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Unconstrained state-space predictive controller for non-minimum phase MIMO systems

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Abstract Non-minimum phase systems are known to be difficult to control especially when Generalize Predictive Control (GPC) based on state-space representation is considered. To solve this problem, an innovative synthesis method is proposed. It appeals: first, to estimate unmeasured states in order to stabilize the process with a state feedback; second, to decouple the system inputs and outputs in order to control separately the dynamics of the obtained subsystems closed loops. Some illustrative simulation results for a given system are presented and discussed. The proposed method makes possible to optimize separately the GPC parameters for each subsystem.

1 INTRODUCTION

For several years, the industrial community has expressed a growing interest in Generalized Predictive Control (GPC) which has also influenced significantly process control [1]. Simultaneously, this type of control continues to be the subject of many theoretical works in the linear and non-linear domains aiming to extend its potential fields of applications. [2] explains nonlinear Model Predictive Control (MPC) and moving horizon estimation and includes numerical solution techniques, [3] contributes in the comprehension of the theoretical results on the closed-loop behavior of MPC algorithms. Notable past analysis of MPC theory include also those of [4], [5], [6], [7], [8], [9]. The GPC using a state-space approach was studied by [10], [6], [7], [11], [12], since it offers interesting prospects:[13] and [14] demonstrate the possibility to put the closed-loop system in a form amenable for applying the perturbation analysis. [15] points out, that the state-space controller has better disturbance rejection when a set-point change occurs in another interconnected loop, and is capable of running significantly faster than the polynomial approach.
Though the state-space approach requiring to have access to all states variables of the considered system, it yields the use of an observer [10], [12], [2].
Unfortunately, the state-space approach always runs up against the problem of non-minimum phase discrete Multi-Input Multi-Output (MIMO) systems [12], [16], [17]. The flatness property has made this problem solvable only for GPC based on output [18].
This paper describes an innovative method to design GPC dedicated to unstable

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non-minimum phase systems [19], [20]. It appeals: first, to estimate unmeasured states in order to stabilize the process with a state feedback; second, to decouple the system inputs and outputs in order to control separately the dynamics of the obtained subsystems closed loops.
This paper is organized as follows : in section 2 the state-space GPC approach and its instability problem are briefly reminded. In section 3 the various steps of the method suggested are detailed. Some illustrative simulations are given in section 4 and section 5 concludes the paper.

2 BRIEF REMINDER ON THE STATE-SPACE GPC APPROACH

2.1 Control Principle

Let considered a linear time invariant discrete MIMO system described by

$$\begin{cases} x(k+1) = Ax(k) + B\triangle u(k) \\ \quad y(k) \quad = \qquad Cx(k) \end{cases} \tag{1}$$

where $x(k) \in \Re^n$, $\triangle u(k) \in \Re^q$ and output $y(k) \in \Re^s$ are the state, input and output vectors at time k respectively.
The matrices A, B and C have respectively the dimensions $(n \times n)$, $(n \times q)$ and $(s \times n)$.
The control signal

$$\triangle u = [\triangle u(k), \triangle u(k+1), \cdots, \triangle u(k+N_2)] \tag{2}$$

is chosen to minimise the quadratic cost function [10]

$$J = \sum_{j=N_1}^{N_2} [\hat{y}(k+j) - y_c(k+j)]^2 + \lambda \sum_{j=1}^{N_u} [\Delta u(k+j-1)]^2 \tag{3}$$

where $\hat{y}$ is the predicted output and y_d the desired output. The parameters N_1, N_2 and N_u are refereed, respectively, to the minimum prediction horizon, the maximum prediction horizon, and the control horizon, and $\lambda \geq 0$ is the control increment weighting.
There are no rules to set those parameters, but some directive lines are given by [4], [21], [16]:

- N_1 is equal to 1+ system's delay (if this delay exists).
- N_2 is chosen to satisfy $N_2 T_s$ equal to the response time of the system (where T_s is the sampling time). Generally, the value of N_2 is greater than the numerator degree of the transfer function of the system.
- N_u and λ considerably effect the stability of the system. They are much more difficult to set.

