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Stabilization of uncertain polytopic system by switched command law: multiobserver based

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Abstract. *This paper presents a new approach to stabilize robustly, by state feedback control, an uncertain polytopic system with non measurable states. The control law is obtained by switching between estimated outputs generated by different observers designed from the tops of polytope. Every controller is synthesized to stabilize a sub region into the polytope. All the stabilized sub regions describe the whole polytope. The computing of state feedback gains and observers gains are based on the use of LMI formulation. This work is, also, presenting techniques for resolving the non convexity of obtained LMI's by using a relaxation algorithm. The used switched law guaranties the robust plant stabilization by choosing the optimal controller.*

Keywords. *uncertain system, polytopic uncertainty, observers, robust stabilization, non convex LMI, LMI relaxation, switched command system, switched laws.*

1. Introduction

During the last couple of decades, the switched systems were the subject of several publications in the theory of system commands. In fact, several industrial applications impose a changing in the regime of functioning generally which are leading to a modification of system dynamics. Take for example the regime of changing speed in vehicle [5], the control of some robots and the flexible workshops [3], [14], [11] the functioning of human heart can be also modelized by the hybrid system [17].

Many researchers were interested in the stabilization of switched systems that have uncontrollable switched law between subsystems [16], [7], [8], [9]. The interest

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of such approach is based on the formulation of a necessary and a sufficient condition to stabilize asymptotically a switched system for an arbitrary switching law.

Other approaches are interested in determining a switched law among many supervisors who guarantee systems stability [21], [15], [13] whose constraints of functioning impose the switching between many controllers [20], [23]. We can quote the work of Petterson and Lennarston [20] who assume a perfect knowledge of a linear system and the command laws. They present a method for a synthesis, if it is possible, for a stabilized switching law.

In 1994, wicks [23] assumed a switched law for the stabilization of a linear system by using two switched commands.

In 1997, Skafidas [21] presents a necessary and sufficient condition for the robust stability of systems commanded by synchronous switched controllers.

In 1998, Branicky [4] presents the use of several Lyapunov functions as a tool for the study of some switched command systems.

In 1998, Savkin [21] laid down sufficient conditions for the robust stabilization by output feedback with switched synchronous controllers.

In the second part of this paper, we'll start by the problem formulation of an uncertain system stabilization having an uncertainty of a polytopic form and we'll equally represent the principle of a restrained command figure.

In the third part, we'll assume a developed command from a single barycentric observer using the formulation *L.M.I*. An illustrative example is presented to show the limits of using a single observer for the stabilization of these particular types of systems.

In the fourth part, we'll start by presenting the construction of a multiobserver as well as the conditions for polyquadratic stabilization of every sub region of observation. Then, we'll show the criterion of switching between different observers guaranteeing the stabilization of an uncertain system. An illustration is provided below this part.

2. Problem formulation

The discrete and uncertain polytopic L.T.I system can be described by the following representation:

$$\begin{cases} x(k+1) = \sum_{i=1}^{i=n} \lambda_i A_i x(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (1)$$

with :A_i : one of the vertices of the polytop, B: input matrix, C: observation matrix, λ_i : activation function satisfying :$0 \le \lambda_i \le 1$ et $\sum_{i=1}^{i=n} \lambda_i = 1$.

Every activation function is uncertain but time invariant.
In the rest of this paper, we assume that some of the states of this system are not measurable. We proposed to synthesize a switched command law based on the use of many observers designed from the vertices of polytop. We are equally determining that in case of an uncertain system, the principle of separation is not applicable, and to get round this difficulty, many researchers [22] were oriented towards the use of a single observer in an uncertain system case. This approach does not anyway allow the stabilization of the system in a great deal of uncertainty [18].

We consider P, points of polytop from which we build P observers , one observer is written in the following form :

$$\begin{cases} \hat{x}_j(k+1) = \hat{A}_j \hat{x}_j(k) + Bu(k) + L_j(y(k) - \hat{y}_j(k)) \\ \hat{y}_j(k) = C\hat{x}_j(k) \end{cases} \quad (2)$$

L_j : gain matrix of observer j

In order to stabilize the uncertain system commanded by the group of observers' j as $j \in \{1..P\}$ and described in (2): there must be:

- Every observer j stabilizes a region P_j of polytop as the groups of regions Pj hide the global region described by the polytop.
- The switching law guarantees the global system stabilization by choosing the best observer and then the best controller.

