

Iterative identification of Wiener model using hysteresis memory-less nonlinearity.

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ABSTRACT. *In this paper, we are interested in the parametric identification of the Wiener model. In this model, the used memory-less nonlinearity is the saturation with hysteresis. Equation is obtained by the decomposition technique and the superposition of the two switched saturation preposition is leading to pseudo linear regression. In this identification problem, we assume that the structure of the model is known whereas all the parameters of the discontinuous memory-less nonlinearities are unknown. The identification procedure applies the least-squares algorithm. The simulation results using MATLAB on academic example for different Signal-to-Noise Ratio (SNR) are given.*

KEYWORDS: *Nonlinear systems, Wiener Model, Discontinuous nonlinearities, Saturation with hysteresis, identification.*

1. Introduction

The saturation phenomena with hysteresis appear frequently in the industrial systems such as the electromagnetic systems (winds with core of iron, electric machines), electro pneumatic systems (actuators, pneumatic transmitters, servo valves), temperature control with thermostat e.g. [2], [6-9], [11], [16].

Many works in the literature deal with the modeling and the identification of nonlinear systems having memory-less hysteresis devices. Among these works, E.W. Bai [12] has studied the Hammerstein model based on several discontinuous memory-less nonlinearities. Typically, in [12], the authors assume that the nonlinear model of the hysteresis may be characterized by only one unknown parameter.

The iterative approach is frequently used in the literature review to parametric identification of Hammerstein and Wiener systems. e.g. K.S. Narendra et al., 1966 [1], P. Stoïca, 1981 [3], J. VÖRÖS, 1997, 2001, 2003-a and 2007 [5, 10, 13, 19], S. Rejeb et al., 2006 [17] and Y. Liu et al., 2007 [18]. This approach is simple to implement and leads good results. The local convergence analysis of the iterative least-square algorithm is studied, in general cases, by P. Stoïca, 1981 [3]. Concerning the convergence analysis of the iterative approach to identify the Wiener model, there exist few papers in the literature dealing with this problem. On the other hand, for the Hammerstein model some recent works exist so far e.g. E.W. Bai et al., 2004 [15] and Y. Liu et al., 2007 [18].

Among these recent publications, many authors treat the identification of Hammerstein and Wiener models with discontinuous nonlinearity, when the points of discontinuity are assumed known. In this paper, our main contribution is to consider that these points of discontinuity are unknown and their estimation is the main purpose of the proposed work. However, this problem has been worked out by some authors such as E.W. Bai, 2002 [12] and from J. Vörös, 1997, 2001, 2003-b and 2007 [5, 10, 14, 19]. The estimation of the memory-less discontinuity points of the nonlinearity remains an open field of research.

In this work, our interest is focused on modeling and parametric identification of Wiener model using hysteresis based saturation nonlinearity. This nonlinearity is characterized by four unknown parameters. In describing the system, we focus on the decomposition technique and the superposition principle of two saturations switched characteristics. The basic idea is to formulate this model into linear regression form which depends on the parameter vector. Consequently, an iterative least-square parametric identification algorithm is proposed.

In section 2, a Wiener model with static nonlinearity saturation with hysteresis is given and developed. In section 3, the iterative least-squares algorithm parameter identification with stopped criteria is formulated. Finally, in section 4 simulation results on an academic example are shown which highlight the superiority of the proposed algorithm.

2. Wiener model

The Wiener model is a special kind of nonlinear system where the linear block is followed by a static nonlinear block. This model is described by the bloc scheme of figure 1.

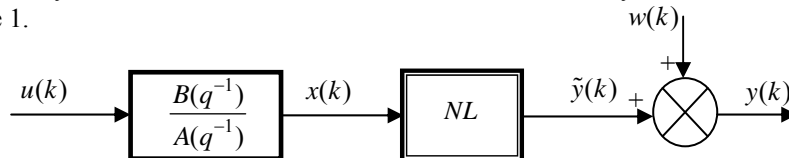


Fig. 1. Bloc scheme of Wiener Model

In the different blocs of figure 1, the variables and the polynomials are given as follows:

$u(k)$: Input model.

