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Constrained Fuzzy Based Model Predictive Control with Disturbances

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Abstract: Using a linear model predictive controller (MPC) as a controller of nonlinear system may not give a satisfactory dynamic performance. This has led to the development of a number of nonlinear MPC (NMPC) approaches that permit the use of first principles based nonlinear models. This paper presents an algorithm to solve the problem of non-linear predictive control in the presence of constraint, where the nonlinear system presented by a fuzzy system. Our strategy reduced the original and complex nonlinear quadratic optimization problem into a quadratic program (QP). The approach utilizes the Takagi–Sugeno modeling methodology to generate a fuzzy convolution model which is in fact represents our non-linear system. With this method, a novel hierarchical control design approach is described. The method is applied to control a Stirred Tank reactor and the simulation results demonstrate the attractiveness of the FMPC.

Keywords. *Model Based predictive control (MBPC), non-linear system, Takagi−Sugeno (T−S) fuzzy models, fuzzy implication (FI), QP.*

I. Introduction

Model Based predictive Control (MBPC) has been an active field of research during the last three decades, driven both by numerous successful applications of the technology [1, 2] and by the research interests of the academia. The main reason of this success is the ability of MBPC to control systems under constraints in an optimal way. In model predictive control, the control action is computed by solving an optimization problem on line in each sampling period. A major bottleneck in practical applications of MBPC is obtaining a reliable predictive model. Fuzzy models of Takagi-Sugeno (T-S) type proved to be suitable for the use in nonlinear MPC because of their ability to accurately approximate complex nonlinear systems.

Many applications of MBPC based on fuzzy prediction models have been reported [3-5]. Recently, several papers appeared where different fuzzy MBPC algorithms are

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analyzed and compared [6]. The methods discussed in these references can generally be classified into two groups: methods utilizing directly the fuzzy model in the optimization procedure [7], [6], [4] and methods using a linearized model obtained form the fuzzy model, whereas Abonyi *et al.* [5] apply Jacobian linearization. Other possibilities are to compute the control signals for the different fuzzy submodels separately and to weigh them [8], or to use only the submodel with the highest membership degree [3], [6].

The predictive controller discussed in this paper uses local linear models derived from the fuzzy model, as described in. A linear model can be extracted from a T-S model at very low computational costs at a given working point or along a trajectory. The control signal is then obtained by solving a constrained quadratic program. A Convergent iterative optimization scheme can be used to reduce the effect of linearization errors.

The fuzzy model proposed by Takagi and Sugeno [9] is described by fuzzy IF-THEN rules, which represent local linear input- output relations of a nonlinear system. The *i*th rules of T-S fuzzy models are of the following form, where DFS denote the discrete fuzzy system.

Plant Rule i:

IF z_1 is B^1 and … and z_p is B^p

$$\text{THEN} \begin{cases} \dot{x}(t+1) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} i = 1,2,...,r. \quad (1)$$

B^i is the fuzzy set and r is the number of IF-THEN rules. $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^q$ is the output vector, $A \in R^{n\times n}$, $B \in R^{n\times m}$, and $C \in R^{q\times n}$.$z_1...z_p$ are the premise variables.

The organization of this paper is as follows: In section II, we present some preliminary results. Section III introduces the T-S model. Section IV presents the incorporation of the constraints in the model predictive control. In Section V, a simulation example is presented. Finally, Section VI concludes the paper.

II. Preliminaries

Consider the time-invariant nonlinear system described by:

$$\begin{aligned} \dot{x} &= f(x,u,v,t) \\ y &= g(x,t) \end{aligned} \quad (2)$$

where $x \in R^n$, $u \in R^m$, $v \in R^k$ and $y \in R$, f and g are smooth function, v is the external disturbance that caused by the variation of the system variables. For many cases the linearization of the non-linear system is required to avoid the complexity of the system and to evolve a simple linear model that might be used to control the original system. Consider the set of points

$$P = \{(x_{0i}, x_{0i}) : x_{0i} \in R^n, x_{0i} \in R^m, i = 1,2,...,r\}$$

which is known such as the linearization around each (x_{0i}, u_{0i}) $\in P$ and the resulting models are interpolated to generate the non linear system described above. The first order linearization for the above system around the point (x_{0i}, u_{0i}) $\in P$ results the following system

