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Generalized Predictive Control for Single-Link Flexible Joint Robot

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Abstract. *This paper presents an application of the generalized predictive control to a single link flexible joint robot. The objective is to track some predefined profiles for angular displacement of the link. The model of the robot is approximated to linear (ARIMAX) model using the recursive least squares (RLS) identification algorithm for parameters estimation. Simulation results show the performance of the control scheme.*

keywords. *Flexible joint robot, generalized predictive control, ARIMAX model.*

1. Introduction

Model based predictive control (MPC) has received a great deal of attention and is considered by many to be one of the most promising methods in control engineering. The predictive control strategy belongs to the optimal control methods. The difference is that the cost function to be optimized is defined over a future horizon [1], [2]. Generalized predictive control (GPC) can be applied for nonlinear systems approximated by linear models with a good performance. The predictive model is carried out based on the solving of the Diophantine equations [3], [4].

In the development of a modern robot manipulator, it is required that the robot controller has the capability to overcome unmodeled dynamics, variable payloads, friction torques, torque disturbances, parameter variations, measurement noises which can be often presented in the practical environments. For this raison, control of mobile robot is currently among the main subjects of scientific research in robotic area. Many

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strategies have been developed in last years for robot control [5], [6]. Predictive control has been applied to robot manipulator with good performance [7], [8], [9], [10], [11], and can be generalized to single link flexible joint robot, which is the contribution of this work.

The aim of this paper is to apply a generalized predictive controller for the single-link flexible joint robot. It is organized as follows. Section 2 describes the generalized predictive control algorithm. Section 3 derives the linear model, which is the approximation of the robot model. Section 4 presents simulation results for tracking problem of the robot using the proposed controller. Section 5 concludes the paper.

2. Generalized Predictive Control

A square linear system ($n\times n$) can be described by an ARIMAX model given by

$$\mathbf{A}(q^{-1})y(k) = \mathbf{B}(q^{-1})u(k-1) + \frac{\mathbf{C}(q^{-1})}{\Delta(q^{-1})}\xi(k) \tag{1}$$

where, y is the output, u is the input and ξ is white noise of zero mean.
A, **B** and **C** are polynomials, of degrees n_a, n_b and n_c respectively, in the backward shift operator q^{-1}.

$$\begin{aligned}
\mathbf{A}(q^{-1}) &= I_{n\times n} + A_1 q^{-1} + A_2 q^{-2} + \ldots + A_{n_a} q^{-n_a} \\
\mathbf{B}(q^{-1}) &= q^{-d}\left(B_0 + B_1 q^{-1} + \ldots + B_{n_b} q^{-n_b}\right) \\
\mathbf{C}(q^{-1}) &= I_{n\times n} + C_1 q^{-1} + \ldots + C_{n_c} q^{-n_c} \\
\Delta(q^{-1}) &= (1 - q^{-1})
\end{aligned}$$

d is time delay.

The GPC cost function is taken the form

$$\Im(N_1, N_2, N_u, \lambda) = E\{\sum_{j=N_1}^{N_2}[y(k+j) - r(k+j)]^2 + \sum_{j=1}^{N_u}\lambda(j)[\Delta u(k+j-1)]^2\} \tag{2}$$

where N_1 and N_2 are the minimum and maximum cost horizons and N_u the control horizon, which are finite. $\lambda(j)$ is a weighting sequence for the input, which is usually set to a constant λ. $r(k+j)$ is the future set point j-step ahead.

The model (1) can be considered as

$$\mathbf{A}(q^{-1})\Delta y(k) = \mathbf{B}(q^{-1})\Delta u(k-1) + \mathbf{C}(q^{-1})\xi(k) \tag{3}$$

The j-step ahead prediction of $y(k)$, $\hat{y}(k+j)$, is based on the solving the Diophantine equation

$$\mathbf{C}(q^{-1}) = \mathbf{E}_j(q^{-1})\mathbf{A}(q^{-1})\Delta(q^{-1}) + q^{-1}\mathbf{F}_j(q^{-1}) \tag{4}$$

The predictor can be carried out by

$$\hat{y}(k+j) = \mathbf{C}^{-1}\mathbf{F}_j y(k) + \mathbf{C}^{-1}\mathbf{E}_j\mathbf{B}\Delta u(k+j-1) \tag{5}$$

Then, by introducing the second Diophantine equation

$$\mathbf{E}_j(q^{-1})\mathbf{B}(q^{-1}) = \mathbf{G}_j(q^{-1})\mathbf{C}(q^{-1}) + q^{-j}\mathbf{H}_j(q^{-1}) \quad (6)$$