Like feedback control, GPC state-space approach requires the estimation of all unmeasurable state variables. For this reason, the observer theory proposed by [10] has been chosen as a first approach among all existing ones [2], [22], [23], [24]

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + B\triangle u(k) + L[y(k) - C\hat{x}(k)] \\ \hat{y}(k) \quad = \quad C\hat{x}(k) \end{cases} \tag{4}$$

where L is the observer gain matrix and $\hat{x}(k) \triangleq \hat{x}(k/k-1)$. According to [25], an equivalent representation of (4) becomes

$$\hat{y}(k) = G\triangle u(k) + MA\hat{x}(k) + ML\xi(k) \tag{5}$$

with

$$\xi(k) = y(k) - \hat{y}(k) \tag{6}$$

where $\hat{y}(k) \triangleq \hat{y}(k/k-1)$ and

$$G = \begin{bmatrix} CB & & & \\ CAB & CB & & \\ \vdots & CAB & \ddots & \\ \vdots & \vdots & \ddots & CB \\ \vdots & \vdots & \vdots & \vdots \\ CA^{N_2-1}B & CA^{N_2-2}B & \cdots & CA^{N2-Nu-1}B \end{bmatrix} \tag{7}$$

$$M = \begin{bmatrix} C \; CA \cdots \; CA^{N_2-1} \end{bmatrix}^T \tag{8}$$

Finally the minimization of criteria (3) leads to:

$$\triangle u(k) = [G^T G + \lambda I]^{-1} G^T [y_d(k) - MA\hat{x}(k) - ML\xi(k)] \tag{9}$$

where only the first row of $\Delta u(k)$ is considered and applied on the system to control.
The control vector (9) required clearly the estimation of the state variable vector and differs from the control vector of output GPC approach (10)

$$\triangle u(k) = [\tilde{G}^T \tilde{G} + \lambda I]^{-1} \tilde{G}^T [y_d(k) - f] \tag{10}$$

where $f = [\hat{y}(k+1|t), \cdots, \hat{y}(k+N_2|t)]^T$
and

$$\tilde{G} = \begin{bmatrix} g_1 & 0 & \ldots & 0 \\ g_2 & g_1 & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{hp} & g_{hp-1} & \cdots & g_1 \end{bmatrix} \tag{11}$$

$\tilde{G}$ is a matrix of $N_2 \times N_u$ elements. Each g_i is the i^{th} polynomial coefficient defined by [4]. The equation of estimate output is written:

$$\hat{y} = \tilde{G}\triangle u(k) + f \tag{12}$$

2.2 GPC Instability Problems

The application of the state-space GPC requires to lean with attention on the choice of the various GPC parameters N_1, N_2, N_u and λ. Indeed, these parameters influence considerably the closed-loop stability. There exist, in the literature, straightforward guidelines to tune the GPC parameters [16], [26].
The application runs also up against the system problem bound to poles and zeros outside the unit circle [27], [28], [12], [29]. In the case of unstable poles, a usual solution consists on doing pole placement [30], [31], [27]. What returns to stabilize the system and choose its dynamics, before applying the GPC.
In the case of unstable zeros, some solutions are proposed in [12], [17], [23], those method are generally applied on a constrained system or a non-linear one.

The following section is entirely devoted to the presentation of a method solving the problem of the unstable zeros when the state-space GPC is considered.

3 PROPOSED METHOD

Under the hypothesis that the system to control is a MIMO-square system with the inputs number equal to the outputs number ($dim\ \triangle u = dim\ y = s$), the suggested method is based on the following stages.

3.1 Decoupling MIMO System

The first stage is inspired from the decoupling method of continuous systems matrix presented in [32] and [33]. It aims to obtain independent chains of input-output subsystems.