$$\frac{dx}{dt} = A_{ci}x + B_{ci}u + R_{ci}v + D_{ci}$$
$$y = C_i x + E_{ci} \qquad (3)$$

where the matrixes A_{ci}, B_{0i}, C_{ci} and R_{ci} are respectively given by the following: $\partial f/\partial x|_{(x_{0i},x_{0i},t)}$, $\partial f/\partial u|_{(x_{0i},x_{0i},t)}$, $\partial g/\partial x|_{(x_{0i},t)}$ and $\partial f/\partial v|_{(x_{0i},u_{0i},t)}$.
D_i and E_i are respectively given by

$$D_{ci} = f(x_{0i}, u_{0i}, t) - A_i x_{0i} - B_i u_{0i}$$
$$E_{ci} = g(x_{0i}) - C_i x_{0i}$$

In the special case where $f(X_{0i}, U_{0i}, t) = 0$ for (X_{0i}, U_{0i}) $\in P$, the point (X_{0i}, U_{0i}) $\in P$ is called the equilibrium point of the system (2) and the system (3) can be written as

$$\dot{x} = A_{ci}x + B_{ci}u \qquad (4)$$
$$y = C_{ci}x + Y_{0i}$$

Since we use discrete time controllers, we need to have a discrete time plant, so the next step is to discretize the system (4), and here, for simplicity, we use a first order Euler approximation [10].

The discretized system can be expressed as

$$x(k+1) = \overline{A}_i x(k) + \overline{B}_i u(k) + \overline{R}_i v(k) + \overline{D}_i$$
$$y(k) = \overline{C}_i x + \overline{E}_i \qquad (5)$$

where $\overline{A}_i = I + T_e A_{ci}$, $\overline{B}_i = T_e B_{ci}$, $\overline{R}_i = T_e R_{ci}$, $\overline{D}_i = T_e \overline{D}_{ci}$, $\overline{C}_i = C_i$ and $\overline{E}_i = E_i$, for i=1,2,…,r.

III. Takagi–Sugeno Fuzzy Systems (TSFS)

Consider a highly nonlinear system described above. The system is decomposed into subsystems such that each subsystem demonstrates a linear behaviour as demonstrated into the above section. By Takagi-Sugeno's modelling methodology [12], a fuzzy quasi-linear model, or FI, was developed for each subsystem. In such a model, the cause-effect relationship between control u and output y at the sampling time t is established in a discrete time representation.

An FI is rule based on and consists of a set of symbolic antecedents in the IF part (premise) and a linear numerical expression in the THEN part (consequence). Each FI is generated based on a subsystem. Thus, it can be called a fuzzy convolution sub-model that has the following structure:

IF x_1 is B^1 and … and x_p is B^p, *and u is* B^u

$$\text{THEN} \begin{cases} x(k+1) = \overline{A}_i x(k) + \overline{B}_i u(k) + \overline{R}_i v(k) + \overline{D}_i \\ y(k) = \overline{C}_i x(k) + \overline{E}_i \end{cases} \quad i = 1,2,\dots,r. \tag{6}$$

where

B^i fuzzy set corresponding to states $x_i(t)$ in the *ith* FI;

B^u fuzzy set corresponding to output $u(k)$ in the *ith* FI;

A complete fuzzy convolution model for the system consists of p FIs. Let $w_i(x,u)$ be the truth value for the *ith* FI; it can be calculated based on the fuzzy sets in the IF part, where $w_i(x,u)$ is defined in way to give the following

$$\sum_{i=1}^{p} w_i(x,u)=1 \tag{7}$$

So that at each $(x,u)\in R^n \times R^m$ we are perfectly certain that it can be represented by the set of models that are "on" (i.e., the set $w_i(x,u) > 0$). Using (5) and the definition of $w_i(x,u)$, the contribution of the i^{th} affine system in constructing the TSFS can be expressed as:

$$\begin{aligned} x(k+1) &= A(x,u)\ x(k) + B(x,u)u(k) + R(x,u)v(k) + D(x,u) \\ y(k) &= C(x,u)x(k) + E(x,u) \end{aligned} \tag{8}$$

where

$$A(x,u) = \sum_{i=1}^{r} w_i(x,u)\overline{A}_i,\ B(x,u) = \sum_{i=1}^{r} w_i(x,u)\overline{B}_i,\ R(x,u) = \sum_{i=1}^{r} w_i(x,u)\ \overline{R}_i,$$

$$C(x,u) = \sum_{i=1}^{r} w_i(x,u)\ \overline{C}_i,\ \ E(x,u) = \sum_{i=1}^{r} w_i(x,u)\ \overline{E}_i \text{ and } D(x,u) = \sum_{i=1}^{r} w_i(x,u)\ \overline{D}_i.$$

The TSFS described above can approximate the non-linear system (2) to any degree of accuracy by increasing the number of local models so long as the points (x_i,u_i) are properly distributed across the space of interest.