The predictor (6) becomes

$$\hat{y}(k+j) = \mathbf{C}^{-1}\mathbf{F}_j y(k) + \mathbf{C}^{-1}\mathbf{H}_j \Delta u(k-1) + \mathbf{G}_j \Delta u(k+j-1) \quad (7)$$

Under matrix form, we have

$$\hat{\mathbf{y}} = \mathbf{G}\Delta\mathbf{u} + \mathbf{L} \quad (8)$$

where

$$\hat{\mathbf{y}} = \left[y(k+N_1), y(k+N_1+1),...., y(k+N_2)\right]^T, \hat{\mathbf{y}} \in \Re^{(N_2-N_1+1)n}$$

$$\Delta\mathbf{u} = \left[\Delta u(k), \Delta u(k+1),...., \Delta u(k+N_u-1)\right]^T, \Delta\mathbf{u} \in \Re^{N_u n}$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{C}^{-1}\left[\mathbf{H}_{N_1}\Delta u(k-1) + \mathbf{F}_{N_1} y(k)\right] \\ \mathbf{C}^{-1}\left[\mathbf{H}_{N_1+1}\Delta u(k-1) + \mathbf{F}_{N_1+1} y(k)\right] \\ . \\ . \\ \mathbf{C}^{-1}\left[\mathbf{H}_{N_2}\Delta u(k-1) + \mathbf{F}_{N_2} y(k)\right] \end{bmatrix}, \mathbf{L} \in \Re^{(N_2-N_1+1)n}$$

$$\mathbf{G} = \begin{bmatrix} G_{N_1-1} & .... & G_0 & .... & ..... & 0 \\ G_{N_1} & G_{N_1-1} & .... & G_0 & .... & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ G_{N_u-1} & G_{N_u-2} & G_{N_u-3} & ..... & .... & G_0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ G_{N_2-1} & G_{N_2-2} & G_{N_2-3} & .... & G_{N_2-N_u+1} & G_{N_2-N_u} \end{bmatrix}, \mathbf{G} \in \Re^{(N_2-N_1+1)n \times N_u n}$$

The cost function (2), under matrix form, becomes

$$\Im = (\hat{\mathbf{y}} - \mathbf{r})^T(\hat{\mathbf{y}} - \mathbf{r}) + \lambda \Delta\mathbf{u}^T \Delta\mathbf{u} \quad (9)$$

where $\mathbf{r} = [r(k+N_1),...., r(k+N_2)]^T$

Form (9) and (10), the solution the this problem of minimization is

$$\Delta\mathbf{u} = (\mathbf{G}^T\mathbf{G} + \lambda I)^{-1}\mathbf{G}^T(\mathbf{r} - \mathbf{L}) \quad (10)$$

In fact, only the first input is applied to the system. Then, the control input is given by

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \overline{\mathbf{g}}^T(\mathbf{r} - \mathbf{L}) \tag{11}$$

where $\overline{\mathbf{g}}$ is the n first rows of matrix $(\mathbf{G}^T\mathbf{G} + \lambda I)^{-1}\mathbf{G}^T$.

The detail about the solution Diophantine equations (4) and (6) is given in appendix.

3. System Identification

The equations of motion of a single-link flexible joint robot are [5], [6]

$$\begin{aligned} I\ddot{\theta}_1 + Mgl\sin(\theta_1) + K(\theta_1 - \theta_2) = 0 \\ J\ddot{\theta}_2 + K(\theta_2 - \theta_1) = u \end{aligned} \tag{12}$$

where, θ_1 is the link angular displacement and considered as the output to be controlled, θ_2 is the motor angular position, I is the link inertial, J is the rotor inertia, K is the stiffness, M is the link mass, g is the gravity constant, and l is the center of mass. The control input u is the torque delivered by the motor.

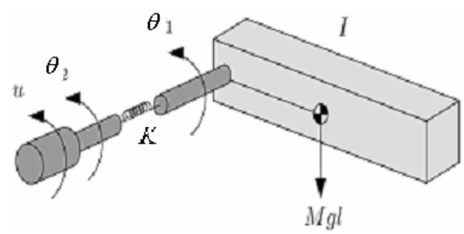

Fig. 1. Single-link flexible joint robot

Using some simplifications, the transfer function, in s-domain, of the system is given by

$$\frac{\theta_1(s)}{u(s)} = \frac{K}{IJs^4 + (IK + MglJ + KJ)s^2 + MglK} \tag{13}$$

For off-line Identification, a random input is applied to the robot. Input and output data are used while applying the recursive least-squares (RLS) identification algorithm; with a sampling frequency of 50 Hz. This technique is based on computation of the optimal value of the parameter vector based on minimization of a scalar function of the squared equation error [12], [13], [14], [15].

In each case the system order was adjusted empirically until the modeling error was minimized.