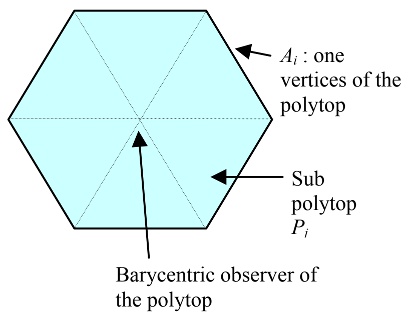

Fig.1. Polytop of uncertain system

The scheme of control bases on the use of multiobserver and switched law is represented on figure 2.

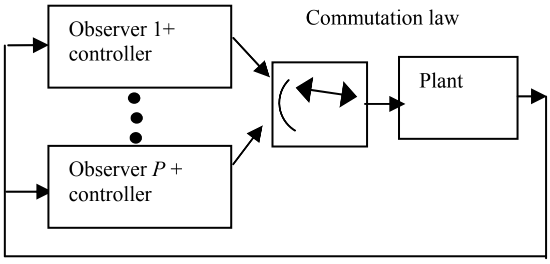

Fig.2. Scheme of control: multiobserver based

3. Stabilization by barycentric observer

In this paragraph, we are propose to use a single observer, barycenter of the polytop to stabilize an uncertain system.

3.1. Augmented system equations

We consider the system described by (1), the state equations of the barycentric polytop observer are written in an analogue way to (2):

$$\begin{cases} \hat{x}_b(k+1) = \hat{A}_b \hat{x}_b(k) + Bu(k) + L_b(y(k) - \hat{y}_b(k)) \\ \hat{y}_b(k) = C\hat{x}_b(k) \end{cases} \tag{3}$$

Let: $\varepsilon(k) = x(k) - \hat{x}_b(k)$ (4)

The dynamic of error ε $(k+1)$ is, then, written as follows:

$$\varepsilon(k+1) = \sum_{i=1}^{n} \lambda_i (A_i - \hat{A}_b)\, x(k) + \sum_{i=1}^{n} \lambda_i (A_i - LC)\, \varepsilon(k) \tag{5}$$

The control law $u(k)$ is :

$$u(k) = K\hat{x}_b(k) \tag{6}$$

The augmented system equation:

$$\begin{pmatrix} x(k+1) \\ \varepsilon(k+1) \end{pmatrix} = \sum_{i=1}^{i=n} \lambda_i \begin{pmatrix} A_i + BK & -BK \\ A_i - \hat{A}_b & \hat{A}_b - LC \end{pmatrix} \begin{pmatrix} x(k) \\ \varepsilon(k) \end{pmatrix} \tag{7}$$

Let: $\phi_i = \begin{pmatrix} A_i + BK & -BK \\ A_i - \hat{A}_b & \hat{A}_b - LC \end{pmatrix}$ (8)

It's not possible to use the separation principle due to the presence of the term $A_i - \hat{A}_b$.

Taking into account equality (8), equality (7) is then written as follows:

$$\begin{pmatrix} x(k+1) \\ \varepsilon(k+1) \end{pmatrix} = \sum_{i=1}^{i=n} \lambda_i \phi_i \begin{pmatrix} x(k) \\ \varepsilon(k) \end{pmatrix} = \Phi(\lambda) \begin{pmatrix} x(k) \\ \varepsilon(k) \end{pmatrix} \quad (9)$$

with

$$\Phi(\lambda) = \sum_{i=1}^{i=n} \lambda_i \phi_i \begin{pmatrix} x(k) \\ \varepsilon(k) \end{pmatrix} \quad (10)$$

