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A New Design Method for Fractional PI D Controller

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Abstract. *Our objective is to apply the fractional $PI^{\lambda}D^{\mu}$ controller to enhance the system control performances. Unlike Proportional Integral Derivative (PID) controller, there is no systematic and yet rigor design or tuning method existing for this $PI^{\lambda}D^{\mu}$ controller. In this paper a method to design $PI^{\lambda}D^{\mu}$ controller is presented. The basic ideas of the tuning method are based, in the first place, on the conventional tuning methods for setting the parameters of the fractional PI D controller for $\lambda=1$ and $\mu=1$ which means setting the parameters of the conventional PID controller, and on the minimum Integral Squared Error (ISE) criterion for setting the fractional integration action order λ and the fractional differentiation action order μ. The simulation results show the control quality enhancement using this proposed $PI^{\lambda}D^{\mu}$ controller.*

Résumé. *Notre objectif est d'appliquer le contrôleur $PI^{\lambda}D^{\mu}$ fractionnaire pour perfectionner les performances de commande des systèmes. Par différence au contrôleur Proportionnel Intégral Dérivé (PID), il n'y a aucune méthode systématique de conception des contrôleurs $PI^{\lambda}D^{\mu}$. Dans ce papier une méthode de conception du contrôleur $PI^{\lambda}D^{\mu}$ est présentée. Les idées de base de la méthode de conception sont basées, en premier lieu, sur les méthodes conventionnelles de conception pour le réglage des paramètres du contrôleur $PI^{\lambda}D^{\mu}$ fractionnaire pour $\lambda=1$ et $\mu=1$ qui signifie réglage des paramètres du contrôleur PID conventionnel, et sur la minimisation du critère de l'intégral du carré de l'erreur (ISE) pour le réglage de l'ordre fractionnaire de l'action intégral λ et l'ordre fractionnaire de l'action dérivée μ. Les résultats de simulation montrent le perfectionnement de la qualité de commande en utilisant le contrôleur $PI^{\lambda}D^{\mu}$ proposé.*

Keywords. *PID controller, Ziegler-Nichols tuning, ISE criteria, Fractional order controller, PI D controller.*

Mots-clés. *Contrôleur PID, Réglage de Ziegler-Nichols, Critère ISE, Contrôleur d'ordre Fractionnaire, Contrôleur PI D*

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1. Introduction

Despite the development of more advanced control strategies, the majority of industrial control systems still use PID controllers because they are standard industrial components, and their principle is well understood by engineers [1], [2]. The development of PID controller tuning dated back to the early work of Ziegler and Nichols [3].

Although all the existing techniques for the PID controller parameter tuning, a continuous and an intensive research work is still underway towards system control quality enhancement and performance improvement. One of the possibilities to improve PID controllers is to use fractional order controllers with fractional order differentiation and integration parts.

Fractional calculus is a generalization of integration and derivation to non-integer order fundamental operator ${}_aD_t$, where *a* and *t* are the limits of the operation. The two definitions used for the general fractional differintegral are Grunwald definition and Riemann-Liouville definition [4].

In recent years we observe an increasing number of studies related with the application of the fractional calculus (FC) theory in many areas of science and engineering [5], [6], and [7]. This fact is due to a better understanding of the FC potentialities revealed by many phenomena. In what concerns the area of automatic control systems the application of the FC concepts is still scarce and only in the last two decades appeared the first applications [8], [9].

The first who really introduced a fractional order controller was Oustaloup. He developed the so-called Commande Robuste d'Ordre Non Entier (CRONE) controller and applied it in various fields of control systems [10]. More recently, Podlubny proposed a generalization of the PID controller, namely the fractional $PI^{\lambda}D^{\mu}$ controller, involving an integration action of a fractional order λ and differentiation action of a fractional order μ [11], [12]. Since, many researchers have been interested in the use and tuning of this fractional PI D controller [13]–[16]. The interest of this kind of controllers is justified by a better flexibility, since it has two more parameters which are the fractional integration action order λ and the fractional differentiation action order μ. These parameters can be used to fulfill additional specifications for the design or other interesting requirements for the controlled system, than in the case of a conventional PID controller (λ=1, μ =1).

In this paper we propose the design of the fractional PI D controller of a classical unity feedback control system, where the controller is the fractional PI D controller whose transfer function is given as:

$$C(s) \quad K_p \;\; 1 \quad \frac{1}{T_I s} \quad T_D s \qquad \textbf{(1)}$$

With K_P is the proportional constant, T_I is the integration constant, T_D is the differentiation constant, is the fractional integration action order such that 0< <2 and is the fractional differentiation action order such that 0< <2.