The relative degree d_i is exactly equal to the minimum number of increments of the output $y_i(k)$ which lets the input $\triangle u(k)$ appears in (1), where $y_i(k)$ is the i^{th} output component of the vector output. Thus, the integer d_i is characterized by the condition [32]:

$$c_i A^{d_i - 1} B \neq 0 \tag{13}$$

where c_i is the i^{th} row of the matrix C associated to the output $y_i(k)$.
After d_i increments, the i^{th} component of the output vector is:

$$y_i(k + d_i) = c_i A^{d_i} x(k) + c_i A^{d_i - 1} B \triangle u(k) + \cdots + c_i B \triangle u(k + d_i - 1) \tag{14}$$

From (13) and (14), one can deduce:

$$\begin{bmatrix} y_1(k + d_1) \\ \vdots \\ y_s(k + d_s) \end{bmatrix} = \begin{bmatrix} c_1 A^{d_1} \\ \vdots \\ c_s A^{d_s} \end{bmatrix} x(k) + \begin{bmatrix} c_1 A^{d_1 - 1} B \\ \vdots \\ c_s A^{d_s - 1} B \end{bmatrix} \triangle u(k) \tag{15}$$

which leads to:

$$\begin{bmatrix} y_1(k+d_1) \\ \vdots \\ y_s(k+d_s) \end{bmatrix} = \Delta_0 x(k) + \Delta_1 \triangle u(k) \tag{16}$$

Regarding [32], the necessary and sufficient condition for decoupling the system

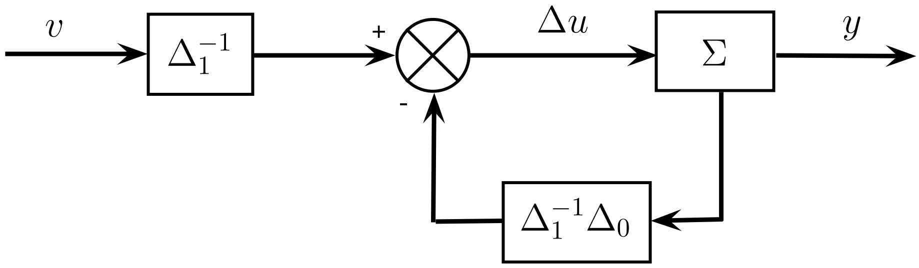

Figure1. Decoupled MIMO system

(1) is the invertibility of matrix Δ_1.
So, it is possible to apply to system (1) the feedback law

$$\triangle u(k) = \Delta_1^{-1}(\nu(k) - \Delta_0 x(k)) \tag{17}$$

Where $\nu(k)$ is the input of the decoupled system depicted in Figure 1.

Let the decoupled system be given by the new representation:

$$\begin{cases} x(k+1) = \bar{A}x(k) + \bar{B}\nu(k) \\ \quad y(k) \quad = \quad Cx(k) \end{cases} \tag{18}$$

where $\bar{A} = A - B\Delta_1^{-1}\Delta_0$ and $\bar{B} = B\Delta_1^{-1}$.
Note also that the system is now decoupled in s independent chains of integrators.

3.2 Base-change Matrices

The system being now decoupled, it is possible to fix independently the dynamics of the s various input-output chains. In this intention, a base-change is carried out:

$$\begin{cases} \xi(k+1) = \begin{bmatrix} \xi_1(k+1) \\ \vdots \\ \xi_s(k+1) \end{bmatrix} = Tx(k+1) \\ = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_s \end{bmatrix} \xi(k) + \begin{bmatrix} \beta_1 & & \\ & \ddots & \\ & & \beta_s \end{bmatrix} \nu(k) \\ y(k) = \begin{bmatrix} \gamma_1 & & \\ & \ddots & \\ & & \gamma_s \end{bmatrix} \xi(k) \end{cases} \tag{19}$$

with

$$T = \begin{bmatrix} T_1, \cdots, T_i, \cdots, T_s \end{bmatrix}^T \tag{20}$$

and

$$T_i = \begin{bmatrix} c_i, c_i A, \cdots, c_i A^{d_i - 1} \end{bmatrix}^T \tag{21}$$

