International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 3, N° 1, July 2009, pp. 926–941.

Identification of the Structure and the Parameters of Volterra models using crosscumulants

Kamel Abderrahim[1], Houda Mathlouthi[1], Faouzi Msahli[2] and Gérard Favier[3]

[1] National School of Engineers of Gabès, Route de Medenine
6029 Gabès, Tunisie
[2] National School of Engineers of Monastir, Avenue Ibn Jazzar,
5019, Monastir, Tunisia.
[3] I3S Laboratory, UNSA/CNRS, 2000 Route des Lucioles,
Sophia-Antipolis, Biot, France.

Abstract. *In this paper, we address the problem of structure and parameter identification of Volterra models driven by symmetric input with four levels. It consists in estimating the model order, the memory length of each kernel and the parameters. The proposed approaches are based on the crosscumulants between the input and the output using the statistics proprieties of the input sequence. The structure identification method consists in estimating the order of the Volterra model that will be used to identify the length of each kernel. A closed form solution has been developed to estimate the parameters of the Volterra models. Simulations are presented to illustrate the performance of the proposed methods.*

keywords. *Structure Identification, Parameter Estimation, Cross-cumulant, Volterra system.*

1. Introduction

The truncated Volterra model has been the most popular since it can represent any nonlinear system time invariant with fading memory [1]-[5]. Moreover, the parameters of this model are linearly related to the output which allows the extension of some results of linear systems to nonlinear ones [12], [19]. For these reasons, the truncated Volterra model has found applications in many fields such as signal processing and control [1], [12], [15], [17], [19], [22].

Two main problems must be taken into account for the identification of truncated Volterra model: one is the identification of the model kernels and two is the identification of the model structure defined by the model order and the kernel memory lengths. Several methods have been proposed in the literature for the identification of the kernels of Volterra models [6]-[12]. They can be classified into two great families. The

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

methods of the first family consist in expressing the parameters of the model in function of the output and input signals (cumulants, polyspectras, crosscumulants) [5], [6], [21], [24], [25]. These methods are very interesting from a theoretical point of view because they show the possibility of identifying the system from only the knowledge of the statistics of the output and input signals. The methods of the second approach exploit the property of linearity of the Volterra models to their parameters. This makes it possible to extend the identification algorithms of the linear models, like RLS and LMS algorithms, to the Volterra models [12], [19].

For the sake of simplicity, the existing methods suppose that the structure of the model is a priori known. So, the problem of structure identification did not attract much attention and few solutions were proposed in the literature [11]. These solutions treat particular cases, for example, the approach suggested in [11] is valid only for the quadratic Volterra model. Recently, a structure identification of Volterra models excited by Gaussian input has been proposed in [13].

The excitation signal represents, also, a fundamental problem for the identification of Volterra systems. Most of the existing methods suppose that the input signal is Gaussian. This type of signal is very interesting from a theoretical point of view. However, it is not recommended in practice because it requires a relatively high number of measurements. Moreover, it causes the wear of the actuators. To overcome this problem, new sequence, namely *"plant friendly"*, has been developed for the excitation of the Volterra models [23]. This sequence supposes the knowledge of the structure of the model. Indeed, it contains $(r+1)$ levels where r represents the model order [18]. This excitation allows the identification of the linear diagonal and under diagonal parameters of the Volterra model. The other parameters are estimated using a random binary sequence. This solution presents the possibility of identifying the parameters using an explicit solution. Moreover, it insures very satisfactory results in simulation with models having modest order and memory lengths. However, its performance degrades with the increase in the complexity of the model. This is due to the propagation of the errors in estimation.

In this work, we address the problems of identification of the structure and the parameters of Volterra models excited by symmetric input with four levels are addressed. This input insures the identifiabilily of the parameters of second and third order Volterra models [18]. Moreover, it allows to overcome the problems of Gaussian sequence in practical situations.

This paper is organized as follows. Section 2 presents the model and its assumptions. The basic relations are described in section 3. Section 4 and 5 propose the structure and the parameters algorithms. Results of simulations are presented in the last section.

2. Problem formulation

In the following section, we address the problem of estimating the order, the memory lengths and the parameters of a discrete, causal stationary Volterra system described by:

$$x(n) = \sum_{i=1}^{r} \sum_{k_1=0}^{K_i} \cdots \sum_{k_i=0}^{K_i} h_i(k_1, \cdots, k_i) u(n-k_1) \cdots u(n-k_i) \tag{1}$$

where $\{u(n)\}$ is the input sequence, r is the Volterra model order, $h_i(\cdot)$ are the coefficients of the i^{th} order Volterra kernel with memory K_i and $\{x(n)\}$ is the noiseless output sequence.