$\tilde{y}(k)$: Noiseless Output model.

$y(k)$: Noisy Output model.

$w(k)$: Noise measurement.

$x(k)$: Internal variable (considered unknown).

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_mq^{-m} \tag{1}$$

$$B(q^{-1}) = b_1q^{-1} + \dots + b_nq^{-n} \tag{2}$$

Where m and n are respectively the orders of the polynomials $A(q^{-1})$ and $B(q^{-1})$.

The recurrent equation of the linear dynamic block is described as:

$$x(k) = B(q^{-1})u(k) + [1 - A(q^{-1})]x(k) \tag{3}$$

Equation (3) can be developed as follows:

$$x(k) = \sum_{i=1}^n b_i u(k-i) - \sum_{i=1}^m a_i x(k-i) \tag{4}$$

The nonlinear output block $\tilde{y}(k)$ can be written as:

$$\tilde{y}(k) = NL[x(k)] \tag{5}$$

Thereafter, we propose to describe the Wiener model in linear pseudo-regression form.

$$y(k) = \varphi^T(k, \theta)\theta + w(k) \tag{6}$$

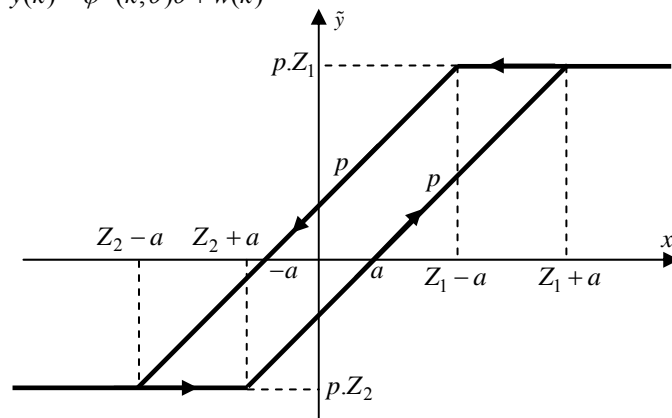


Fig. 2. Nonlinearity saturation with hysteresis shape

This nonlinearity is characterized by the following parameters:

- $Z_1 - a$, $Z_1 + a$, $Z_2 - a$ et $Z_2 + a$ which represent the discontinuity points.
- a characterizes the hysteresis width.
- p denotes the slope of the linear saturation segment.

These parameters are unknown and have to be identified.

The relationship between $\tilde{y}(k)$ and the internal variable $x(k)$ is described as:

$$\tilde{y}(k) = \begin{cases} p.Z_1 & \text{if } x(k) > Z_1 + a \text{ or } Z_1 - a < x(k) < Z_1 + a \text{ and } \Delta x < 0 \\ p.(x(k) - a) & \text{if } Z_2 + a \leq x(k) \leq Z_1 + a \text{ and } \Delta x > 0 \\ p.(x(k) + a) & \text{if } Z_2 - a \leq x(k) \leq Z_1 - a \text{ and } \Delta x < 0 \\ p.Z_2 & \text{if } x(k) < Z_2 - a \text{ or } Z_2 - a < x(k) < Z_2 + a \text{ and } \Delta x > 0 \end{cases} \quad (7)$$