The equation for the fractional order PI D controller's output in time domain is:

$$u(t) \quad K_p \quad e(t) \quad \frac{1}{T_I}.D \quad e(t) \quad T_D.D \quad e(t) \tag{2}$$

One of the most important advantages of the fractional $PI^{\lambda}D^{\mu}$ controller is its possibility for better control of the fractional order dynamical systems. Another advantage lies in the fact that the fractional $PI^{\lambda}D^{\mu}$ controllers are less sensitive to changes of parameters of a controlled system. This due to the fact that having two extra degrees of freedom can better adjust the dynamical properties of fractional order control systems.

This paper is organized as follows. Section 2 introduces the basic ideas and the derived formulations of the new design method of the fractional $PI^{\lambda}D^{\mu}$ controller. In section 3, some illustrative examples are presented to demonstrate the control enhancement of the design method. In section 4, robustness to model errors of the fractional controllers is presented. In section 5, implementation considerations of the PI D controller are presented. Finally, section 6 draws the main conclusion.

2. Fractional PI D Controller Design

We propose the design of the fractional PI D controller of a classical unity feedback control system shown in Fig. 1:

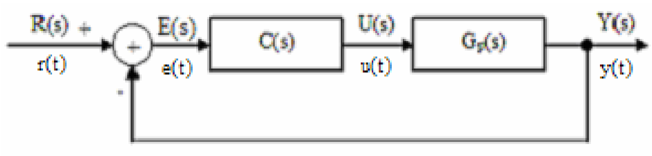

Fig. 1. Classical unity feedback control system

The plant's transfer functions $G_p(s)$ considered is a first order plant with a time delay, or a first order plant with an integrator with a time delay, and C(s) is the transfer function of the fractional PI D controller of (1).

2.1. Design of the Parameters K_P, T_I and T_D

Our design method is based, in the first place, on any existing classical tuning rules for setting the parameters K_P, T_I and T_D of the fractional PI D controller for $\lambda=1$ and $\mu=1$ which means setting the parameters of a simple conventional PID controller.

2.2. Design of the Parameters λ and μ

The proposed method consists of using the parameters K_P, T_I and T_D obtained in the first step for setting the parameters λ and μ minimizing the integral square error (ISE) of the classical unity feedback control system of Fig.1 for a unit step input.

The integral square error (ISE) is given as:

$$J = \int_0^{\infty} [e(t)]^2 dt = \int_0^{\infty} [r(t) - y(t)]^2 dt \qquad \textbf{(3)}$$

Where e(t)=[r(t) – y(t)] is the error signal. According to Laplace transform properties the integral J can be written as [17]:

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E(s)E(-s)ds \qquad \textbf{(4)}$$

Then, for $E(s) = \frac{N_E(s)}{D_E(s)}$ a rational function in s, the complex integral J will be:

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{N_E(s)N_E(-s)}{D_E(s)D_E(-s)} ds \qquad \textbf{(5)}$$

From Fig. 1, the error signal E(s) is given as:

$$E(s) = \frac{1}{1 + C(s)G_p(s)} R(s) = \frac{1}{1 + C(s)G_p(s)} \cdot \frac{1}{s} \qquad \textbf{(6)}$$

The settings of the fractional integration action order λ and the fractional differentiation action order μ of the fractional $PI^{\lambda}D^{\mu}$ controller consists in finding these two parameters that minimize the ISE index J of (5). For the minimization task, we varied the values of the parameters λ and μ from 0.05 to 1.95 each with a step of 0.05 and for each value of the couple (λ, μ) we calculate the corresponding ISE index J using the Hall-Sartorius method given in [17]. With a simple comparison test of all the ISE index J(λ, μ) calculated we can obtain the minimum ISE index J and the corresponding optimum settings of the two parameters λ and μ of the fractional $PI^{\lambda}D^{\mu}$ controller. In order to calculate the complex integral J using the Hall-Sartorius method [17], E(s) must be a rational function. But the fractional $PI^{\lambda}D^{\mu}$ controller's transfer function C(s) given in (1) is an irrational function and the plant's transfer function $G_p(s)$ is also an irrational function because of the time delay. To circumvent this problem, the time delay of the plant's transfer function $G_p(s)$ is approximated by a rational function using the Padé approximation method.