The robot output for a random input, to be used for parameters identification, is shown in figure 2.

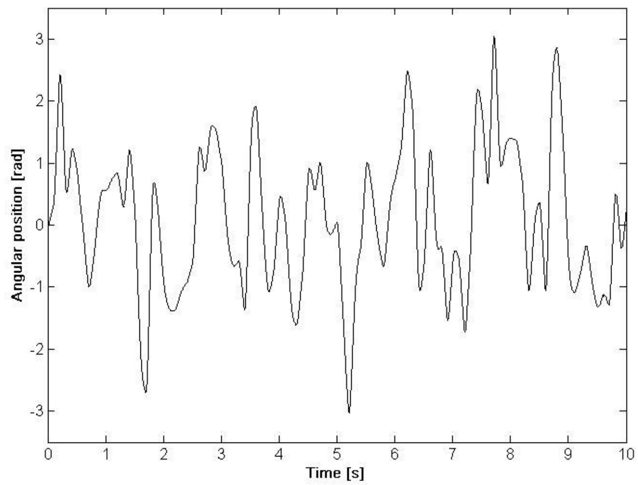

Fig. 2. System output data used in parameters identification

An illustration of the results of the time domain parameterization (ARMA modeling) follows. Identification of the discrete domain transfer function for motor input u_1 to link angular displacement θ_1 resulted in fifth order models, given by

$$\frac{\theta_1}{u_1} = \frac{-0.002\,z^3 + 0.0053\,z^2 - 0.0055\,z - 0.0124}{z^5 - 0.7562\,z^4 - 0.9848\,z^3 + 1.1352\,z^2 + 0.2888\,z - 0.6830}$$

This transfer function is reorganized in order to get the ARIMAX model (1), which will be used in GPC. The degrees of polynomials are $n_a = 5$, $n_b = 3$, $n_c = 1$ and $d = 1$.

In case of on-line identification, for adaptive control, the model parameters a_i and b_i are time varying – their values are to be identified at each time interval. The RLS algorithm can be used for online identification. At each sampling time, the parameters of the plant discrete model are to be estimated based on the latest data pair (u_1, θ_1).

4. Simulation results for GPC System Control

The objective of the robot control is to track a reference some profiles (reference angle value of $\pi/6$ until 2.5 s, then, a value of $\pi/3$). The prediction controller parameters (N_1, N_2, N_u, and λ) are chosen by trial and error in order to get an acceptable tracking. The values are: $N_1 = 1$, $N_2 = 10$, $N_u = 5$ and $\lambda = 10^{-3}$.

First, the non-disturbed robot system is controlled by a GPC controller to track an angular reference. The tracking response is shown in Fig. 3, where, it can be seen that the tracking performance is successfully achieved.

Then, a step disturbance, of value $\pi/20$, is injected in the robot output. This quantity has been included in the output response as a forcing term to represent unmeasured disturbances including payload, frictional effect and uncorrelated random noise sequence if it exists. The system response is given in Fig. 4, where the tracking performance is achieved successfully and the effect of disturbance is well rejected.

Finally, the robot output is affected by a small random disturbance as shown in Fig. 5. It can be seen that the system response follows the reference. However, the effect of the disturbance remains in the response.

