure is changed discontinuously to maintain the system on the sliding surface. At this stage, the system is in the sliding phase. The control law may be linear or nonlinear during the whole or parts of the control mission. Its structure changes according to a preselected switching logic. The switches in the control structure

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depends on the instantaneous values of the system state along the trajectory, see [2], [3] and [8].

High frequency control switching leads to the so-called chattering effect, which results in potentially harmful high frequency vibration of the controlled plant. Several methods have been proposed to overcome these difficulties; for example [15] proposed an interpolation of the control inside boundary layer, replacing in the expression of u the term sign (S) by sat(S). Our strategy consists in adding an integral corrector to the sliding mode control when the trajectory of the state goes into a boundary layer and approaches its reference.

The matching condition assumption is unfortunately fairly restrictive and not satisfied by the majority of real-world systems. Hence the non-matching disturbances and uncertainties will affect the sliding mode dynamics and may cause unacceptable deterioration in the performance of some regulated output. Most of the references in the VSC literature such as Emelyanov et al [6] consider the stability of a nonlinear system with VSC where the output dependent nonlinearity is treated as a non-matching disturbance, and Spurgeon and Davies [16] consider a modified VSC (to provide a continuous control and eliminate chattering) to achieve uniform ultimate boundedness in the neighbourhood of the ideal sliding surface for a class of unmatched uncertainty. More recently, a technique for constructing a storage function for a broad class of nonlinear passive systems with mismatched uncertainties has been proposed by Weiping [18]

The paper is organized as follows: the non linear system model is presented in section two; section three and four describes the sliding mode control. Section five will be devoted to the experimental results. Finally some concluding remarks end the paper.

2. Problem Statement

We consider a nonlinear SISO controllable system given by

$$\begin{cases} \dot{x}_1 = f_1(x) + B(x)u + w_1 \\ \dot{x}_2 = f_2(x) + w_2 \\ y = x_2 \end{cases} \tag{1}$$

$x = [x_1, x_2]^T \subset \mathrm{X} \in \Re^2$ is the state vector, $u \in U \subset \Re$ is the control input to be bounded by $|u| \le U_0$, $f_1 : \Re^2 \to \Re$ and $f_2 : \Re^2 \to \Re$ are continuous and smooth functions, we assume that $\dfrac{\partial f_2}{\partial x_1} \neq 0$ and $B(x) \neq 0$, $y \in Y \subset \Re$ is the output, $w = [w_1, w_2]^T \in W \subset \Re^2$ represent the disturbances and parameter variations, where $|w_1| < w_{10}$, $|w_2| < w_{20}$ and $|\dot{w}_2| < dw_{20}$.

The design procedure for a state based sliding mode controller can be divided into two parts:

Step 1: Finding the switching function $S(x) \in \Re$, such that the internal dynamics in the sliding mode are stable.

Step 2: Designing a controller *u*, which ensures that the sliding mode is reached and subsequently maintained.

$$u = \begin{cases} u^{+} & if \quad S > 0 \\ u^{-} & if \quad S < 0 \end{cases} \tag{2}$$

3. Sliding surface design

The switching surface plays the major role in defining the reduced-order system dynamics. Many techniques have been devised for designing the switching surface. These techniques include: Filoppov's method [7], Utkin's equivalent control method [17], pole placement, and quadratic minimization methods.

To achieve the robustness property with respect to unmatched disturbances and uncertainty parameters, the sliding surface should be designed such that:

$$S(t) = h(\tau_{sm}\dot{\tilde{x}}_2 + \tilde{x}_2) \tag{3}$$

where $\tau_{sm} > 0$ is the time constant in sliding mode, $h > 0$ is a constant parameter and $\tilde{x}_2 = y - y_r$ .is the output error.

Definition 1. *The system (1) is in presence of an unmatched disturbance, if the exists a disturbance w(t) such that the motion in the sliding mode of system (1) depends on the w(t).*

Notations The expression a_1, a_2, a_3 and υ are intermediate parameters given by $a_1 = \tau_{sm}\dfrac{\partial f_2}{\partial x_1} \neq 0$, $a_2 = (\tau_{sm}\dfrac{\partial f_2}{\partial x_2} + 1)$, $a_3 = \tau_{sm}\dfrac{\partial f_2}{\partial x_2}$, and $\upsilon = -\tau_{sm}\ddot{y}_r - \dot{y}_r$.