And the irrational function of the fractional $PI^{\lambda}D^{\mu}$ controller C(s) of (1) is also approximated by a rational function [18], [19].

The transfer function of the fractional $PI^{\lambda}D^{\mu}$ controller is given in (1), taking λ=1 and μ=1, we obtain a conventional integer order PID controller. If λ=1, μ =0, we obtain PI controller. If λ=0 and μ=1, we have PD controller. All these conventional types of PID controllers are the particular cases of the fractional order $PI^{\lambda}D^{\mu}$

controller given by (1). Because the orders λ and μ can be arbitrary real number, the fractional order PI D controllers is more flexible and given an opportunity to better adjust the dynamical properties of systems. If the relation of (1) becomes:

$$C(s) \quad \frac{K_p}{T_I} \frac{(T_D T_I s^{(\quad)} \quad T_I s \quad 1)}{s} \tag{7}$$

In the final expression of (7) it can be noted the presence of zeros that are related to (1). Taking into account the previous discussion, the asymptotic magnitude Bode diagram of the fractional order PI D controller in (1) can be obtained.

The exact diagrams of the PI D (magnitude and phase) are reported in Fig. 2.

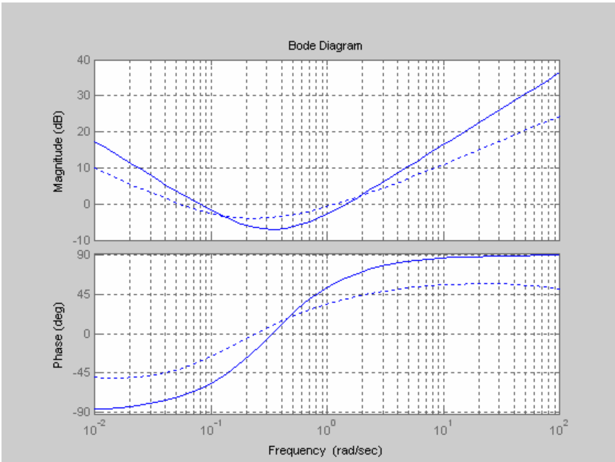

Fig. 2. An example of PID (solid line) and PI D dashed line, with λ=0.8, μ =0.7 magnitude and phase plot

Next statements are important to be considered. First of all, the fact of being < 1 makes the output converge to its final value more slowly than in the case of an integer controller. Furthermore, the fractional effects need to be band-limited when it is implemented. Therefore, the fractional integrator must be implemented as $1/s \quad (1/s)s^{1}$, ensuring this way the effect of an integer integrator $1/s$ at very low frequency. Similarly to the fractional integrator, the fractional differentiator, s , has also to be band-limited when implemented, ensuring this way a finite control effort and noise rejection at high frequencies.

On the other hand, when fractional controllers have to be implemented or simulations have to be performed, fractional transfer functions are usually replaced by integer transfer functions with a behaviour close enough to the one desired, but much easier to handle. There are many different ways of finding such approximations but unfortunately it is not possible to say that one of them is the best, because even though some of them are better than others in regard to certain characteristics.

In this work one way to approximate fractional order operators to an integer transfer function have been used: the Singularity Function Approximation [18].

With λ and μ are such that 0 < λ < 2 and 0 < μ < 2, there are four cases depending on the parameters λ and μ:

Case 1: 0 < λ < 1 and 0 < μ < 1: The rational function approximation of C(s), in a given frequency band of practical interest [ω_L, ω_H], is given as:

$$C(s) = \mathrm{K_P}\left(1 + \frac{K_I}{T_I}\frac{\prod_{i=0}^{N_I-1}\left(1+\frac{s}{z_{Ii}}\right)}{\prod_{i=0}^{N_I}\left(1+\frac{s}{p_{Ii}}\right)} + T_D\mathrm{K_D}\frac{\prod_{i=0}^{N_D}\left(1+\frac{s}{z_{Di}}\right)}{\prod_{i=0}^{N_D}\left(1+\frac{s}{p_{Di}}\right)}\right) \tag{8}$$

Case 2: 1 < λ < 2 and 0 < μ < 1: The rational function approximation of C(s), in a given frequency band of practical interest [ω_L, ω_H], is given as:

$$C(s) = \mathrm{K_P}\left(1 + \frac{K_I}{sT_I}\frac{\prod_{i=0}^{N_I-1}\left(1+\frac{s}{z_{Ii}}\right)}{\prod_{i=0}^{N_I}\left(1+\frac{s}{p_{Ii}}\right)} + T_D\mathrm{K_D}\frac{\prod_{i=0}^{N_D}\left(1+\frac{s}{z_{Di}}\right)}{\prod_{i=0}^{N_D}\left(1+\frac{s}{p_{Di}}\right)}\right) \tag{9}$$