IV. MBPC with Constraints

The main objective of an *FMPC* controller is to minimize the predictive error between an output and a given reference trajectory in the next N_2 steps through the selection of N_u-step optimal control policies.

From [11], the optimization problem can be formulated as

$$V(k)=\sum_{i=N_1}^{N_2}\left\|\hat{y}(k+i\mid \mathrm{k})\text{-}r(\mathrm{k+i})\right\|_{Q(i)}^{2}+\sum_{i=0}^{N_u-1}\left\|\Delta\hat{u}(k+i\mid \mathrm{k})\right\|_{R(i)}^{2} \quad (9)$$

where, *r(k)* is the reference signal, *Q(i)* and *R(i)* are respectively, the weighting matrixes for the prediction error and control energy;

In order to optimize the const function, the optimal prediction of $y(k+i)$ for $i\geq N_1$ and $i\leq N_2$ will be obtained according to [11], as follow

$$Y(k)=\Psi x(k)+\Xi u(k-1)+\Omega\Delta U(k)+\Xi_v v(k-1)+\Omega_v\Delta V(k)+D_c+E_c,$$

where

$$Y(k)=\begin{bmatrix}\hat{y}(k+N_1\setminus \mathrm{k})\\ \vdots\\ \hat{y}(k+N_2\setminus \mathrm{k})\end{bmatrix} \quad (10)$$

The TSFS is used to predict the process output, that is, subject to inputs and outputs constraints [11].

The constraints form is given according to the following expressions:

$$M\begin{bmatrix}\Delta U(k)\\ 1\end{bmatrix}\leq 0 \quad (11)$$

$$T\begin{bmatrix}U(k)\\ 1\end{bmatrix}\leq 0 \quad (12)$$

$$G\begin{bmatrix}Y(k)\\ 1\end{bmatrix}\leq 0 \quad (13)$$

where $U(k)=[u(k/k)^T,\ldots,u(k+H_u-1/k)^T]^T$.

Then we can assemble inequalities (11), (12), and (13) into the single inequality

$$\begin{bmatrix}Te\\ \Gamma\Omega\\ W\end{bmatrix}\Delta U(k)\leq\begin{bmatrix}-Te_1u(k-1)-f\\ -\Gamma[\Psi x(k)+\mathrm{Z}u(k-1)+P_c]-g\\ w\end{bmatrix} \quad (14)$$

We have to solve the following constrained optimization problem:

$$min\ \ \Delta U(k)^T\mathrm{H}\Delta U(k)-\mathrm{P}^T\Delta U(k) \quad (15)$$

Subject to the inequality constraint (14), which is a standard optimization problem known as the Quadratic Programming (QP) problem, and standard algorithms are available for its solution.

V. Simulation Examples

The benchmark plant represents a continuous-Stirred Tank reactor. The model was represented in [9] and is described by the following differential equations

$$\begin{aligned} \dot{C}_a(t) &= \frac{q}{V}(C_{a0} - C_a(t)) - k_0 C_a(t) e^{-\frac{E}{RT(t)}} \\ \dot{T}(t) &= \frac{q}{V}(T_0 - T(t)) - k_1 C_a(t) e^{-\frac{E}{RT(t)}} + k_2 q_c(t)(1 - e^{-\frac{k_3}{q_c(t)}})(T_{c0} - T(t)) \end{aligned} \tag{22}$$

The process consists of an irreversible, exothermic reaction, *A ->B*, in a constant volume reactor cooled by a single coolant stream. The product concentration is represented by $C_a(t)$ and this concentration is controlled by manipulating the coolant flow rate $q_c(t)$. The temperature of the mixture is represented by $T(t)$. The heat generated acts to slow the reaction. C_{a0} is the inlet feed concentration, q is the process flow rate, T_0 and T_{c0} the inlet feed and coolant temperatures. All these values are assumed constant at the nominal values. In the same way, k_0, E/R, V, k_1, k_2 and k_3 are thermodynamic and chemical constants. The numerical values of these parameters are given in *table 1.*