Theorem 1. *Consider the sliding surface (3) and let us suppose that the trajectory of state reaches the surface in a finite time t_r and confined to this surface $S(t) = \dot{S}(t) = 0$ for $t>t_r$.*

Then, for all time $t > t_r$ (t_r is reaching time), the solution of (1) is given by

$$\begin{cases} x_1(t) = x_1(t_r) + \int_{t_r}^{t} -\frac{a_2}{a_1}\dot{\tilde{x}}_2 - \frac{\tau_{sm}}{a_1}\dot{w}_2(t) + \\ \qquad\qquad + \frac{\tau_{sm}}{a_1}\ddot{y}_{re} - \frac{a_3}{a_1}\dot{y}_r \, d\tau \\ \tilde{x}_2(t) = \tilde{x}_2(t_r)\, e^{-\frac{1}{\tau_{sm}}(t-t_r)} \end{cases} \tag{4}$$

So, the output error converges exponentially to zero.

Proof. At the surface $S(x) = \dot{S}(x) = 0$, we have

$$\begin{aligned} B(x)u_{eq} &= -(f_1(x) + w_1 + \frac{a_2}{a_1}\dot{\tilde{x}}_2 - \frac{\tau_{sm}}{a_1}\dot{w}_2(t)) \\ &+ \frac{\tau_{sm}}{a_1}\ddot{y}_{ref} - \frac{a_3}{a_1}\dot{y}_{ref} \end{aligned} \tag{5}$$

According to equations (5) to (1), for all $t \geq t_r$, the motions equations are of the form,

$$\begin{cases} \dot{x}_1 = -\frac{a_2}{a_1}\dot{\tilde{x}}_2 - \frac{\tau_{sm}}{a_1}\dot{w}_2(t) + \frac{\tau_{sm}}{a_1}\ddot{y}_{ref} - \frac{a_3}{a_1}\dot{y}_{ref} \\ \dot{\tilde{x}}_2 = -\frac{1}{\tau_{sm}}\tilde{x}_2 \end{cases} \tag{6}$$

this means that (4) is well verified □

So, in the course of sliding mode, it is important that its output error ($\tilde{x}_2(t) = \tilde{x}_2(t_r)\, e^{-\frac{1}{\tau_{sm}}(t-t_r)}$) depends neither on the parameter nor disturbance.

From the theorem 1 and definition 1, the system (1) is a system with mismatch disturbance ($x_1(t)$ depend on the $w_2(t)$).

4. Sliding Mode Control Design

In the design of a Sliding Mode Control (SMC), there exist a number of approaches, in particular the method based on the selection of a Lyapunov function. The control should be chosen such that the following candidate Lyapunov function satisfies Lyapunov stability criteria:

$$V = \frac{1}{2} S^2 \tag{7}$$

The derivative of the Lyapunov function is negative definite, thus guaranteeing motion of the state trajectory to the manifold:

$$\dot{V} = S\dot{S} < 0 \tag{8}$$

The time derivative of $S(x)$ is

$$\begin{aligned}\dot{S}(t) = h\big[&a_1(f_1(x) + w_1) + a_1 B(x)u + a_2 \dot{x}_2 \\ &+ \tau_{sm}\dot{w}_2(t) + \upsilon\big]\end{aligned} \tag{9}$$

The control input signal can be given as

$$u = u_c + \Delta u \tag{10}$$

where u_c is the continuous control

$$u_c = \frac{-1}{B(x)}\left(f_1(x) + \frac{a_2}{a_1}\dot{x}_2 + \frac{\upsilon}{a_1}\right) \tag{11}$$

and Δu is the corrective control term

$$\Delta u = \frac{-1}{B(x)}\frac{K}{a_1} sign(S) \tag{12}$$

The constant K *is chosen so that* a sliding phase ($S(x) = 0$) occurs.

According to (8) this implies

$$K > w_{01} + \tau_{sm} dw_0 \tag{13}$$

Then, after a finite time t_r which depends only on the parameter τ_{sm} , the system state reaches the sliding surface.