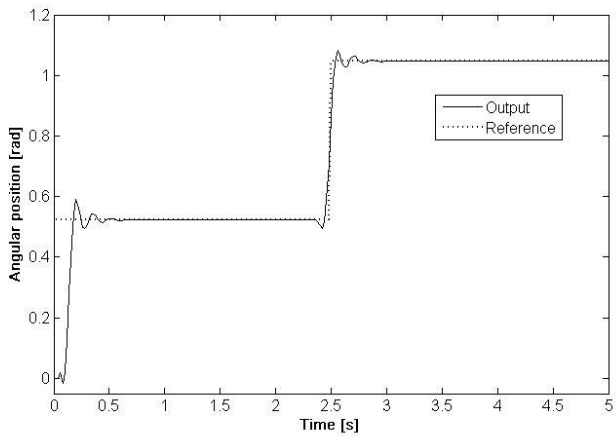

Fig. 3. GPC control response of non-disturbed system

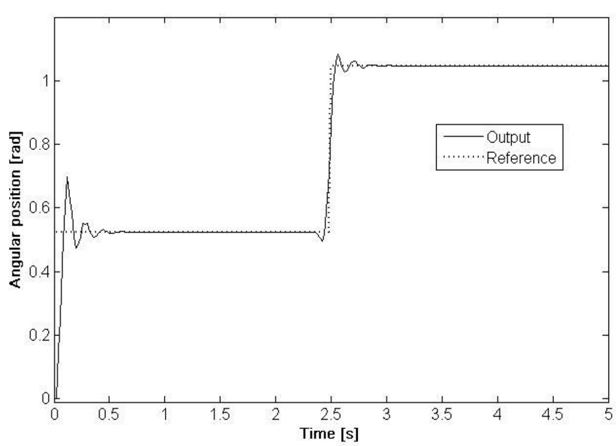

Fig. 4. GPC control response of the system affected by a step disturbance

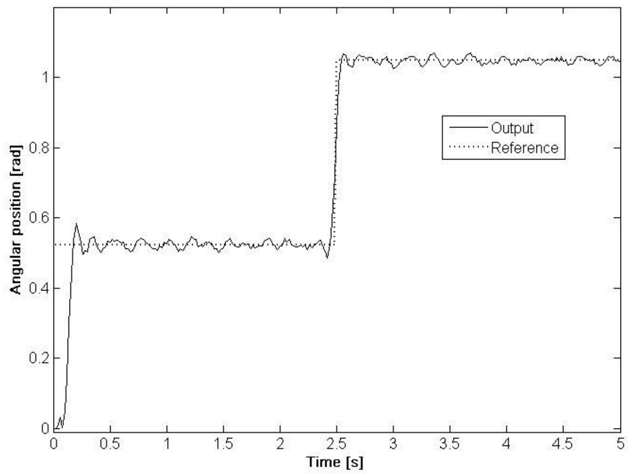

Fig. 5. GPC control response of the system affected by a random disturbance

5. Conclusions

In this paper, a GPC control strategy is applied to single-link flexible joint robot for trajectory tracking. The prediction model is designed using the Diophantine equations solution.

The nonlinear model of the robot is approximated by an ARIMAX model using the recursive least-squares (RLS) identification algorithm for parameters estimation.

Results show that the single-link flexible-joint robot under GPC control has a good output tracking performance and the step disturbance is well rejected.

Appendix

The polynomials couple ($\mathbf{E}_j$, $\mathbf{F}_j$) is the unique solution of the Diophantine equation (4).

$$\mathbf{E}_j = I_{n\times n} + E_1^{(j)} q^{-1} + \ldots + E_{j-1}^{(j)} q^{-j+1}$$

$$\mathbf{F}_j = F_0^{(j)} + F_1^{(j)} q^{-1} + \ldots + F_{n_f^{(j)}}^{(j)} q^{-n_f^{(j)}}$$

$n_f^{(j)} = \max(n_a, n_c - j)$. The matrices $\mathbf{E}_j$ and $\mathbf{F}_j$ have dimension $n\times n$.

The solution can be carried out in recurrent manner from (4).

For j=1

$$\begin{cases} E_0 = I \\ F_i^{(1)} = C_{i+1} - E_0 (A_{i+1} - A_i) \quad 0 \le i \le n_f^{(1)} \end{cases}$$

For $j \ge 2$

$$\begin{cases} E_{j-1} = F_0^{(j-1)} \\ F_i^{(1)} = F_{i+1}^{(j-1)} - E_{j-1}(A_{i+1} - A_i) \quad 0 \le i \le n_f^{(1)} \end{cases}$$

The polynomials couple ($\mathbf{G}_j$, $\mathbf{H}_j$) is the unique solution of the Diophantine equation (6).

$$\mathbf{G}_j = G_0^{(j)} + G_1^{(j)} q^{-1} + \ldots + G_{j-1}^{(j)} q^{-j+1}$$
$$\mathbf{H}_j = H_0^{(j)} + H_1^{(j)} q^{-1} + \ldots + H_{n_h}^{(j)} q^{-n_h}$$

$n_h = \max(n_c, n_b + d) - 1$. The matrices $\mathbf{G}_j$ and $\mathbf{H}_j$ have dimension $n \times n$.
The solution can be carried out in recurrent manner from (6).

For j=1

$$\text{if} \quad d = 0 \quad \begin{cases} G_0 = E_0 B_0 \\ H_i^{(1)} = E_0 B_{i+1} - C_{i+1} G_0 \qquad 0 \le i \le n_h \end{cases}$$

$$\text{if} \quad d > 0 \quad \begin{cases} G_0 = 0 \\ H_i^{(1)} = E_0 B_{i-d+1} \qquad 0 \le i \le n_h \end{cases}$$

For $j \ge 2$

$$\begin{cases} G_{j-1} = \begin{cases} H_0^{(j-1)} + E_{j-1} B_0 & \text{if} \quad d = 0 \\ H_0^{(j-1)} & \text{if} \quad d > 0 \end{cases} \\ H_i^{(j)} = H_{i+1}^{(j-1)} - C_{i+1} G_{j-1} + E_{j-1} B_{i+1-d} \qquad 0 \le i \le n_h \end{cases}$$

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