Parameters ;	Description;	Nominal value
q	Process flue rate	*100 l/min*
V	Reactor volume	*100 l*
k_0	Reaction rate constant	7.2×10^{10} min^{-1}
E/R	Activation energy	10^4 K
T_0	Feed temperature	*350 K*
T_{c0}	Inlet coolant temperature	*350 K*
ΔH	Heat of reaction	-2×10^5 *cal/mol*
C_p, C_{pc}	Specific heats	*1 cal/g/K*
ρ, ρ_c	Liquid densities	10^3 *g/l*
h_a	Heat transfer coefficient	7×10^5 *cal/min/K*
C_{a0}	Inlet feed concentration	*1 mol/l*

with:

$$k_1 = \frac{\Delta H k_0}{\rho C_p}, \quad k_2 = \frac{\rho_c C_{pc}}{\rho C_p V}, \quad and \quad k_1 = \frac{h_a}{\rho_c C_{pc}}$$

The nominal conditions for a product concentration C_a *=0.1 mol/l* are: $T = 438.54\ K\ q_c = 103.41\ l/min$. The number of the affine systems used here is $R = 10$. The desired range of the state is specified to be between $C_a^{\min}$ *= 0.1* and $C_a^{\max}$ *= 0.3,* in other word the set of point *P* might be written as following

$$P = \left\{ (C_a^1, T^1, q_c^1), \ldots, (C_a^R, T^R, q_c^R) \right\}$$

where $C_a^1 = C_a^{\min}$ and $C_a^i = C_a^{i-1} + \frac{C_a^{\max} - C_a^{\min}}{R}$. For each value of C_a^i , the correspondence T_i and q_c are founded by solving the following equations $f(x_i, u_i)$= 0, and $g(x_i) = 0$.

A series of simulations are conducted to examine the control quality by the FMPC controller. The response of the system with multi-step reference is given in Fig.1; the reference of C_a was changed from the initial point 0.08 mol/l to 0.085 to 0.087 and then to 0.089 (see the red line in Fig.1). The dynamic response of the system is depicted in the same figure. The control dynamics is represented in Fig. 2. Fig. 4 illustrates the disturbance that we were used for investigate the performance of the FMPC controller. In simulation, we consider that the plant is subject to a Normal sequence of disturbances that we added to the system, and constraints. The dynamic response in the Fig.1 shows that the FMPC system has a strong disturbance rejection capability.