4.1 Smooth control interpolation in the boundary layer

The continuous sliding mode control has been selected commonly in SMC problems to avoid chattering phenomena. To solve this problem, the signum function is replaced in (12) by a smooth function. Many functions have been used, introducing a thin boundary layer around the sliding surface [15]. For example a saturation function has been used

$$sat(S) = \begin{cases} sign(S) & if\ |S| > \varphi \\ \dfrac{S}{\varphi} & if\ |S| \le \varphi \end{cases} \tag{14}$$

where φ is a positive constant, $\varphi \in R^+$. φ defines the thickness of the boundary layer.

4.2 Behavior inside the boundary layer

The presence of a boundary layer is required to completely eliminate the chattering phenomenon. In order to have a robust feedback system, the use of a small φ is needed, but a too small φ will increase the chattering problem. Thus, it is very important to know the system behavior inside the boundary layer to be able to set an appropriate value of φ.

4.2.1 Nominal system behavior in the boundary layer

In this section we show that the sliding mode control algorithm guarantees the convergence of the nominal system to equilibrium point.

Lemma 1. *Consider the sliding surface (3) and the control input (10) with smooth function (14), for the nominal case* ($w_1(t)=0$ *and* $w_2(t)=0$), *we have*

$$\lim_{t \to +\infty} S(t) = 0 \tag{15}$$

The convergence of $y(t)$ *to* y_r *is guaranteed.*

Proof. In the nominal case, with the continuous sliding mode control, outside of the boundary layer, the derivative of the sliding surface is

$$\dot{S}(t) = -K\ sign(S(t)) \tag{16}$$

where $t_r = \dfrac{|S(0)| - \varphi}{K}$

In the boundary layer the derivative of S is

$$\dot{S}(t) = -K \frac{S(t)}{\varphi} \tag{17}$$

K is a positive constant. The solution of expression (17) is given by

$$S(t) = \varphi\, sign[S(t_r)]\, e^{-\frac{K}{\varphi}(t-t_r)} \tag{18}$$

which implies (15)
Then, the convergence of $y(t)$ to y_r is guaranteed.

□

The following figure (1) shows the trajectory of the sliding surface in the boundary layer.

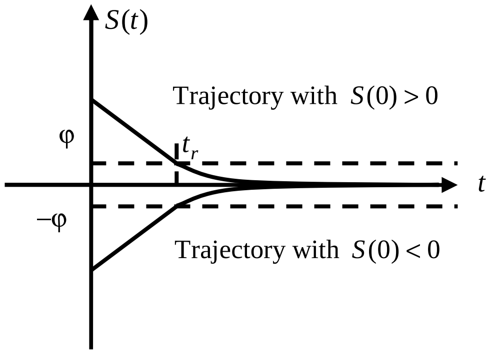

Fig. 1. Nominal system behavior in boundary layer.

4.2.2 Uncertain and disturbed system behavior in the boundary layer

Notation Knowledge of the maximum disturbance bound is needed to design a sliding mode controller. The expressions of $\varepsilon(t)$ and ξ are used to simplify our analysis. $\varepsilon(t) \in \Re$ is defined by:

$$\varepsilon(t) = a_1 w_1(t) + \tau_{sm} \dot{w}_2(t) \tag{19}$$

$\xi \in \Re$ is constant and defined such that:

$$\xi = |a_{1\max}| w_{01} + \tau_{sm} dw_{02} \tag{20}$$

Lemma 2. *Consider the sliding surface* (3) *and the control input* (10). *If the parametric uncertainty and disturbance are bounded, then for all time* $t > t_r$ (t_r *is the reaching time to the boundary layer:* $S(t_r) = \varphi sign[S(t_r)]$), *then the surface* $S(t)$ *is bounded inside the boundary layer:*

$$|S(t)| < \varphi\,, \quad \forall\, t > t_r \tag{21}$$

Proof. For $\left|S(x,t)\right| > \varphi$ the system trajectories are guaranteed to converge to the boundary layer by the law (8), but when $\left|S(x,t)\right| \le \varphi$ the structure of control is changed and the $\mathrm{sign}(S)$ becomes $\dfrac{S}{\varphi}$, we have then:

$$\frac{\varphi}{K}\dot{S}(t) + S(t) = \frac{\varphi}{K}\varepsilon(t) \tag{22}$$

Equation (22) is a first-order differential equation with a second member, the general solution to this equation for $t>t_r$, is

$$S(t) = \varphi\, sign[S(t_r)] + e^{-\frac{K}{\varphi}t}\int_{t_r}^{t}\varepsilon(\tau)e^{\frac{K}{\varphi}\tau}\,d\tau \tag{23}$$