With $\Delta x = x(k) - x(k-1)$

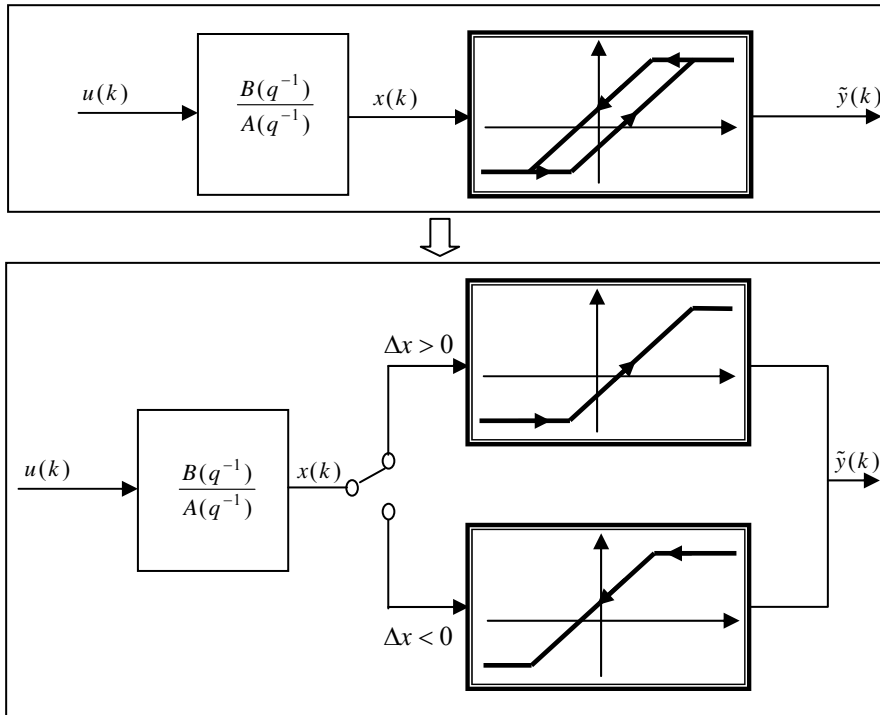


Fig. 3. Representation of nonlinearity saturation with hysteresis by the superposition of two saturations

Following, it is possible to introduce a switching sequence $h(\alpha)$ which is defined as:

$$h(\alpha) = \begin{cases} 1 & \text{si } \alpha > 0 \\ 0 & \text{si } \alpha \leq 0 \end{cases} \quad (8)$$

Using this switching sequence, it is possible to rewrite the nonlinearity underhand in the following form:

$$\tilde{y}(k) = \begin{cases} px(k)f_1(k) - paf_1(k) + pZ1h_1(k) + pZ2h_2(k) & \text{if } \Delta x > 0 \\ px(k)f_2(k) + paf_2(k) + pZ1h_3(k) + pZ2h_4(k) & \text{if } \Delta x < 0 \end{cases} \quad (9)$$

With:

$$\begin{aligned} h_1(k) &= h[x(k) - (Z_1 + a)] \\ h_2(k) &= h[(Z_2 + a) - x(k)] \\ f_1(k) &= 1 - (h_1(k) + h_2(k)) \\ h_3(k) &= h[x(k) - (Z_1 - a)] \\ h_4(k) &= h[(Z_2 - a) - x(k)] \\ f_2(k) &= 1 - (h_3(k) + h_4(k)) \end{aligned} \quad (10)$$

Hence, substituting (4) in equation (9) it follows:

$$\tilde{y}(k) = \begin{cases} \sum_{i=1}^n pb_i u(k-i)f_1(k) - \sum_{i=1}^m pa_i x(k-i)f_1(k) \\ - paf_1(k) + pZ1h_1(k) + pZ2h_2(k) & \text{if } \Delta x > 0 \\ \sum_{i=1}^n pb_i u(k-i)f_2(k) - \sum_{i=1}^m pa_i x(k-i)f_2(k) \\ + paf_2(k) + pZ1h_3(k) + pZ2h_4(k) & \text{if } \Delta x < 0 \end{cases} \quad (11)$$

In order to obtain a single solution of the vector of parameters, we assume that b_1 is equal to 1 ($b_1 = 1$).