3.2. Robust stability analysis

We consider the system defined by:

$$x(k+1) = \Phi(\lambda) x(k) \quad (11)$$

The matrix Φ(λ) to the set of convex polytopic Φ defined by :

$$\Phi := \left\{ \Phi(\lambda) : \Phi(\lambda) = \sum_{i=1}^{i=n} \lambda_i \phi_i \ avec \sum_{i=1}^{i=n} \lambda_i = 1 \ et \ \lambda_{i \geq 0} \right\} \quad (12)$$

3.2.1. Definition 1 (Oliveira et al, 1999)[19]

The system described by (11) is robustly stable in the uncertain domain described by (12) if all the characteristic values of the matrix $\Phi(\lambda)$ have a magnitude inferior to 1 for all the values of λ as $\Phi(\lambda)$ belongs to Φ.

The theorem 1 presents *LMI* formulation of the robust stability problem of polytopic uncertain system.

3.2.2. Theorem 1 (Oliveira et al, 1999)[19]

The uncertain system (11) is robustly stable in the uncertain domain described by (12) if the symmetric matrices P_i exist and a matrix G satisfying the following inequalities:

$$\begin{pmatrix} P_i & \phi_i^T G^T \\ G\phi_i & G+G^T - P_i \end{pmatrix} > 0 \quad for\ i=1,..,n \tag{13}$$

The matrix G is introduced to synthesize one only command that stabilizes the whole of polytop taking into consideration the Lyapunov functions proper to every top.

3.3. Application in case of barycentric observer

The application of theorem 1 in case of system (9) allowed, by substituting:

$$G = \begin{pmatrix} G_1 & G_2 \\ G_3 & G_4 \end{pmatrix} \tag{14}$$

with G_i of nxn dimension, to obtain the following conditions of robust stability :

$$\begin{pmatrix} P_i & \begin{pmatrix} A_i + BK & -BK \\ A_i - \hat{A}_b & \hat{A}_b - LC \end{pmatrix}^T \begin{pmatrix} G_1 & G_2 \\ G_3 & G_4 \end{pmatrix}^T \\ \begin{pmatrix} G_1 & G_2 \\ G_3 & G_4 \end{pmatrix} \begin{pmatrix} A_i + BK & -BK \\ A_i - \hat{A}_b & \hat{A}_b - LC \end{pmatrix} & \begin{pmatrix} G_1 & G_2 \\ G_3 & G_4 \end{pmatrix} + \begin{pmatrix} G_1 & G_2 \\ G_3 & G_4 \end{pmatrix}^T - P_i \end{pmatrix} > 0$$

$$\forall\ i = 1,..,n \tag{15}$$

The development of expressed condition (15) leads to (16)

$$\begin{pmatrix} P_i & (\bullet)T \\ \begin{pmatrix} G_1 A_i + BU_1 + G_2(A_i - \hat{A}_b) & -BU_1 + G_2\hat{A}_b - Y_2 C \\ G_3 A_i + BU_3 + G_4(A_i - \hat{A}_b) & -BU_3 + G_4\hat{A}_b - Y_4 C \end{pmatrix} & G + G' - P_i \end{pmatrix} > 0$$

$$\forall\ i = 1,..,n \tag{16}$$

$$with: BV_1 = G_1 B,\ BV_3 = G_3 B,\ \begin{bmatrix} U_1 \\ U_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_3 \end{bmatrix} K \quad and \quad \begin{bmatrix} Y_2 \\ Y_4 \end{bmatrix} = \begin{bmatrix} G_2 \\ G_4 \end{bmatrix} L$$

The research of *LMI* solutions to (16) with constraints of equality is not a convex problem. The K and L solution of imposed equalities exist only if the following conditions were checked [12]:

$$\begin{aligned} & rang\begin{pmatrix} G_2 \\ G_4 \end{pmatrix} = rang\begin{pmatrix} Y_2 & G_2 \\ Y_4 & G_4 \end{pmatrix} \ and \\ & rang\begin{pmatrix} V_1 \\ V_3 \end{pmatrix} = rang\begin{pmatrix} U_1 & V_1 \\ U_3 & V_3 \end{pmatrix} \end{aligned} \tag{17}$$