From equations (19) and (20), we have $-\xi \le \varepsilon(t) \le \xi$, then

$$S_m(t) \le S(t) \le S_M(t) \tag{24}$$

where

$$S_m(t) = -e^{-\frac{K}{\varphi}t}\int_{t_r}^{t}\xi e^{\frac{K}{\varphi}\tau}\,d\tau$$

$$S_M(t) = e^{-\frac{K}{\varphi}t}\int_{t_r}^{t}\xi e^{\frac{K}{\varphi}\tau}\,d\tau$$

In the case $S(t_r) > 0$, we have

$$S_m(t) = -\varphi\,[\frac{\xi}{K} - (1+\frac{\xi}{K})e^{-\frac{K}{\varphi}(t-t_r)}]$$

$$S_M(t) = \varphi\,[\frac{\xi}{K} + (1-\frac{\xi}{K})e^{-\frac{K}{\varphi}(t-t_r)}]$$

Then

$$-\varphi < S(t) < \varphi$$

In the case $S(t_r) < 0$, we have

$$S_m(t) = -\varphi\,[\frac{\xi}{K} + (1-\frac{\xi}{K})e^{-\frac{K}{\varphi}(t-t_r)}]$$

$$S_M(t) = \varphi\,[\frac{\xi}{K} - (1+\frac{\xi}{K})e^{-\frac{K}{\varphi}(t-t_r)}]$$

Then

$$-\varphi < S(t) < \varphi$$

Thus, for all $t>t_r$, $S(t)$ is bounded such that

$$\left|S(t)\right| < \varphi$$

hence equation (21) of lemma 2 is satisfied.

□

The system behavior is not determined inside a boundary layer (see figure 2); further convergence to zero depends on the parameter $\varepsilon(t)$, equation (22).

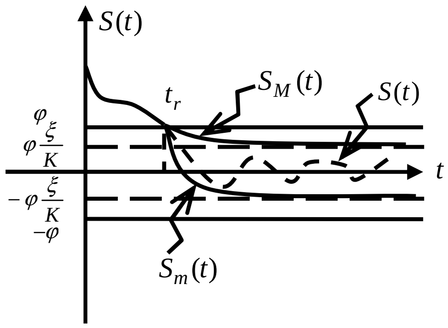

Fig. 2. Behavior inside the boundary layer (case $S(t_r)$>0)

4.3 Integrator corrector in the boundary layer

In order to solve a steady-state error problem, an integral sliding manifold was proposed in [5] and [13]. This development is introduced and justified only by tests on specific systems. Our idea consists in reconstituting a control law to eliminate the steady-state error created by the disturbance. We added an integrator when the trajectories of states approach the reference [9], [10] and [11]

$$u = u_c + \Delta u + u_r \tag{25}$$

where u_c and Δu are defined in (11,12) and u_r is given by

$$u_r = \begin{cases} -k_i \int (y(\tau) - y_{réf}(\tau))\, d\tau & when\ |\tilde{y}| \le \delta \\ 0 & when\ |\tilde{y}| > \delta \end{cases} \tag{22}$$

where k_i is the integral constant and δ is a positive constant satisfying

$$\delta \ge \frac{\xi}{hK}\varphi \tag{27}$$

5. Experiments Results

In order to illustrate the above design techniques and demonstrate the performances of systems control, the proposed algorithm is applied to a physical laboratory plant consisting of a three tank system figure (3 & 4); the objective is to control the liquid level of tank three while introducing a leakage (external disturbance) in the outflow pipe of tanks 1 and 3. The experimental schemes have been done under Matlab/Simulink, using Real-Time Workshop interface, and run on the DS1102 DSPACE system, which is equipped with a Power PC processor. The control algorithm is implemented on a DSP (TMS 320C31).

Fig. 3. Benchmark (three-tank)

The system consists of three liquid tanks that can be filled with two identical, independent pumps acting on the outer tanks 1 and 2. The pumps deliver the liquid flows Q_1 and Q_2 and they can be continuously manipulated from a flow of 0 to a maximum flow Q_{max}. The tanks are interconnected to each other through lower pipes. The flow through these pipes can be interrupted with switching valves C_{13}, C_{32}, C_{20}, L_1, L_2 and L_3. The out flowing liquid is collected in a tank, which supplies the pumps 1 and 2. Here the circle is closed. The liquid levels $h_1(t)$, $h_2(t)$ and $h_3(t)$ in each tank can be measured with continuous valued level sensors.