From equation (11) and using the equation (6), it possible to define the vector of parameters θ as:

$$\theta = [p, pb_2, \dots, pb_n, pa_1, \dots, pa_m, pa, pZ_1, pZ_2]^T \quad (12)$$

and the data vector defined by:

$$\varphi(k, \theta) = \begin{cases} [u(k-1)f_1(k), \dots, u(k-n)f_1(k), -x(k-1)f_1(k), \dots \\ \dots, -x(k-m)f_1(k), -f_1(k), h_1(k), h_2(k)]^T & \text{if } \Delta x > 0 \\ [u(k-1)f_2(k), \dots, u(k-n)f_2(k), -x(k-1)f_2(k), \dots \\ \dots, -x(k-m)f_2(k), f_2(k), h_3(k), h_4(k)]^T & \text{if } \Delta x < 0 \end{cases} \quad (13)$$

3. Iterative identification algorithm

Since the variable $x(k)$ in (11) is unmeasurable and must be estimated, an internal variable estimation iterative identification procedure should be launched, similarly as in the case of hysteresis nonlinearity saturation estimation.

Consequently, inspecting equation (13), the data vector $\varphi^T(k, \theta)$ contains unmeasurable internal signals and it is necessary to estimate them together with estimation of the nonlinear parameters.

The following quantities should be estimated jointly in the iterative algorithm:

$$\begin{aligned}
 {}^{(s)}\hat{x}(k) &= \sum_{i=1}^n {}^{(s)}\hat{b}_i u(k-i) - \sum_{i=1}^m {}^{(s)}\hat{a}_i {}^{(s)}\hat{x}(k-i) & (14) \\
 {}^{(s)}\hat{h}_1(k) &= h[{}^{(s)}\hat{x}(k) - ({}^{(s)}\hat{Z}_1 + {}^{(s)}\hat{a})] \\
 {}^{(s)}\hat{h}_2(k) &= h[({}^{(s)}\hat{Z}_2 + {}^{(s)}\hat{a}) - {}^{(s)}\hat{x}(k)] \\
 {}^{(s)}\hat{f}_1(k) &= 1 - ({}^{(s)}\hat{h}_1(k) + {}^{(s)}\hat{h}_2(k)) \\
 {}^{(s)}\hat{h}_3(k) &= h[{}^{(s)}\hat{x}(k) - ({}^{(s)}\hat{Z}_1 - {}^{(s)}\hat{a})] & (15) \\
 {}^{(s)}\hat{h}_4(k) &= h[({}^{(s)}\hat{Z}_2 - {}^{(s)}\hat{a}) - {}^{(s)}\hat{x}(k)] \\
 {}^{(s)}\hat{f}_2(k) &= 1 - ({}^{(s)}\hat{h}_3(k) + {}^{(s)}\hat{h}_4(k))
 \end{aligned}$$

The error to be minimized in the s -th step is defined from (6) as:

$${}^{(s)}e(k) = y(k) - {}^{(s)}\hat{\varphi}^T(k, {}^{(s-1)}\hat{\theta}_{(N)}) {}^{(s)}\hat{\theta}_{(N)} \quad (16)$$

Where ${}^{(s)}\hat{\varphi}^T(k, {}^{(s-1)}\hat{\theta}_{(N)})$ is the data vector with the corresponding estimates of internal variable according and the functions which depends of this one to (14)–(15) and ${}^{(s)}\hat{\theta}_{(N)}$ is the (s) th estimate of the parameter vector. Where N is the total number of data.

The iterative algorithm is based on the minimization of the mean-square error (MSE).

$${}^{(s)}MSE = \frac{1}{N} \sum_{k=1}^N {}^{(s)}e(k)^2 \quad (17)$$

From where the estimator of least-squares:

$${}^{(s)}\hat{\theta}_{(N)} = \arg \min_{\theta} {}^{(s)}MSE \quad (18)$$

The steps in the iterative procedure may be now stated as follows:

- Step1): initialize ${}^{(0)}\theta$, ${}^{(0)}\hat{x} = y$, ε . Where ε is a specified threshold chosen by the user.
- Step2): for $s = s + 1$, form the vector of observation ${}^{(s)}\hat{\varphi}^T(k, {}^{(s-1)}\hat{\theta}_{(N)})$.
- Step3): identify the vector θ by minimizing a proper criterion based on (18).