The new state-space representation is thus expressed in the new base by the following equation:

$$\begin{cases} \xi(k+1) = \tilde{A}\xi(k) + \tilde{B}\nu(k) \\ \quad y(k) \quad = \quad \tilde{C}\xi(k) \end{cases} \tag{22}$$

so that

$$\tilde{A} = T\bar{A}T^{-1} \tag{23}$$

$$\tilde{B} = T\bar{B} \tag{24}$$

$$\tilde{C} = \bar{C}T^{-1} \tag{25}$$

where every triplet (A_i, β_i, γ_i) is given by the controllable canonical form.

3.3 Dynamics Modification of The Decoupled System

The feedback gain $\tilde{K}_i$ is deduced from the coefficients of the characteristic polynomial (26) determinated by the pole placement:

$$\Phi(z) = z^{d_i} + \alpha_{id_{i-1}} z^{d_i - 1} + \cdots + \alpha_{i0} \tag{26}$$

It leads to the new form of control expression ν:

$$\nu(k) = -\tilde{K}\xi(k) + y_d = -\tilde{K}Tx(k) + y_d = -Kx(k) + y_d \tag{27}$$

with

$$K = \begin{bmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_s \end{bmatrix} \tag{28}$$

where

$$\alpha_i = \begin{bmatrix} \alpha_{i0} \; \alpha_{i1} \; \cdots \; \alpha_{id_{i-1}} \end{bmatrix} \tag{29}$$

The combination of the decoupling technique with equation (27) transforms the control equation into:

$$\triangle u = -\mathit{\Delta}_1^{-1}(\mathit{\Delta}_0 + \tilde{K}T)x(k) + \mathit{\Delta}_1^{-1} y_d \tag{30}$$

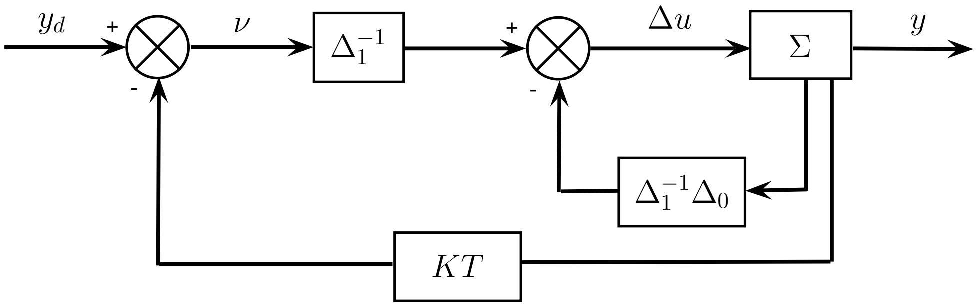

Figure2. Stabilization of the decoupled MIMO system

If $\sum d_i = n$, the closed-loop control system, shown in the Figure (2), has the same order of the initial state-space system given by the equation (1). The zero dynamics does not exist and the problem of non-minimum phase is then solved. The process (18) is thus observable and controllable.
If $\sum d_i < n$ the zero dynamics exists and is stabilized. The system given by (18) can not represent the totality of the system (1). There are then $(n - \sum d_i)$ unobservable modes. It is then important to highlight them (by an adequate base-change) before applying the pole placement.
According to the nature of the system modes, the control law $\triangle u(k)$ is calculated to modify the (un)observable modes with(out) vanishing the interaction between the control laws $\triangle u_i(k)$.

3.4 Finding Prediction Matrices

Once the system is stabilized, the next stage consists in establishing the predictive control for the whole obtained SISO subsystems.