From the conservation of mass in the tanks we obtain the differential equations

$$\begin{aligned} \dot{h}_1(t) &= \frac{1}{A}(Q_1 - Q_{13} - Q_{10}) \\ \dot{h}_2(t) &= \frac{1}{A}(Q_2 + Q_{32} - Q_{20}) \\ \dot{h}_3(t) &= \frac{1}{A}(Q_{13} - Q_{32} - Q_{30}) \end{aligned} \tag{28}$$

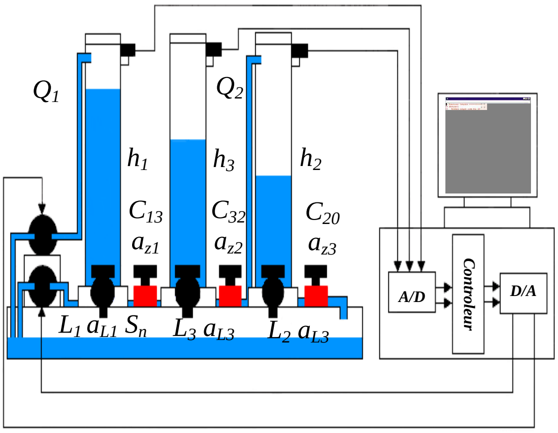

Fig. 4. Benchmark (three-tank)

where

$$Q_{13} = S_n a_{z1} sign(h_1(t) - h_3(t))\sqrt{2g|h_1(t) - h_3(t)|}$$

$$Q_{32} = S_n a_{z2} sign(h_3(t) - h_2(t))\sqrt{2g|h_3(t) - h_2(t)|}$$

$$Q_{20} = S_n (a_{L3} + a_{z3})\sqrt{2gh_2}$$

$$Q_{i0} = S_n a_{Li}\sqrt{2gh_3} \quad i = 1,2,3$$

h_i(t): the level in the first tank T_i i=1,2,3;
$Q_{13}(t)$: the flow rate from tank T_1 to tank T_3;
$Q_{32}(t)$: the flow rate from tank T_3 to tank T_2;
Q_{20}(t): outflow due to leak in tank T_2;
Q_{10}(t): outflow due to leak in tank T_1;
Q_{30}(t): outflow due to leak in tank T_3

In this paper, we propose a sliding mode control with integral action for the coupled tanks T_1 and T_3. The tank T_2 is used to simulate a disturbance.

The outflows through valves C_{20}, L_1, and L_2 are zero and they are used to model failures of the system.

The dynamic model of the coupled tank can be written as

$$\begin{cases} \dot{x}_1 = f_1(x) + u + w_1 \\ \dot{x}_2 = f_2(x) + w_2 \\ y = x_2 \end{cases} \tag{29}$$

where

$x = [x_1, x_2]^T = [h_1, h_3]^T$

$$f_1(x) = -\beta_1 sign(x_1(t) - x_2(t))\sqrt{|x_1(t) - x_2(t)|}$$
$$f_2(x) = \beta_1 sign(x_1(t) - x_2(t))\sqrt{|x_1(t) - x_2(t)|} - C_3\sqrt{h_2}$$

$$u = \frac{Q_1}{A}$$
$$w_1(t) = -\frac{Q_{10}}{A} + \Delta f_1$$

$$w_2(t) = -\frac{Q_{32}}{A} + \Delta f_2$$
$$\beta_i = \frac{S_n a_{zi}\sqrt{2g}}{A} \quad \text{i= 1, 2, 3,}$$
$$C_3 = \frac{S_n a_{L3}\sqrt{2g}}{A}$$

The parameters of three tank system are defined in the following table

Table 1. Model parameters of the three tank system

Symbol	Value	Meaning
A	0.0154 m^2	tank section
S_n	$2.5*10^{-5}$ m^2	cross-section of valves
a_{Zi}	$0 \le a_{zi} \le 1$	flow correction term (i=1, 2,3)
a_{Li}	$0 \le a_{Li} \le 1$	leakage flow correction term (i=1, 2,3)
g	9.81 m/s^2	gravity constant m/s^2
h_{max}	0.6 m	maximum water level in each tank
Q_{imax}	1.17 10^{-4} m^3/s	maximum inflow through pump i (i = 1, 2)