Step4): estimate the internal variable $^{(s)}\hat{x}(k)$ and the functions $^{(s)}\hat{h}_1(k)$, $^{(s)}\hat{h}_2(k)$, $^{(s)}\hat{h}_3(k)$, $^{(s)}\hat{h}_4(k)$, $^{(s)}\hat{f}_1(k)$ and $^{(s)}\hat{f}_2(k)$ based on (14) and (15).

Step5): if $\|^{(s)}\hat{\theta}_{(N)} - ^{(s-1)}\hat{\theta}_{(N)}\| > \varepsilon$ repeat steps 2, 3 and 4, else $\hat{\theta}_{(N)} = ^{(s)}\hat{\theta}_{(N)}$ and stop.

4. Results of simulation

In order to test the proposed identification procedure, some simulation results on an academic example containing a hysteresis saturation memory-less nonlinearity are show in the following section.

The linear dynamic block is described by the following equation as:

$$x(k) = u(k-1) + 0.5u(k-2) + 0.2x(k-1) - 0.35x(k-2)$$

The parameters of the nonlinearity are given as follows:

$$Z_1 = 1.5; Z_2 = -1; a = 0.5; p = 1.5$$

Hence, the least-squares method was used for the repeated estimations of all the model parameters and the internal variables. The identification was carried out with 2000 samples, using uniformly distributed random inputs with $|u(k)| \leq 3$ and simulated outputs. The initial values of all parameters were chosen as: $^{(0)}\theta = [2.25, 0, 0, 0, 0.75, 2.25, -1.5]^T$.

The results of the Wiener model identification parameters are obtained for a chosen threshold $\varepsilon = 10^{-2}$.

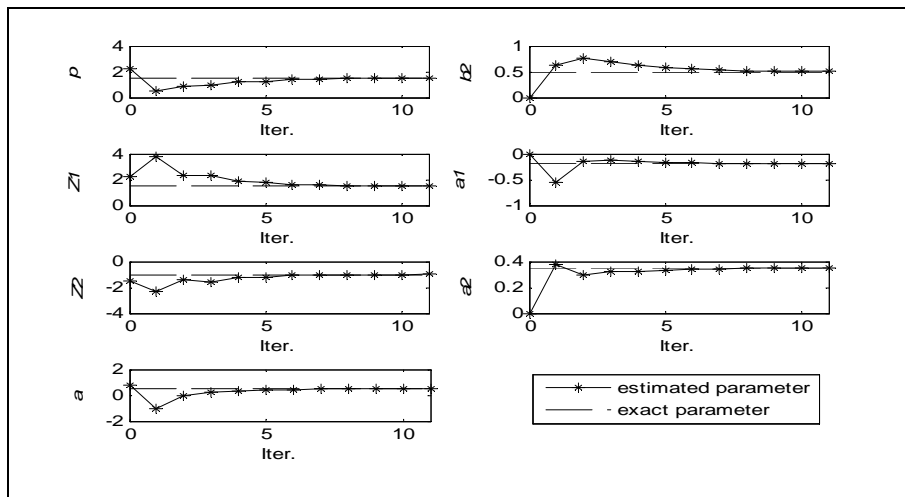


Fig. 5. Evolution of the parameter estimates of the Wiener model without noise $w(k) = 0$ according to the iteration steps

From figure 5, we can see that all the parameters converge towards their exact values. This convergence is around their nominal values with a deviation calculated such that $\|^{(s)}\hat{\theta}_{(N)} - ^{(s-1)}\hat{\theta}_{(N)}\|$ has value lower than $\varepsilon=10^{-2}$. Figure 6 depicts the convergence of $\|^{(s)}\hat{\theta}_{(N)} - ^{(s-1)}\hat{\theta}_{(N)}\|$ with respect to the iteration steps.