The relation (1) can be rewritten as follows:

$$x(n) = \sum_{i=1}^{r} y_i(n) \tag{2}$$

where

$$y_1(n) = \sum_{k_1=0}^{K_1} h_1(k_1) u(n-k_1),$$

$$y_2(n) = \sum_{k_1=0}^{K_2} \sum_{k_2=0}^{K_2} h_2(k_1, k_2) u(n-k_1) u(n-k_2)$$

…

$$y_r(n) = \sum_{k_1=0}^{K_r} \cdots \sum_{k_r=0}^{K_r} h_r(k_1, \cdots, k_r) u(n-k_1) \cdots u(n-k_r)$$

The observed output process $\{y(n)\}$ is given by:

$$y(n) = x(n) + v(n) \tag{3}$$

where $\{v(n)\}$ is the noise sequence.

The following assumptions are assumed to be verified:

A1. The input sequence $\{u(n)\}$ is zero mean, independent and identically distributed (i.i.d), stationary process and symmetric with four levels.
A2. The additive noise $\{v(n)\}$ is independent from the input sequence and with unknown variance.
A3. The Volterra Kernels $h_i(\tau_1, \tau_2, \cdots, \tau_i)$ are absolutely summable sequences. Moreover $h_i(\tau_1, \tau_2, \cdots, \tau_i)$ for $i \geq 2$ are symmetric, that is :
$h_i(\tau_1, \tau_2, \cdots, \tau_i) = h_i(\pi(\tau_1, \tau_2, \cdots, \tau_i))$
where $\pi(\cdot)$ is a permutation function.

Next, we recall the definition of cumulants. Let $V=\left(v_1,v_2,\cdots,v_k\right)^T$ be a complex vector and $X=(x_1,x_2,...,x_k)^T$ a random vector with $E\left|x_j\right|^k<\infty,\ j=1,\cdots,k$. The p^{th} order cumulant of these random variables is defined as a coefficient of $v_1,v_2,\cdots,v_k$ in the Taylor series expansion of the cumulant generating function $\Psi_X\left(V\right)=Ln\left(E\left\{exp\left(jV^TX\right)\right\}\right)$. The p^{th} order cumulant sequence of a stationary random signal $\left\{x(k)\right\}$ is written as [16]-[19]:

$$C_{p,x}\left(\tau_1,\tau_2,\cdots,\tau_{p-1}\right)=Cum\left(x\left(n\right),x\left(n+\tau_1\right),\cdots,x\left(n+\tau_{p-1}\right)\right) \quad \textbf{(4)}$$

Cross-cumulants are defined in a similar way:

$$C_{p,x_1,x_2,\cdots,x_p}\left(\tau_1,\tau_2,\cdots,\tau_{p-1}\right)=Cum\left(x_1\left(n\right),x_2\left(n+\tau_1\right),\cdots,x_p\left(n+\tau_{p-1}\right)\right) \quad \textbf{(5)}$$

3. Basic relations

If the input $u\left(k\right)$ is a symmetric sequence, the m^{th} order moments and cumulants are defined as [3]:

$$Mom_{m,u}\left(\tau_1,\tau_2,\cdots,\tau_{m-1}\right)=Mom\left[u\left(k\right),u\left(k+\tau_1\right),\cdots,u\left(k+\tau_{m-1}\right)\right] \quad \textbf{(6)}$$

$$C_{m,u}\left(\tau_1,\tau_2,\cdots,\tau_{m-1}\right)=\begin{cases}\gamma_{m,u} & if \quad m\ is\ even\ and\quad all\quad \tau_i=0\\ 0 & otherwise\end{cases} \quad \textbf{(7)}$$

The second, fourth and sixth order cumulants of the input are expressed as follows:

$$\gamma_{2,u}=C_{2,u}\left(0\right)=Mom_{2,u}\left(0\right) \quad \textbf{(8)}$$

$$\gamma_{4,u}=C_{4,u}\left(0,0,0\right)=Mom_{4,u}\left(0,0,0\right)-3\left(Mom_{2,u}\left(0\right)\right)^2 \quad \textbf{(9)}$$

$$\begin{aligned}\gamma_{6,u}&=C_{6,u}\left(0,0,0,0,0\right)\\&=Mom_{6,u}\left(0,0,0,0,0\right)-15Mom_{4,u}\left(0,0,0\right)Mom_{2,u}\left(0\right)+30\left(Mom_{2,u}\left(0\right)\right)^2\end{aligned} \quad \textbf{(10)}$$