During operation of the system, high-frequency noise corrupts measurements from the pressure sensors, to alleviate this problem; a sliding mode differentiator is used to obtain better measurements of the water height.
To obtain realistic results, the experimentations are carried out using the following uncertain parameters variations and perturbations:

$0{,}004 \le \beta_1 \le 0.0073$, $\beta_2 \le 0.0073$, $\beta_3 \le 0.0073$. $|h_1 - h_3|_{\min} = 0.001m$, $|h_1 - h_3|_{\max} < 0.2m$,

We choose the following sliding surface:
$S(t) = h(\tau_{sm}\dot{\tilde{x}}_2 + \tilde{x}_2)$, with h= 0.1 and $\tau_{sm} = 40\ s$.
Then

$$a_{1\max} = \tau_{sm} \frac{\beta_{1\max}}{2\sqrt{|h_1 - h_3|}_{\min}} \approx 2.1\,,\ |\Delta f_1|_{\max} = |\Delta f_2|_{\max} = (\beta_{1\max} - \beta_{1\min})\sqrt{|h_1 - h_3|}_{\max}$$

$$w_{1\max} = (|\Delta f_1|_{\max} + 0.0073\sqrt{h_{1\max}}) \approx 0.0051\text{m/s}$$

The disturbance $w_2(t)$ is assumed to be a slowly varying function of time, thus

$$\dot{w}_2 \approx 0$$

Then, from relation (13) we have:

$$K=0.02>0.011.$$

Remarks :

1- We also have:

$$w_{1\max} < \frac{Q_{1\max}}{A} = 0.0077 m/s$$

2- Note from a_1 and a_2 that there is a singularity if $x_1 = x_2$. In fact, only the $\frac{a_2}{a_1}$ and $\frac{1}{a_1}$ expressions are used in the control design:

$$\frac{a_2}{a_1} = \frac{2\sqrt{|x_1(t) - x_2(t)|}}{\tau_{ms}\beta_1} - (1 + \frac{2C_3\sqrt{|x_1(t) - x_2(t)|}}{\tau_{sm}\beta_1\sqrt{x_2}}) \quad \text{and} \quad \frac{1}{a_1} = \frac{2\sqrt{|x_1(t) - x_2(t)|}}{\tau_{ms}\beta_1}.$$

Furthermore, if $x_2 = 0$ there is a control singularity. In reality, there is no control applied to the system. In our control design we assumed that $x_1(0) \neq x_2(0) \neq 0$.

Discussion. The classical SMC, sliding mode control with smooth function SMGS and the sliding mode control with saturation and integral action SMGSI are shown in figures 5, 6 and 7. The step responses of all the three controllers are shown in figure 5.b, 6.b and 7.b. Generally, it can be seen that the transient response of all the three controllers are good, and their performances are very similar. However, it can be observed that the control signal from the sliding mode control with saturation is smoother than the classical sliding mode control schemes fig. 5.a and 6.a, but we have an error after t=408 s when we introduce a step disturbance w_1 fig 6.c.

Figure 7.b presents the system responses $h_1(t)$ and $h_3(t)$, when the SMCSI is used. In the first two intervals the system responses converge toward the reference fig. 11.c. It is noted that the control input $Q(t)$ is with minimal amounts of chattering caused by output derivative feedback in the boundary layer figure 7. a.

The integral action compensates the parametric uncertainties and the external disturbances, the integral action is almost null $Q_r = A\,u_r(t) \approx 5\times10^{-6} m^3.s^{-1}$ in the first and second intervals. In the third interval, the opening of the valve L_1 creates a disturbance $w_1(t)$. In this part, $Q_r = A\,u_r(t) \approx 4.15\times10^{-5} m^3.s^{-1}$

Therefore, it can be concluded that the proposed control SMCSI schemes are robust against the parameters changes and disturbances acting on the system.

6. Conclusions

A nonlinear sliding mode control design method is described in this paper for systems with mismatched disturbance and uncertainties. We start from a sliding surface design and sliding mode control design. Then, we establish a relationship between disturbance and steady-state error if the dynamics of the expression $\varepsilon(t)$ has negligible dynamics. In this context, we propose an integral action to eliminate the steady-state error.