A number of techniques were used to find solutions to non convex problems such as the algorithm of relaxation proposed by Halabi [12]

The solution of (16) has become a convex problem in the case where $G_2 = G_3 = 0$. In other words, in the case where the matrix G is a diagonal bloc because the matrix G_4 has an inverse and we have in this case (18)

$$\begin{aligned} K &= V_1^{-1} U_1 \\ L &= G_4^{-1} Y_4 \end{aligned} \tag{18}$$

The choice of G_2 null, G triangular inferior, allows eliminating the non convexity on L. In This case we'll start by calculating K stabilizing the pairs (A_i, B) for $i \in \{1..n\}$, then in the second step, we'll calculate L using (18) with checking the feasibility of condition (13).

3.4. Illustration example

We consider the benchmark described by Wie and Bernstein, 1992)[24] and adopted by (M.V. Kothare et al, 1996)[18] as well as (Skafidas et al, 1999)[21] and (Cuzzola et al, 2002)[6].

The system consists of two masses related by a spring (see Figure 3). The discrete state representation (M.V. Kothare et al, 1996) is:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ \dfrac{-0.1r}{m_1} & \dfrac{0.1r}{m_1} & 1 & 0 \\ \dfrac{0.1r}{m_2} & -\dfrac{0.1r}{m_2} & 0 & 1 \end{pmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{0.1}{m_1} \\ 0 \end{bmatrix} u(k) \tag{19}$$

$$y(k) = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix} x(k)$$

$x_1(k)$ and $x_3(k)$ are ,respectively, the position and the speed of truck 1. $x_2(k)$ and $x_4(k)$ are, respectively, the position and the speed of truck 2 .

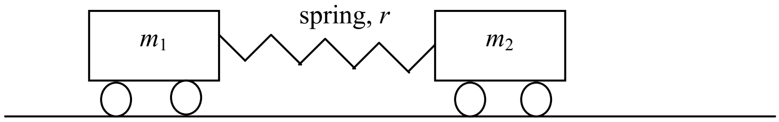

Fig. 3. Two trucks related by spring

Having :
$m = m_1 = m_2 = 1Kg$
$r \in [r_{\min} \;\; r_{\max}] = [0.5 \;\; 15]$ Nm
The computing, in case $G_2 = 0$, of gains K and L leads to (20)

$$\begin{aligned} K &= [-155.9829 \; 148.2237 \; -22.8299 \; -34.5856] \\ L &= [-2.2718 \; 2.2849 \; -11.4529 \; 11.4529]^T \end{aligned} \tag{20}$$

$$G_1 = 1e^{-3}\begin{pmatrix} 0.1698 & -0.1635 & 0.0091 & 0.0380 \\ -0.1638 & 0.1558 & -0.0086 & -0.0348 \\ 0.0094 & -0.0091 & 0.0007 & 0.0022 \\ 0.0517 & -0.0487 & 0.0026 & 0.0237 \end{pmatrix}$$

$$G_3 = 1e^{-3}\begin{pmatrix} -0.1646 & 0.1569 & -0.0088 & -0.0608 \\ -0.2651 & 0.2555 & -0.0176 & -0.0748 \\ -0.6716 & 0.7189 & -0.0420 & -0.6939 \\ -0.6687 & 0.7159 & -0.0417 & -0.6941 \end{pmatrix} \tag{21}$$

$$G_4 = \begin{pmatrix} 0.0039 & 0.0039 & -0.2511 & -0.2511 \\ 0.0045 & 0.0053 & -0.2194 & -0.2194 \\ -0.2486 & -0.2478 & 119.2685 & 119.2684 \\ -0.2487 & -0.2479 & 119.2660 & 119.2659 \end{pmatrix}$$