Case 3: 0 < λ < 1 and 1 < μ < 2: The rational function approximation of C(s), in a given frequency band of practical interest [ω_L, ω_H], is given as:

$$C(s) = \mathrm{K_P}\left(1 + \frac{K_I}{T_I}\frac{\prod_{i=0}^{N_I-1}\left(1+\frac{s}{z_{Ii}}\right)}{\prod_{i=0}^{N_I}\left(1+\frac{s}{p_{Ii}}\right)} + T_D s\,\mathrm{K_D}\frac{\prod_{i=0}^{N_D}\left(1+\frac{s}{z_{Di}}\right)}{\prod_{i=0}^{N_D}\left(1+\frac{s}{p_{Di}}\right)}\right) \tag{10}$$

Case 4: 1 < λ < 2 and 1 < μ < 2: The rational function approximation of C(s), in a given frequency band of practical interest [ω_L, ω_H], is given as:

$$C(s) = \mathrm{K_P}\left(1 + \frac{K_I}{sT_I}\frac{\prod_{i=0}^{N_I-1}\left(1+\frac{s}{z_{Ii}}\right)}{\prod_{i=0}^{N_I}\left(1+\frac{s}{p_{Ii}}\right)} + T_D s\,\mathrm{K_D}\frac{\prod_{i=0}^{N_D}\left(1+\frac{s}{z_{Di}}\right)}{\prod_{i=0}^{N_D}\left(1+\frac{s}{p_{Di}}\right)}\right) \tag{11}$$

3. Illustrative Examples

This section shows the application of the results obtained for the design of the fractional PI /PI D controller for two selected plants.

3.2. First Order Plant with Delay

The plant's transfer function is $G_p \quad \frac{K_0}{(1 \quad \tau s)} e^{\quad Ls} \quad \frac{1}{(1 \quad s)} e^{\quad 0.5s}$ [20]. In [20], a conventional PI controller (λ = 1, μ = 0 and T_D=0) is used to control the above plant. Using the Refined Ziegler-Nichols tuning method, the PI controller's parameters K_P, T_I are found to be K_P = 0.73 and T_I = 0.69. Hence, the conventional PI controller's transfer function $C_1(s)$ is given as:

$$C_1(s) \quad 0.73 \; 1 \quad \frac{1}{0.69s} \tag{12}$$

The smallest ISE index J of section 2 is obtained for the parameter = 0.95. Then the fractional PI controller's transfer function $C_2(s)$ required is given as:

$$C_2(s) \quad 0.73 \; 1 \quad \frac{1}{0.69s^{0.95}} \tag{13}$$

In this example of application, the implementation of the fractional PI controller has been in a frequency band from 0.01 $_u$ to 100 $_u$, where $_u$ is the unity gain crossover frequency of the open loop transfer function $C(s)G_p(s)$ when C(s) is a conventional PI controller.

Fig. 3 shows the Bode plots of the open loop transfer function $C(s)G_p(s)$ when C(s) is the conventional PI controller $C_1(s)$ and when C(s) is the fractional controller $C_2(s)$ in its rational function form.

Fig.4 shows the step responses of the closed loop system when C(s) is the conventional PID controller $C_1(s)$ and when C(s) is the fractional controller $C_2(s)$ in its rational function form.

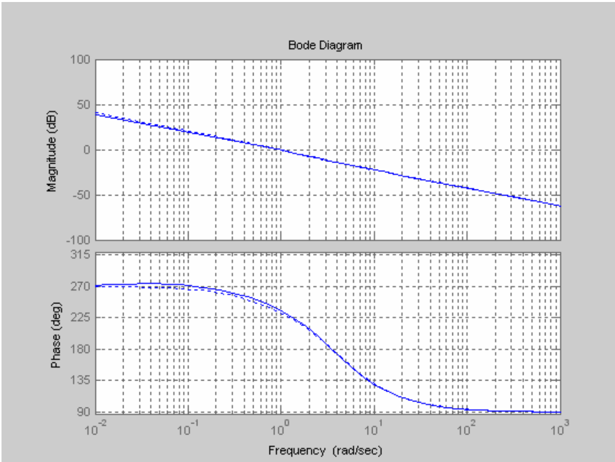

Fig. 3. Bode plots of the open loop transfer function $C(s)G_p(s)$ with C(s) the PI controller $C_1(s)$ (Dotted line) and C(s) the proposed $PI^{0.95}$ controller $C_2(s)$ (solid line).