We validated this method on a nonlinear three-tank system. The obtained results prove the viability of this control method and present good performances in term of robustness to disturbances and system uncertainties.

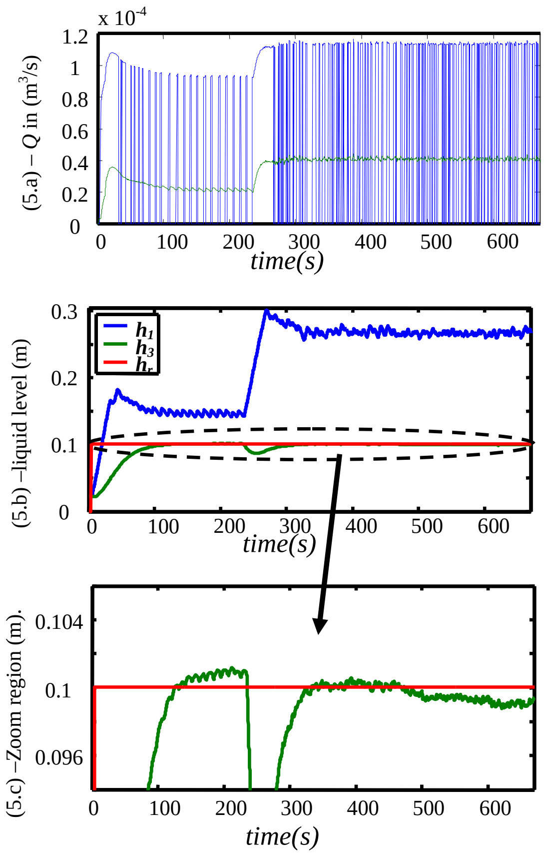

Fig. 5: Experiments results of classical SMC.