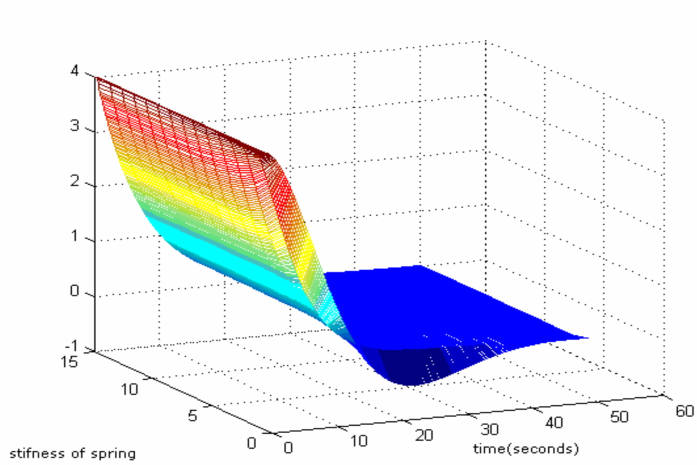

Fig. 4. Evolution, in time, of position of truck 2 for many different values of *r*

4. Stabilization by a switched multiobserver

The defect in using a single observer is the limitation of the uncertain system stability domain. In fact, when the uncertainty increases, it becomes impossible to stabilize the system on the entire region described by the polytop.
That's why we suggest to divide the global polytop into many subpolytops and we build for these subpolytops an adequate observer.

The applied command to the system is the switching result between these observers. The switched law must guarantee the stability of the uncertain system [11].

4.1. Construction of multiobserver

Given that the only two pieces of information, provided by the system, are the input command $u(k)$ and the output $y(k)$. The problem the is n the reconstruction of state $x(k)$. To find a solution to this problem, we use a multiobserver that is similar to the one suggested by Akhenak [1] for the state reconstruction of a system described by a multimodel. :

$$\begin{cases} \hat{x}(k+1) = \sum_{j=1}^{P} \mu_j(k)(\hat{A}_j \hat{x}_j(k) + L_j(y(k) - \hat{y}_j(k)) + Bu(k)) \\ \hat{y}(k) = C\hat{x}(k) \end{cases} \tag{22}$$

μ_j Switching function defined by :

$$\mu_j(k) = \begin{cases} 1 \; if \;\; j = \min\limits_{j} \left\{ J_j(k) \right\} \\ 0 \qquad \text{else} \end{cases} \tag{23}$$

J_j (k) commutation criterion at the discrete time k

$u(k)$ applied to system, at the discrete time k, is then :

$$u(k) = \sum_{j=1}^{j=P} \mu_j(k) K_j \hat{x}_j(k) \tag{24}$$

Taking into account the formulation (2) and the described division in (Figure 1), the condition for robust stabilization of each subpolytop is expressed by the following *LMI*:

$$\begin{pmatrix} P_{j,\alpha_j} & \phi^T{}_{j,\alpha_j} G_j^T \\ G_j \phi_{j,\alpha_j} & G_j + G_j^T - P_{j,\alpha_j} \end{pmatrix} > 0 \tag{25}$$

j: index of subpolytop P_j
α_j : one top of polytop P_j with $\alpha_j \in \{1..n_j\}$
n_j: number of vertices of subpolytop P_j

The resolution of (25) allows to compute one gain K_j and one observation gain L_j for every sub-polytop .

4.2. Commutation Criterion among observers

The switching criterion J_i must allow the convergence to the optimal observer, which generates the minimum errors between the real and estimated states. Such errors are to be computed using a horizon of errors observation. Regarding that in the stage of sampling, the only information provided on the system is its output y, the criterion must choose the best observer and must converge towards the observer who delivers the $\hat{y}_j$ which is the nearest to y on an observation horizon, the length ℓ of which is :

$$n < \ell \tag{26}$$

n *:* rank of system

Observation horizon ℓ must be chosen in a way that it leads to a temporary redundancy for every observer [10].

The criterion J_j is then defined as following

$$J_j = E_j(k-\ell,k)^T E_j(k-\ell,k) \tag{27}$$

where

$$E_j(k-\ell,k) = \begin{pmatrix} \left\| y(k) - \hat{y}_j(k) \right\| \\ . \\ \left\| y(k-\ell+i) - \hat{y}_j(k-\ell+i) \right\| \\ . \\ \left\| y(k-\ell) - \hat{y}_j(k-\ell) \right\| \end{pmatrix} \tag{28}$$

Algorithm

Step 0: Initialization

k=0

Initialization of state vectors for the different observers

Choose the length ℓ of the observation horizon

Initialization of vector E_j

For i=0 to ℓ $E_j(i)$=0

Choose arbitrary one of the states of the different used observers

Apply $u(0) = K_j \hat{x}_j(0)$

Step 1: Choice of the best estimated output

k>0

Collect input data of system and output system and the different observers.