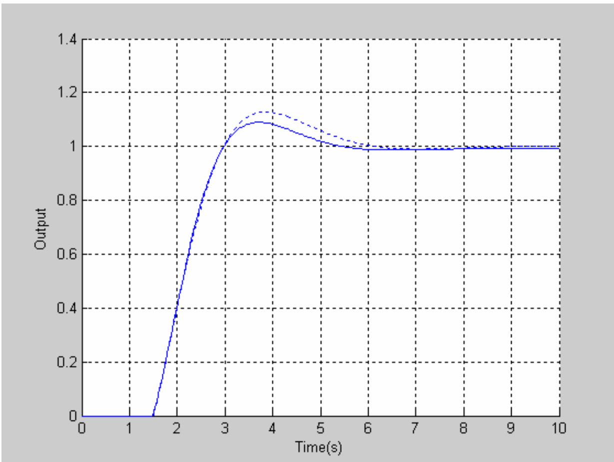

Fig. 4. Step response of the closed loop system C(s) the PI controller $C_1(s)$ (Dotted line) and C(s) the proposed $PI^{0.95}$ controller $C_2(s)$ (solid line)

Some performances characteristics for the feedback control system with both controllers, are summarized in Table 1, in terms of the unity gain crossover frequency $_u$, the phase margin PM, the gain margin GM, the settling time t_s, and overshoot P.

As it can be seen from Table 1, the unity gain crossover frequency has been changed by 1.51 %.We can also see that the settling time with the fractional $PI^{0.95}$ controller is 16.26% smaller than with the conventional PI controller and the overshoot with the fractional $PI^{0.95}$ controller is 33.33% smaller than with the conventional PI controller.

Table 1. Performances characteristics for the conventional PI and the fractional $PI^{0.95}$ controllers

C(s)	$_u$ (rad/s)	PM (deg)	GM (dB)	t_s (s)	P (%)
PI	0.922	53.8	13.5	4.12	12
PI	0.936	57.2	12.9	3.45	8

3.2. First Order Plant with Integrator and Delay

The plant's transfer function, in this case is taken from [21].

$$G_p \quad \frac{K_0}{s(1 \quad \tau s)} e^{\quad Ls} \quad \frac{1}{s(1 \quad 2s)} e^{\quad s} \tag{14}$$

For λ=1, μ=1 and using the Ziegler-Nichols tuning method, the controller's parameters K_P, T_I and T_D are found to be $K_P = 0.444$, $T_I = 6$, $T_D = 1.5$, [21]. Hence, the conventional PID controller's transfer function $C_1(s)$ is given as:

$$C_1(s) \quad 0.444 \; 1 \quad \frac{1}{6s} \quad 1.5s \tag{15}$$

The smallest ISE index J of section 2 is obtained for the couple (,)= (0.1, 1.1). Then the fractional PI D controller's transfer function $C_2(s)$ required is given as:

$$C_2(s) \quad 0.444 \; 1 \quad \frac{1}{6.0s^{0.1}} \quad 1.5s^{1.1} \tag{16}$$

As it can be observed, this controller behaves similarly to a PD controller, since the order of the integral part is very low ( ). However, the fractional integral part guarantees a null steady state error.

The implementation of the fractional integral and derivative parts is carried out as commented in the previous example.

Fig.5 shows the Bode plots of the open loop transfer function $C(s)G_p(s)$ when C(s) is the conventional PID controller $C_1(s)$ and when C(s) is the fractional controller $C_2(s)$ in its rational function form.

Fig.6 shows the step responses of the closed loop system when C(s) is the conventional PID controller $C_1(s)$ and when C(s) is the fractional controller $C_2(s)$ in its rational function form.

Some performances characteristics for the feedback control system with both controllers $C_1(s)$ and $C_2(s)$ are summarized in Table 2, it can be seen that the phase margin has been augmented by 33.38% and the gain margin has increased by 05.40%. We can also see that the settling time and the overshoot with the fractional $PI^{0.1}D^{1.1}$ controller are smaller than with the conventional PID controller.