The criterion to be minimized is:

$$J = \sum_{j=N_1}^{N_{2_m}} [\hat{y}_m(k+j) - y_{d_m}(k+j)]^2 + \lambda \sum_{j=1}^{N_{u_i}} [\Delta u_i(k+j-1)]^2 \tag{31}$$

where N_{2_m} and N_{u_i} correspond respectively to the prediction horizons and the control horizons for each output $m = 1, \cdots, s$ and each input $i = 1, \cdots, q$ of the system.
The control law of each subsystem is given by:

$$\triangle u_i(k) = [G_{mi}^T G_{mi} - \lambda I]^{-1} G_{mi}^T [y_{d_m}(k) - M_{mi} A_{mi} \hat{x}(k) - M_{mi} L_{mi} \xi(k)] \tag{32}$$

with

$$G_{mi} =$$

$$\begin{bmatrix} C_{mi}\tilde{B}_{mi} & & & \\ C_{mi}\tilde{A}_{mi}\tilde{B}_{mi} & C_{mi}\tilde{B}_{mi} & & \\ \vdots & C_{mi}\tilde{A}_{mi}\tilde{B}_{mi} & \ddots & \\ \vdots & \vdots & \ddots & C_{mi}\tilde{B}_{mi} \\ \vdots & \vdots & \vdots & \vdots \\ C_{mi}\tilde{A}_{mi}^{N_2-1}\tilde{B}_{mi} & C_{mi}\tilde{A}_{mi}^{N_2-2}\tilde{B}_{mi} & \cdots & C_{mi}\tilde{A}_{mi}^{N_2-N_u-1}\tilde{B}_{mi} \end{bmatrix}$$

$$M_{mi} = \left[C_{mi} \; C_{mi}\tilde{A}_{mi} \; \cdots \; C_{mi}\tilde{A}_{mi}^{N_2-1} \right]^T \tag{33}$$

where the triplets (A_{mi}, B_{mi}, C_{mi}) represent the new matrices of the decoupled SISO-subsystems.

4 APPLICATION AND RESULTS

As illustrative example of application, the method suggested is used to design the state space predictive controller of the following unstable MIMO and non-minimum phase system:

$$H(z) = \begin{bmatrix} \frac{10.2z-10.1}{z^2-2.031z+1.03} & 0 \\ \frac{0.05084z^2+0.00136z-0.05118}{z^3-3.041z^2+3.082z-1.041} & \frac{10.05}{z-1.01} \end{bmatrix} \tag{34}$$

The sampling time here is $T_s = 0.01s$, the equation (34) is written under the form of the equation (1):

$$\begin{cases} x(k+1) = \begin{bmatrix} 1.0001 & 0.0102 & 0 \\ 0.0203 & 1.0306 & 0 \\ 0.0102 & 0.0103 & 1.0101 \end{bmatrix} x(k) + \begin{bmatrix} 0.0001 & 0 \\ 0.0102 & 0 \\ 0.0001 & 0.0101 \end{bmatrix} \triangle u(k) \\ y(k) = \begin{bmatrix} 1000 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \triangle u(k) \end{cases} \tag{35}$$

Here, the relative degrees d_1 and d_2 of each subsystem are equal to 1.
So:

$$\Delta_0 = \begin{bmatrix} c_1 A \\ c_2 A \end{bmatrix} = \begin{bmatrix} 1020.4 & 1040.7 & 0 \\ 10.2 & 10.3 & 1010.1 \end{bmatrix} \quad (36)$$

and

$$\Delta_1 = \begin{bmatrix} c_1 B \\ c_2 B \end{bmatrix} = \begin{bmatrix} 10.2024 & 0 \\ 0.0508 & 10.0502 \end{bmatrix} \quad (37)$$

which is invertible.
The decoupled system (18) is given by the following matrices:

$$\bar{A} = \begin{bmatrix} 0.9950 & 0.0050 & 0 \\ -0.9950 & -0.0050 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (38)$$

$$\bar{B} = 10^{-3} \begin{bmatrix} 0.0050 & 0 \\ 0.9950 & 0 \\ 0 & 1.0000 \end{bmatrix} \quad (39)$$

$$\bar{C} = \begin{bmatrix} 1000 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix} \quad (40)$$

$$(41)$$

The zero dynamics is then stabilized.
The first base-change matrix T is given by:

$$T = \begin{bmatrix} c_1 \\ c_2 \\ \tau \end{bmatrix} = \begin{bmatrix} 1000 & 1000 & 0 \\ 0 & 0 & 1000 \\ 0 & 1000 & 0 \end{bmatrix} \quad (42)$$