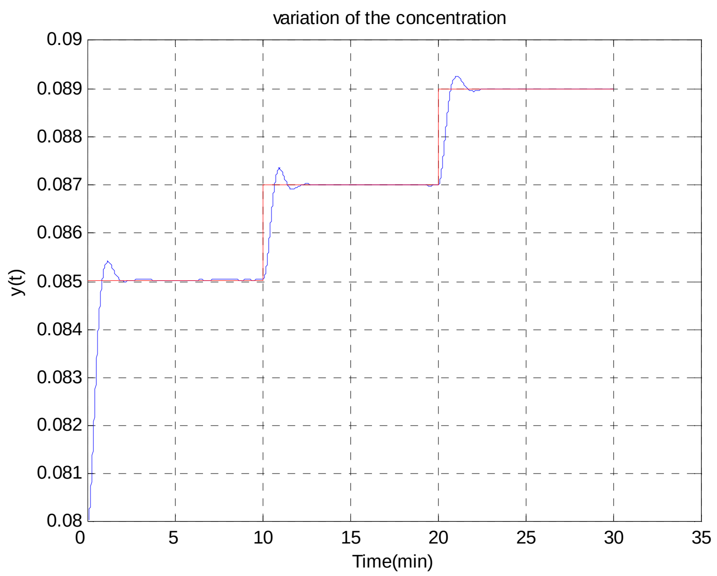

Fig. 1. Output signal

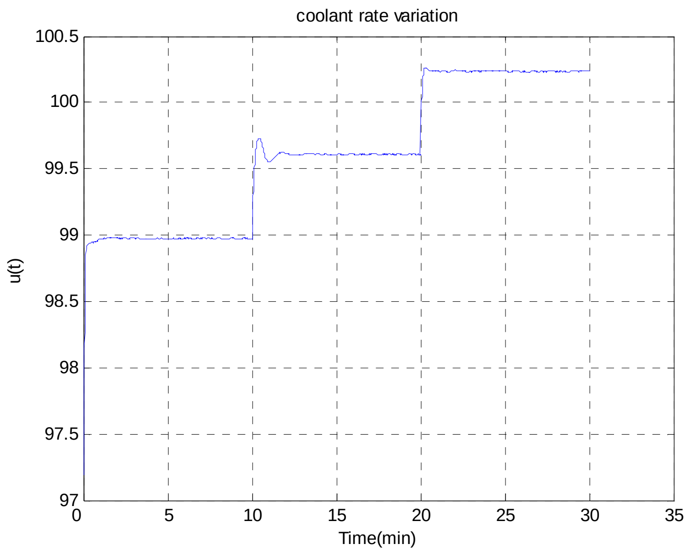

Fig. 2. Control signal

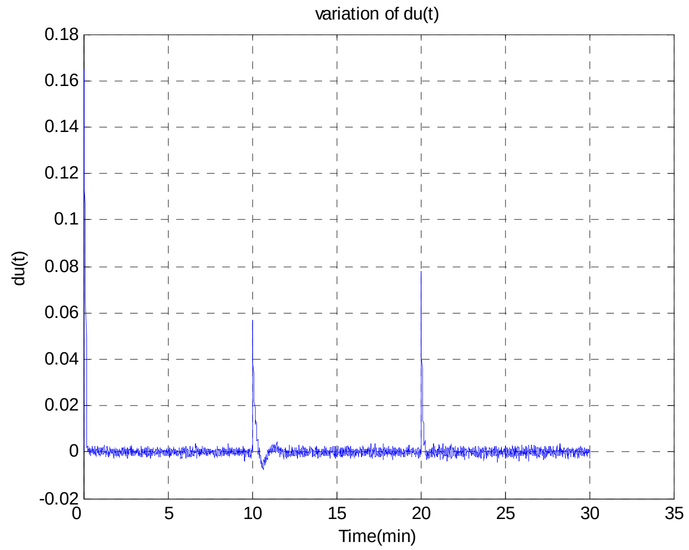

Fig. 3. Increment signal

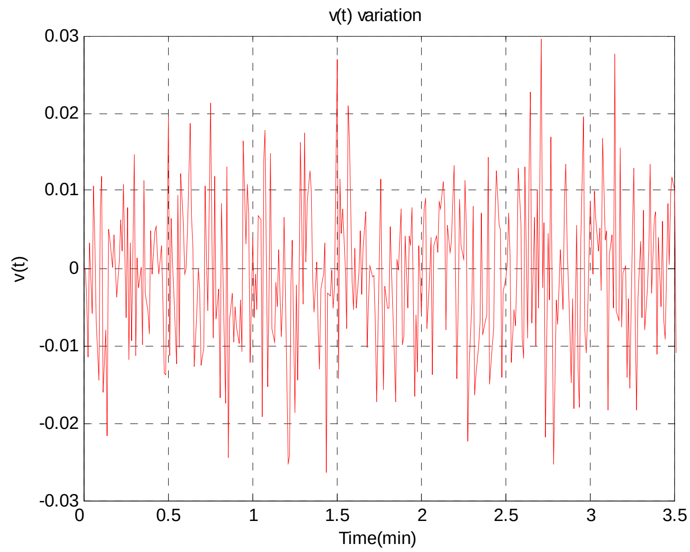

Fig. 4. Disturbance signal

VI. Conclusion

In this work the proposed algorithm is formulated so that it can be solved by QP method (Matlab) . Therefore; there is a need to update the parameters of the FMBPC online so that it becomes a more accurate approximation of the actual system. The final TSFS becomes an interpolation between multiple affine time-varying models. The method handles the constraints explicitly. The proposed methods are applied to a Stirred Tank reactor, the results illustrate the effectiveness of this method and can suggest for solving difficult industrial problems. The introduced several types of constraints are used for improving performance of closed-loop systems. The proposed algorithm can lead to good performance in terms of a reasonable control activity and a good output tracking ability.

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