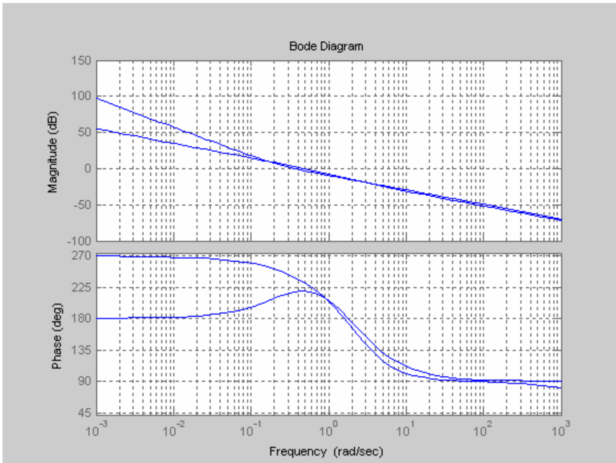

Fig. 5. Bode plots of the open loop transfer function $C(s)G_p(s)$ with $C(s)$ the PID controller $C_1(s)$ (Dotted line) and $C(s)$ the proposed $PI^{0.1}D^{1.1}$ controller $C_2(s)$ (solid line).

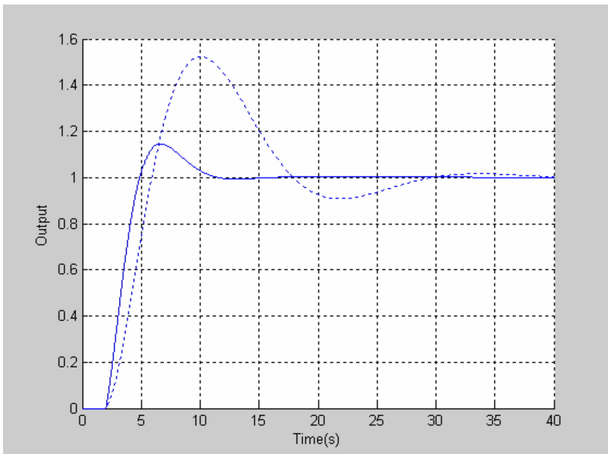

Fig. 6. Step response of the closed loop system $C(s)$ the PID controller $C_1(s)$ (Dotted line) and $C(s)$ the proposed $PI^{0.1}D^{1.1}$ controller $C_2(s)$ (solid line).

Table 2. Performances characteristics for the conventional PID and the fractional $PI^{0.1}D^{1.1}$ controllers

C(s)	$_u$ (rad/s)	PM(deg)	GM (dB)	t_s (s)	P (%)
PID	0.361	38.5	14.80	22.8	41.00
$PI^{0.1}D^{1.1}$	0.414	53.0	15.60	10.1	14.00

4. Robustness to Model Errors

One of the important properties of any controller tuning method is its robustness to model errors. In order to evaluate the robustness of the proposed tuning method, we have evaluated the closed loop responses achieved by the fractional PI D controllers when this controller is applied to a model that is slightly different from the nominal model $G_a(s)$ with which this controller was computed (tuned).

As cases of study we take the plant's transfer function defined in section 3. The three slightly different models used are given as follows:

$$G_a \quad \frac{1.2}{s(1 \quad 2s)} e^{\quad s} \tag{17}$$

$$G_a \quad \frac{1}{s(1 \quad 2.4s)} e^{\quad s} \tag{18}$$

$$G_a \quad \frac{1}{s(1 \quad 2s)} e^{\quad 1.2s} \tag{19}$$

In order to evaluate the robustness of the proposed fractional PI D controller tuned with our method and to compare it with the other method, the same fractional PI D and conventional PID controllers' parameters in section 3 were employed. Firstly, the responses to the model (17), in which the steady state gain of (14) is increased by 20%, are shown in Fig.7.

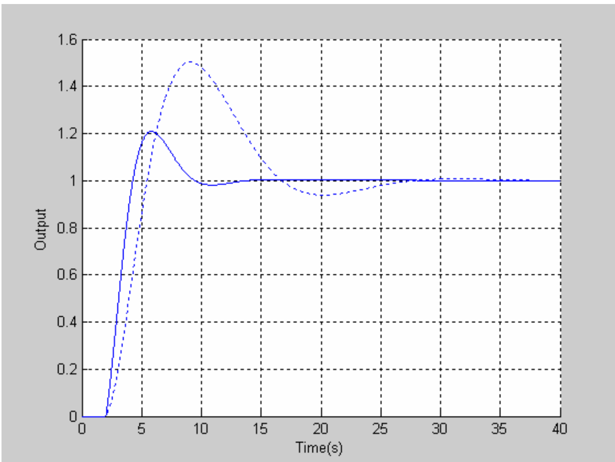

Fig. 7. Step response of the closed loop system C(s) the PID controller $C_1(s)$ (dotted line) and C(s) he proposed $PI^{0.1}D^{1.1}$ controller $C_2(s)$ (solid line), with static gain variation: 1.2

Secondly, the time constant is changed in (18) and its result is shown in Fig. 8.