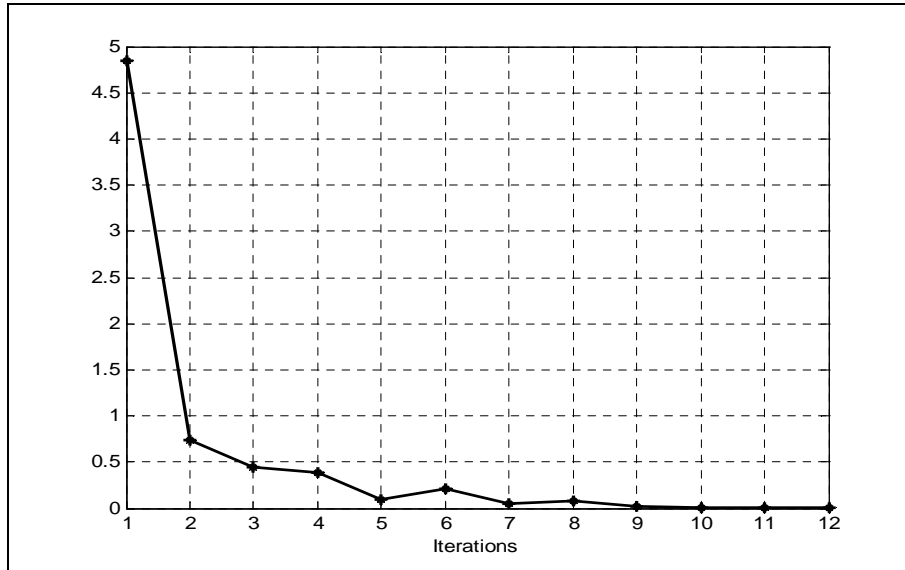


Fig. 6. Evolution of $\|^{(s)}\hat{\theta}_{(N)} - ^{(s-1)}\hat{\theta}_{(N)}\|$ according to the iteration steps

	p	b_2	a_1	a_2	a	Z_1	Z_2	MSE
Iter. 12	1.4988	0.5052	-0.1985	0.3490	0.4934	1.5012	-1.0008	5.6254
True	1.5000	0.5000	-0.2000	0.3500	0.5000	1.5000	-1.0000	-

Table 1. The parameters estimated of the Wiener model without noise

The above experiments have been performed in a noiseless setting, however to test the robustness of the proposed identification method, we shall propose in the sequel to test the algorithm in noisy setting and evaluate the results using a Monte Carlo test. For that, we consider three levels of the noise SNR and for 12 iterations. Table 2 illustrates the average estimate of the parameters obtained by Monte Carlo simulation over 150 random realizations of the input signal and measurement noise.

Parameter		p	b_2	a_1	a_2	a	Z_1	Z_2	MSE
Exact values		1.5000	0.5000	-0.2000	0.3500	0.5000	1.5000	-1.0000	-
SNR=100 dB	Mean	1.4902	0.5098	-0.1969	0.3489	0.4895	1.5096	-1.0062	5.6921
	Std	0.0118	0.0104	0.0033	0.0015	0.0115	0.0119	0.0077	0.1638
SNR=50 dB	Mean	1.4901	0.5094	-0.1971	0.3489	0.4898	1.5096	-1.0064	5.6942
	Std	0.0117	0.0106	0.0034	0.0014	0.0114	0.0118	0.0076	0.1602
SNR=15 dB	Mean	1.4885	0.5095	-0.1981	0.3491	0.4871	1.5120	-1.0074	5.7253
	Std	0.0167	0.0141	0.0059	0.0029	0.0150	0.0180	0.0118	0.1527

Table 2. Values of the parameters estimated by Monte Carlo simulation of the model of disturbed Wiener for several levels of SNR

Table 2 shows the results of the Monte Carlo simulation. These results are given to evaluate the quality of the estimation and its corresponding standard deviation (Std) for three SNR levels. Note that the quality of the estimate depends on the noise amplitude. For instance, the more the noise amplitude is increased more the quality of the estimation is affected. These estimates are almost acceptable under a 15dB level. Beyond this level the convergence is not guaranteed for certain realizations.