τ must be independent of c_1 and c_2.

$$\begin{cases} \xi(k+1) = \left[\begin{array}{cc|c} 0.993 & 0.004 & 0 \\ -1.343 & -0.154 & 0 \\ \hline 0 & 0 & 0.999 \end{array}\right] \xi(k) + \tilde{B}\nu(k) \\ y(k) \quad = \quad \tilde{C}\xi(k) \end{cases} \quad (43)$$

The second base-change is used to make the state matrix on controllability canonical form. The matrix of base-change is given by:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (44)$$

The dynamic of the system is fixed now by:

$$K = \begin{bmatrix} 0.35 & -0.20 & 0 \\ 0 & 0 & -0.999 \end{bmatrix} \quad (45)$$

The choice of the state feedback gain K leads to the following system:

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \left[\begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \left[\begin{array}{c|c} B_1 & 0 \\ \hline 0 & B_2 \end{array}\right] y_d \\ y(k) = \left[\begin{array}{c|c} c_1 & 0 \\ \hline 0 & c_2 \end{array}\right] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \end{cases} \tag{46}$$

where

$$A_1 = \begin{bmatrix} 0.993 & 0.004 \\ -1.343 & -0.154 \end{bmatrix} \tag{47}$$

$$A_2 = 0.999 \tag{48}$$

$$B_1 = 10^{-3} \begin{bmatrix} 0.005 \\ 0.995 \end{bmatrix} \tag{49}$$

$$B_2 = 10^{-3} \tag{50}$$

$$c_1 = \begin{bmatrix} 1000 \; 1000 \end{bmatrix} \tag{51}$$

$$c_2 = 1000 \tag{52}$$

The results of the analysis carried out in section 3 are important as they show the convergence of the different outputs of the subsystems (Figures (3) and (4)). The representation (46) highlights the existence of two decoupled subsystems Σ_1 and Σ_2 defined respectively by (A_1, B_1, c_1) and (A_2, B_2, c_2).
The state-space GPC approach defined by (31) and (33) has been applied on each one of these subsystems, by choosing the increment weighting λ the adequate control and prediction horizons. The simulation results are shown in Figures (3) and (4) where y_{c1}, y_{c2} are the references signals, and y_1, y_2 are the outputs signals. For comparison, simulation results obtained by applying the usual state-space GPC approach on the coupled system (1) are shown in Figures (5) and (6). In the latter case the lack of decoupling does not permit the oscillations attenuation despite the optimization of GPC parameters.

5 Conclusion

In this article the synthesis of predictive controller based on state-space approach is proposed. The method presented here is intended for discrete multivariable non-minimum phase systems, under the hypothesis that the systems can be decoupled.