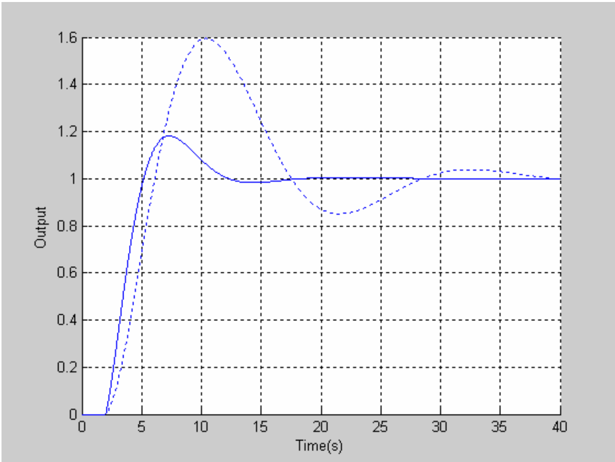

Fig. 8. Step response of the closed loop system C(s) the PID controller $C_1(s)$ (Dotted line) and C(s) the proposed $PI^{0.1}D^{1.1}$ controller $C_2(s)$ (solid line), with time constant variation: 2.4.

Thirdly, the time delay of is changed in (19), and the responses are shown in Fig. 9.

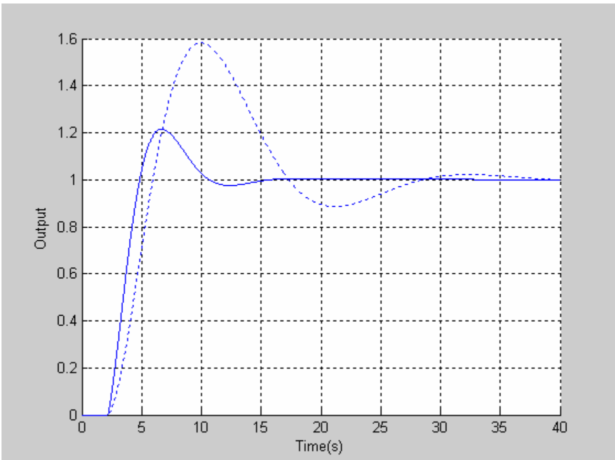

Fig. 9. Step response of the closed loop system C(s) the PID controller $C_1(s)$ (dotted line) and C(s) he proposed $PI^{0.1}D^{1.1}$ controller $C_2(s)$ (solid line), with time delay variation: 1.2

From the results obtained in the case of study, it can be concluded that the compensated systems using the proposed fractional PI and PI D controllers are robust to changes of the static gain and time constant, since variations of the performance characteristics for different values of static gain and time constant are lower for the fractional PI and PI D controllers. In short, it can be said that the use of the fractional PI and PI D controllers provide better responses and robust systems.

5. Implementation Considerations

In spite of using the classical Ziegler-Nichols tuning rules for setting the parameters K_P, T_I and T_D for $\lambda=\mu=1$ of the fractional PI D controller, our proposed conception method can use any other classical parameters tuning technique. Therefore we can use directly the K_P, T_I and T_D parameters of any PID controller of the feedback control system under investigation for performances enhancement. As the PID controllers are the most commonly used in practically all industrial feedback control applications, then our conception method of the fractional PI D controller will be very suitable for already tuned PID controllers. So implementation considerations of the fractional PI D controller with already existing PID controllers will be presented. Suppose that we have an already tuned classical unity gain feedback control system with a regular PID as a controller. This regular PID controller's transfer function is given as:

$$\mathrm{C(s)} \quad K_p \;\; 1 \quad \frac{1}{T_I s} \quad T_D s \qquad K_p \quad \frac{K_I}{s} \quad K_D s \tag{20}$$

Where $K_I=K_p/T_I$ and $K_D= K_pT_D$. The structure of the PID controller of equation (20) consists of the parallel connection of the proportional, integral and derivative parts as given in Fig. 10.