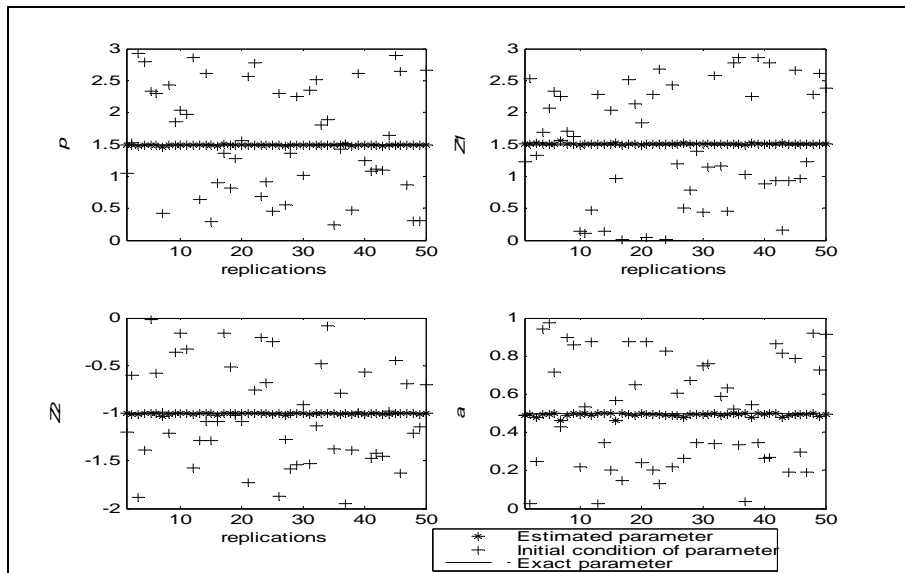


Fig. 7. Behavior of the nonlinearity parameters according to the chosen realizations for an initialization range of $\pm 100\%$ of the parameters nominal values.

range of variation of the Initial conditions		$\pm 10\%$		$\pm 100\%$	
Parameter	True	Mean	Std	Mean	Std
p	1.5000	1.4906	0.0134	1.4910	0.0135
b_2	0.5000	0.5090	0.0129	0.5087	0.0122
a_1	-0.2000	-0.1973	0.0038	-0.1973	0.0035
a_2	0.3500	0.3491	0.0016	0.3491	0.0014
a	0.5000	0.4901	0.0144	0.4907	0.0136
Z_1	1.5000	1.5094	0.0136	1.5089	0.0136
Z_2	-1.0000	-1.0061	0.0089	-1.0058	0.0091
MSE	-	5.6918	0.1475	5.6968	0.1547

Table 3. Results of the Monte Carlo simulation of the model without noise: case of a random initialization of the initial parameter values.

Inspecting Figure 7 and Table 3, the results of the identification algorithm with respect to a random initialization of the parameters according to a uniform distribution are given. The results of this simulation are performed over 150 realizations.

5. Conclusion

In this paper, we have used the iterative least-squares algorithm to estimate the unknown parameters of the Wiener model. The nonlinearity considered here is a static hysteresis memory-less discontinuous and asymmetric nonlinearity. To solve this problem, we have used two techniques. In the first, the superposition principle makes possible to write the nonlinearity equation into saturations switched according to the internal variable and its variation. The second one consists of applying the decomposition technique to obtain the linear pseudo-regression according to parameters vector. It is assumed here that the structure of this model is known and that the parameters all are unknown including the points of discontinuity. An application of this approach is presented by simulation on an academic example. A Monte Carlo simulation is also carried out to test the convergence of the iterative identification algorithm while taking several levels SNR. In conclusion, the proposed algorithm is tested and exhibits good results both in noisy and noiseless environments.

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