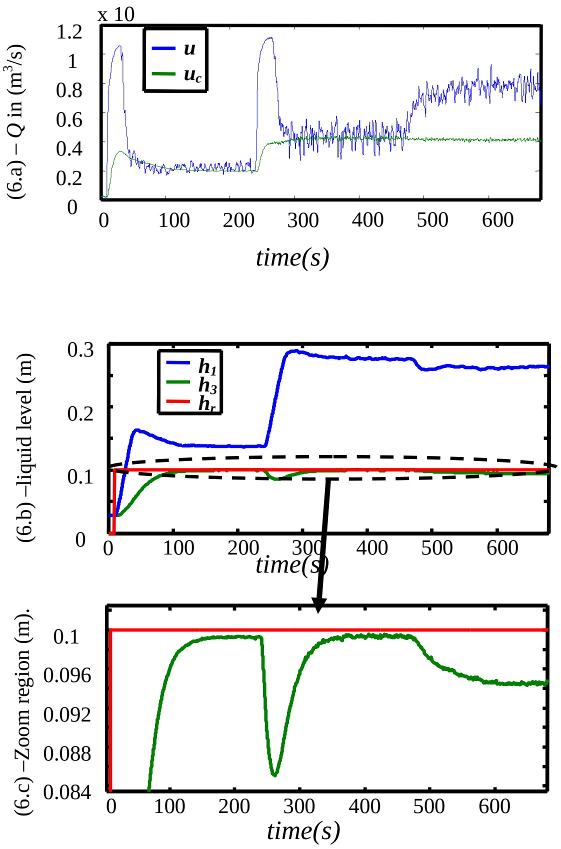

Fig. 6. Experiment result of SMC with smooth control

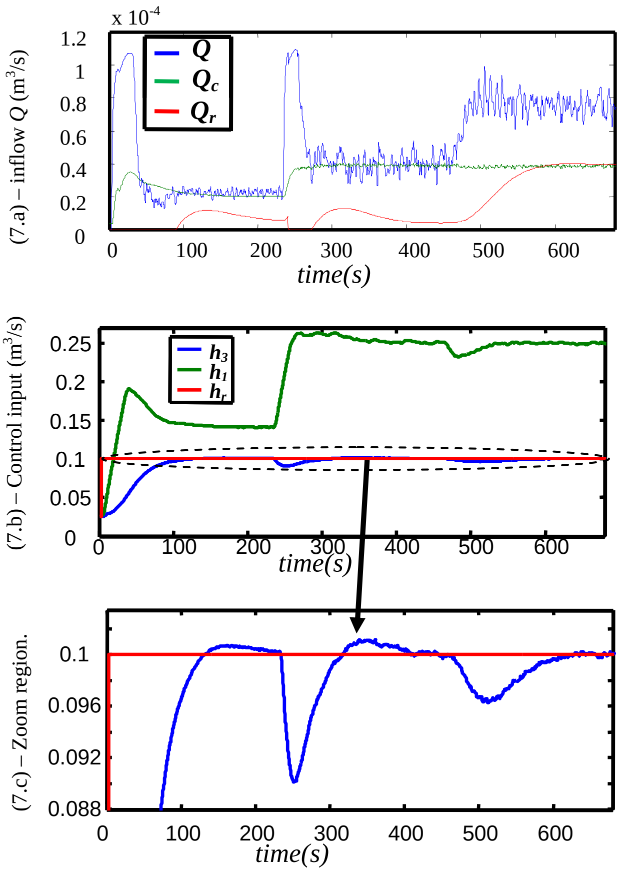

Fig. 7. Experiment results of proposed control.

References

1. H. R. Bühler, “Réglage par mode de glissement”, Presses Polytechniques et Universitaires Romandes, 1986.
2. C. T. Chen, S. T. Peng, “Design of a sliding mode control system for chemical processes”, Journal of Process Control, 2005, p 515–530.
3. B . Drazenovic, “the invariance condition in sliding mode systems”, automatica, Vol.5, p.287-295, Pergamon Press, 1969.
4. C. Edwards and S. K. Spurgeon, “On the limitations of some variable structure output feedback controller designs”, Automatica, 2000, Vol 36, Issue 5, p 743-748.
5. I. Eker and S. A.Akinal, “Sliding mode control with integral action and experimental application to an electromechanical system”, IEEE-ICSC Congress, 2005, 6p.
6. S. V. Emelyanov, S. K. Korovin, A. L. Nersisyan, and YY. Nisenzon, “Discontinuous output feedback stabilizing and uncertain MIMO plant”. *International Journal of Control*, vol. 55, pp. 83-107, 1992.

7. A. F. Fillipov, “Differential equations with discontinuous right hand sides” Mathematicheskii Sbornik, 51, 1960, p 99-128.
8. U. Itkis, “Control systems of variable structure”, Wiley, New-York, 1976.
9. S. Mahieddine-Mahmoud, L. Chrifi-Alaoui, V. Van Assche, “ Commande par mode de glissement en présence de perturbations mal- adaptées d’un système mono-variable”, Huitième conférence internationale des Sciences et Techniques de l’Automatique STA, Tunisie, 05 au 07 novembre 2007, CD pp n° ACS-183.
10. S. Mahieddine-Mahmoud, L. Chrifi-Alaoui, V. Van Assche & J-M. Castelain. “Sliding Mode Control Using Integrator Corrector and Implementation For a Three-Tank System”, IEEE-IECON, France 2006, p 706-711.
11. S. Mahieddine-Mahmoud, L. Chrifi-Alaoui, V. Van Assche & P. Bussy, “Sliding Mode Control with Unmatched Disturbance and Implementation For a Three-Tank System”, IEEE-ICIT, India du 15 au 17 décembre 2006, pp 675-680.
12. W. Perruquetti and J.P. Barbot, “Sliding mode control in engineering”, Marcel Dekker, 2002.
13. S. Seshagiria, HK. Khalil, “Robust output feedback regulation of minimum-phase nonlinear systems using conditional integrators”, Automatica, 2005, 41, 43 – 54.
14. H. Sira-Ramírez and S. K. Spurgeon “Robust Sliding Mode Control Using Measured Outputs” Journal of Mathematical Systems, 1996, Vol. 6, No 3, p 1-13.
15. J. J. E. Slotine and W Li “Applied nonlinear control” Printice-Hall International, Inc. 1991.
16. S. K. Spurgeon and R Davies, “A nonlinear control strategy for robust sliding mode performance in the presence of unmatched uncertainty”. International Journal of Control, vol. 57, pp. 1107- 1124, 1993
17. V. Utkin, “Sliding mode control in electromechanical system”. Taylor & Francis 1999.
18. B. Weiping, X. Mu, “Stability for uncertain nonlinear dissipative systems”, Applied Mathematical Modelling, 2007.