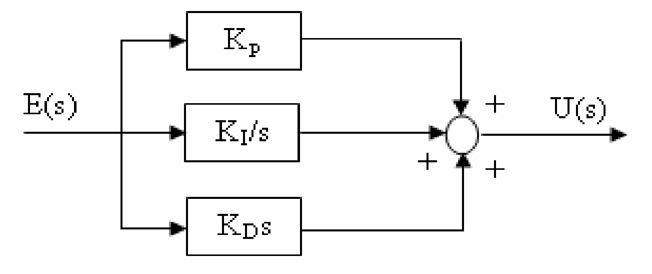

Fig. 10. Structure of the conventional PI D controller

In our design method, we said that we can use directly the already tuned parameters K_P, T_I and T_D of the PID controller of the feedback control system under investigation to set the fractional integration action order (0< <2) and the fractional differentiation action order (0< <2) of the PI D controller. Once the parameters and calculated the PI D controller's transfer function is given as:

$$\mathrm{C(s)} \quad K_p \;\; 1 \quad \frac{1}{T_I s} \quad T_D s \qquad K_p \quad \frac{K_I}{s} \quad K_D s \tag{21}$$

And the structure of this fractional PI D controller of equation (21) consists of the parallel connection of the proportional, fractional integral and fractional derivative parts as given in Fig. 11.

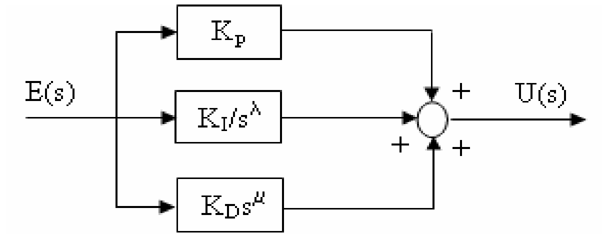

Fig. 11. Structure of the fractional PI D controller

And because we want to use the already existing PID controller to implement our fractional PI D controller therefore we will transform the fractional PI D controller's transfer function of equation (21) to the following equation:

$$C(s) \quad K_p \quad \frac{K_I}{s} s^{(1 \quad)} \quad \frac{K_D s}{s^{(1 \quad)}} \tag{22}$$

It means we just implement the fractional order differentiator $s^{(1 \quad)}$ (0< <2) in cascade with the already implemented regular integration action K_I/s of the regular PID controller to obtain the fractional order integration action K_I/s of the fractional PI D controller and the fractional order integrator $1/s^{(1 \quad)}$ (0< <2) in cascade with the already implemented regular differentiation action $K_D s$ of the regular PID controller to obtain the fractional order differentiation action $K_D s$ of the fractional PI D controller. Therefore the new internal structure of the fractional PI D controller of Fig. 11 with these fractional order operators will be as given in Fig .12.

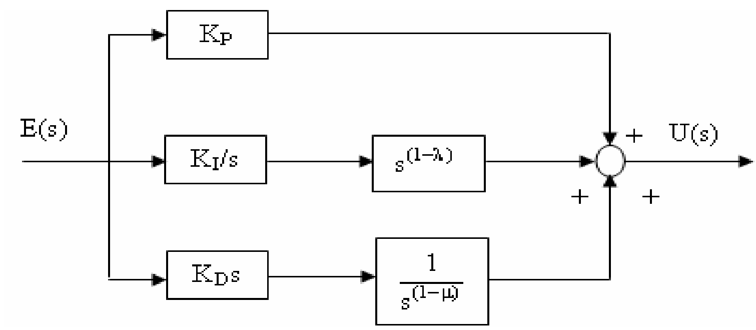

Fig. 12. New Structure of the fractional PI D controller

6. Conclusions

A new design method of the fractional PI D controller is presented towards system control quality enhancement and performances improvements for a classical unity feedback control system. The basic ideas are based on the classical Ziegler-Nichols tuning rules for setting the parameters K_P, T_I and T_D for $\lambda=1$, $\mu=1$ and on the minimum ISE criterion by using the Hall-Sartorius method for setting the parameters λ and μ. The formulations of this new tuning method have been derived using the rational function approximation of the fractional integrator and differentiator operators, in a given frequency band of practical interest.

Through two types of plant models, the fractional order PI PI D controllers design by the proposed method has been demonstrated. The simulation results illustrate that fractional PI and PI D controllers achieve better control performance with the proposed design method and better robustness to error models.

Our design technique will be very suitable for already conventional tuned PI and PID controllers.

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