Calculate the observation vector E_j (k-ℓ, k) on the horizon (k-ℓ,..,k) for every observer j

$$\begin{aligned} &\text{if } k-\ell+i < 0 \text{ then} \\ &\left\| y(k-\ell+i) - \hat{y}_j(k-\ell+i) \right\| = 0 \quad \forall\, i \in [0..\ell] \end{aligned} \tag{29}$$

Calculate the quadratic criterion J_i using (27)

Search $j_{min}(k) = \min\limits_{j} \{ \ J_j(k-\ell,k) \ \}$

Calculate the activation functions $\mu_j(k) = \begin{cases} 1 & \text{if } j = j_{min}(k) \\ 0 & \text{else} \end{cases}$

Calculate $u(k) = \sum_{j=1}^{P} \mu_j(k) K_j \hat{x}_j(k)$

Step 2: Application of command $u(k)$

Apply $u(k)$ to the system and to different observers

Back to step 1

End of algorithm

The algorithm of the estimation ends by converging towards the best observer whose output tends towards the real output of system. The existence of this observer is assured by the fact that the system, which is, by force belongs necessarily to one of the stabilized subpolytops

4.3. Illustration

We reconsider the example introduced in (3.3), the stiffness of the spring $r \in [r_{\min}..r_{\max}]$=[0.5..29.5] *Nm*.

We are proposed to stabilize the system by two observers described by the dynamic matrices $\hat{A}_1$ and $\hat{A}_2$ as the first observer stabilize the interval (15..29.5) and the second observer stabilize the second interval (0.5..15). The calculated feedback state gains K_1, K_2 the observation gains L_1, L_2, and the matrices G_{1i} are:

$$\hat{A}_1 = \begin{pmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -2.225 & 2.225 & 1 & 0.1 \\ 2.225 & -2.225 & 0 & 1 \end{pmatrix}, \hat{A}_2 = \begin{pmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -0.775 & 0.775 & 1 & 0.1 \\ 0.775 & -0.775 & 0 & 1 \end{pmatrix} \tag{30}$$

$$\begin{aligned} K_1 &= [\text{-}42.1491\ 36.2817\ \ \text{-}16.4686\ 5.9471] \\ K_2 &= [\text{-}155.9829\ 148.2237\ \ \text{-}22.8299\ \ \text{-}34.5856] \\ L_1 &= [\text{-}0.8882\ 2.3085\ \ \text{-}12.6027\ 13.3166]^T \\ L_2 &= [\text{-}2.2718\ 2.2849\ \ \text{-}11.4529\ 11.4529]^T \end{aligned} \tag{31}$$

$$G_{11} = \begin{pmatrix} 0.3091 & -0.2586 & 0.0199 & 0.0223 \\ -0.2558 & 0.2401 & -0.0169 & -0.0130 \\ 0.0197 & -0.0169 & 0.0022 & 0.0011 \\ 0.0195 & -0.0109 & 0.0006 & 0.0080 \end{pmatrix} \tag{32}$$

$$G_{13} = \begin{pmatrix} -0.0101 & 0.0069 & -0.0017 & 0.0114 \\ -0.0759 & 0.0871 & -0.0023 & -0.0114 \\ -0.0055 & 0.0128 & 0.0002 & -0.0070 \\ -0.0099 & 0.0127 & -0.0005 & -0.0067 \end{pmatrix} \tag{33}$$

$$G_{14} = \begin{pmatrix} 0.8798 & 0.6383 & -0.3486 & -0.4200 \\ 0.2154 & 4.4385 & 0.0098 & -0.2304 \\ -0.3596 & 0.0306 & 0.5361 & 0.5056 \\ -0.3978 & -0.3693 & 0.5035 & 0.5152 \end{pmatrix} \tag{34}$$

Figures 6, 7, 8 and 9 show respectively the evolution, in time, of the position of truck 2, control law, switching criterion and the switching between observers on the interval [0..3] seconds.