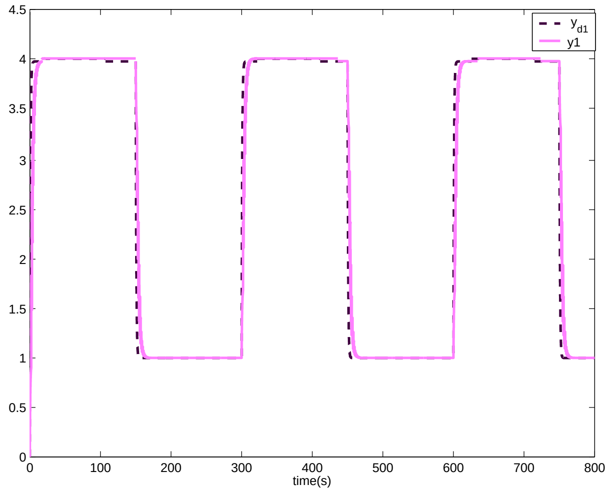

Figure3. Output signal y_1 of the decoupled subsystems vs. time

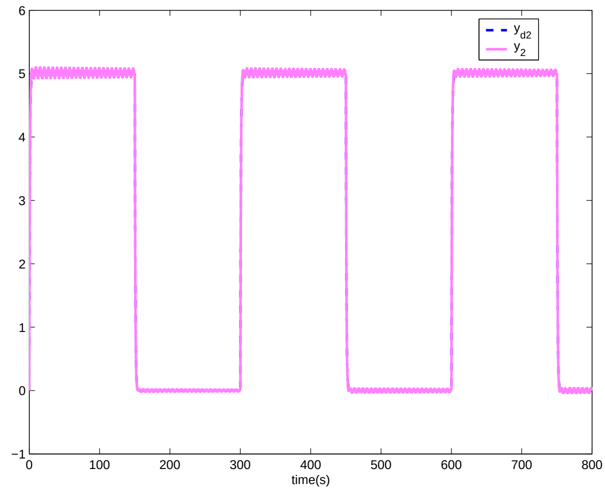

Figure4. Output signal y_2 of the decoupled subsystems vs. time

This approach treats separately the closed loops of the various subsystems of the considered process. The two main advantages are: On the one hand, it leads to fix independently the dynamics of each subsystem and to compensate the unstable zeros. On the other hand, it makes possible to optimize separately the GPC parameters for each subsystem.

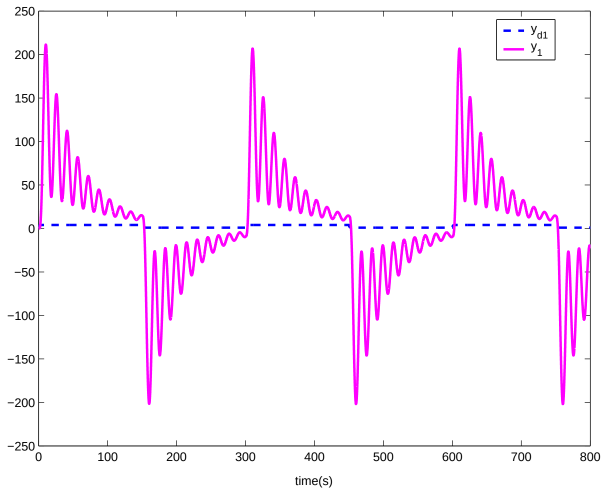

Figure5. Output signal y_1 of the coupled subsystems vs. time

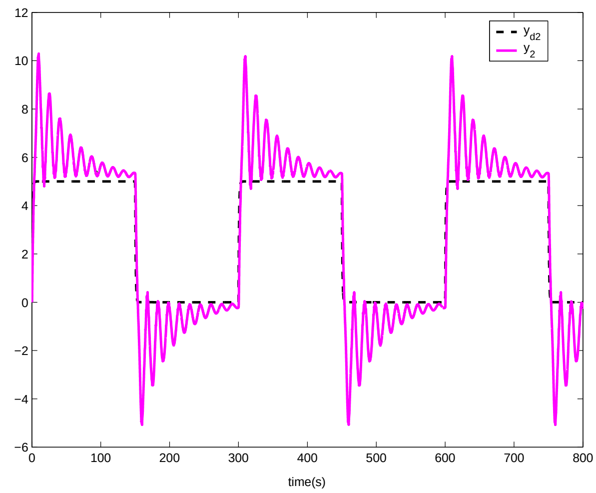

Figure6. Output signal y_2 of the coupled subsystems vs. time

Future work will consist in taking GPC with constraints. This point will permit us to take benefits of all GPC's advantages [34], [3], [35]. The real-time algorithm of the resulting controller will be then developed and tested on a dSpace Simulator.

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