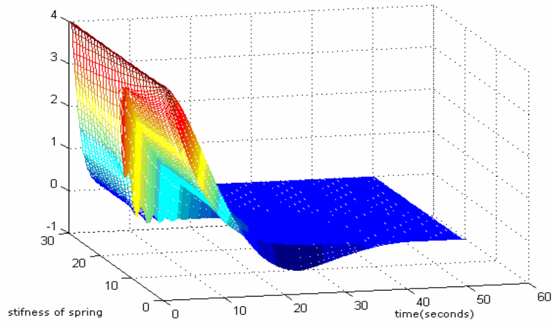

Fig.5. Evolution, in time, of position of truck 2 for many different values of r

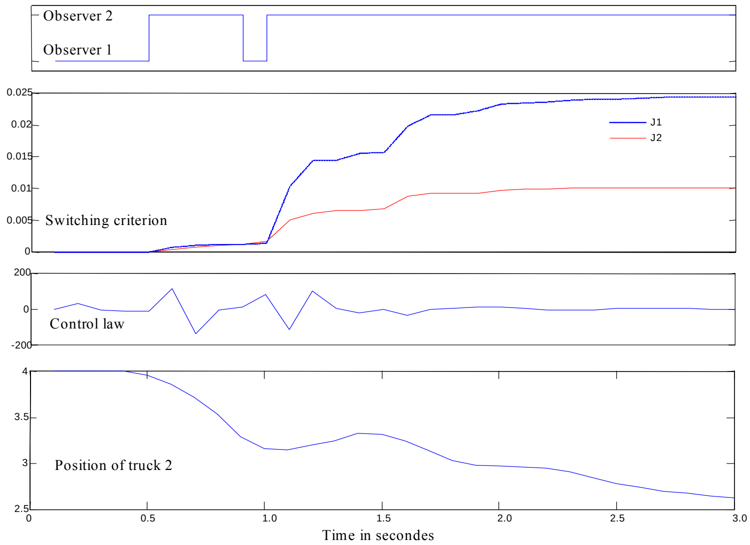

Fig.6. Evolution, in time, of position of truck 2 for *r*=14.25Nm

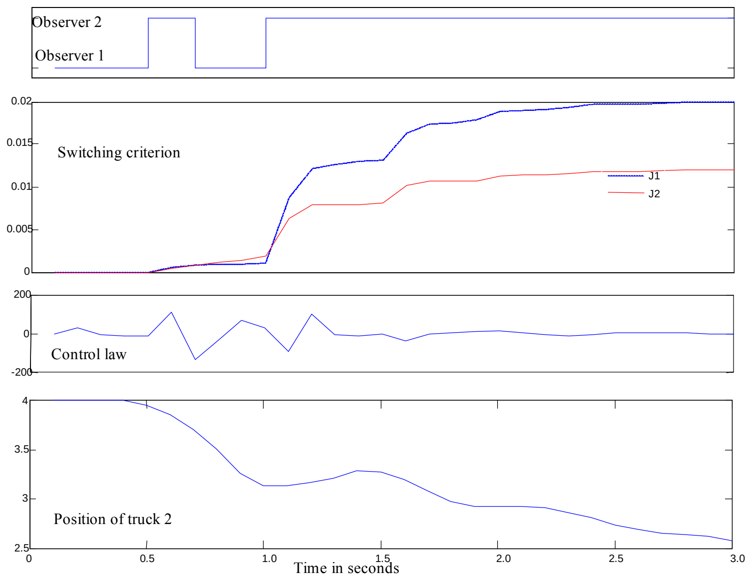

Fig.7. Evolution, in time, of position of truck 2 for *r* =15Nm

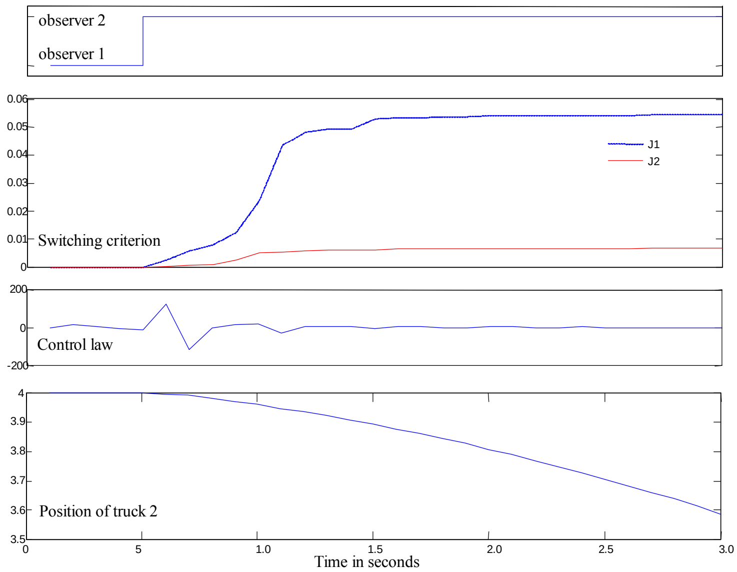

Fig.8. Evolution, in time, of position of truck 2 for r=0.5Nm

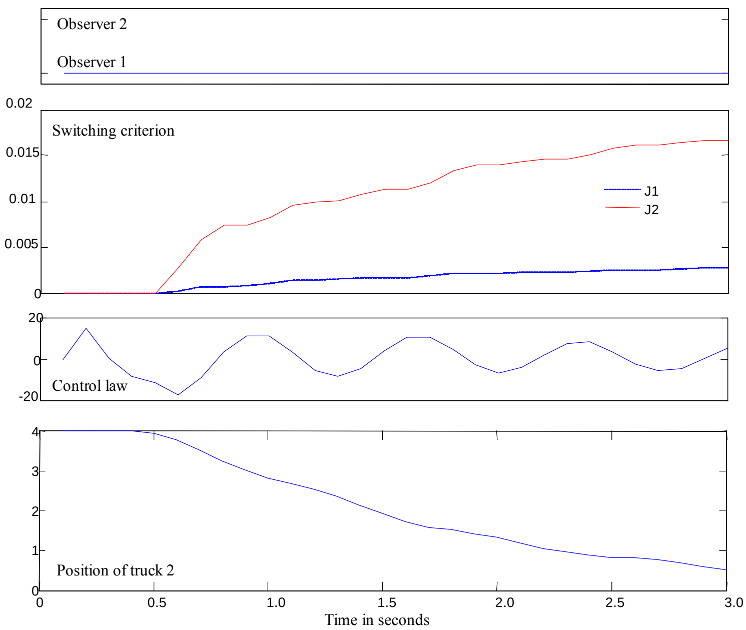

Fig.9. Evolution, in time, of position of truck 2 for r=29.5Nm

Figures 6, 7, 8 and 9 clearly show that the algorithm of choosing the best observer who converges towards the adequate one particularly for the tops of every subpolytop. The developed approach is less conservative and allows stabilization of the plant on a great interval of uncertainty by comparison with the approach proposed in section 3 and other published works [18].
Obviously that in this study the uncertainties are putted only on the state matrix and in case of different input matrices the mathematical development, which is more difficult to carry out due to the presence of a new terms ΔB_i in (7).

5. Conclusion

In this paper a new approach is presented to synthesize a control law and to stabilize a polytopic uncertain system based on observers designed from the tops of polytop.

A *LMI* formulation was presented for the calculation of different gains by state feedback and the gains of observation relative to different observers. A switched control law based on a criterion of quadratic minimization focusing on the distances between the real output of plant and the different outputs generated by the observers. An illustrative example that proves the efficiency of the suggested method is presented.

One of the possible perspectives of this work is the use of switched control laws allowing the stabilization of several subsystem initially unstable.

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