For second order Volterra model, the cross-cumulants between one copy of the output and *i* copies of the input are given by:

$$C_{y,u}\left(i_1\right)=\gamma_{2,u}h_1\left(-i_1\right) \quad \textbf{(11)}$$

$$C_{y,u,u}\left(i_1,i_2\right)=2\gamma_{2,u}^2h_2\left(-i_1,-i_2\right)+\gamma_{4,u}h_2\left(-i_1,-i_2\right)\delta\left(i_1-i_2\right) \quad \textbf{(12)}$$

$$C_{y,u,u,u}\left(i_1,i_2,i_3\right)=\gamma_{4,u}h_1\left(-i_1\right)\delta\left(i_2-i_1\right)\delta\left(i_3-i_1\right) \quad \textbf{(13)}$$

$$C_{y,u,u,u,u,u}\left(i_1,i_2,i_3,i_4,i_5\right)=\gamma_{6,u} \\ h_1\left(-i_1\right)\delta\left(i_1-i_2\right)\delta\left(i_1-i_3\right)\delta\left(i_1-i_4\right)\delta\left(i_1-i_5\right) \quad \textbf{(14)}$$

The cross-cumulants between one copy of the output and i copies of the input, in the case of the third order Volterra model, are given by:

$$C_{y,u}\left(i_1\right)=\gamma_{2,u}h_1\left(-i_1\right) \\ +3\gamma_{2,u}^2\sum_{l=0}^{K_3}h_3\left(l,l,-i_1\right)+\gamma_{4,u}h_3\left(-i_1,-i_1,-i_1\right) \quad \textbf{(15)}$$

$$C_{y,u,u}\left(i_1,i_2\right)=\gamma_{4,u}h_2\left(-i_1,-i_2\right)\delta\left(i_1-i_2\right)+2\gamma_{2,u}^2h_2\left(-i_1,-i_2\right) \quad \textbf{(16)}$$

$$\begin{aligned}C_{y,u,u,u}\left(i_1,i_2,i_3\right)=\;&\gamma_{4,u}h_1\left(-i_1\right)\delta\left(i_2-i_1\right)\delta\left(i_3-i_1\right)\\&+\gamma_{6,u}h_3\left(-i_1,-i_1,-i_1\right)\delta\left(i_2-i_1\right)\delta\left(i_3-i_1\right)\\&+3\gamma_{2,u}\gamma_{4,u}\sum_{l=0}^{K_3}h_3\left(l,l,-i_1\right)\delta\left(i_2-i_1\right)\delta\left(i_3-i_1\right)\\&+6\gamma_{2,u}^3h_3\left(-i_1,-i_2,-i_3\right)\\&+3\gamma_{2,u}\gamma_{4,u}h_3\left(-i_1,-i_2,-i_2\right)\delta\left(i_3-i_2\right)\\&+3\gamma_{2,u}\gamma_{4,u}h_3\left(-i_1,-i_1,-i_2\right)\delta\left(i_3-i_1\right)\\&+3\gamma_{2,u}\gamma_{4,u}h_3\left(-i_1,-i_1,-i_3\right)\delta\left(i_2-i_1\right)\end{aligned} \quad \textbf{(17)}$$

4. The proposed structure identification method

This paragraph proposes a method for the structure identification of Volterra models excited by symmetric input with four levels. This approach consists in estimating the order of Volterra model that will be used to determine the kernel memory lengths. Firstly, we present the principle of the proposed method and secondly, we describe a reformulation allowing to improve its performance.

4.1. Principle

The proposed method is based on the crosscumulants and the statistics proprieties of the input sequence using the vanished statistical input-output information. For example, the values of $C_{y,x}\left(\tau\right)$ are equal to zero for all $\tau > K_1$ where K_1 represents the memory length of the first kernel.
The principle of the proposed method can be summarized by the following steps:

Step 1 : Identification of the Volterra model order
- Construct the diagonal matrix Md defined by:

$$Md = \begin{bmatrix} C_{y,u}(0)/\gamma_{2,u} & 0 & 0 \\ 0 & C_{y,u,u,u}(0,0,0)/\gamma_{4,u} & 0 \\ 0 & 0 & C_{y,u,u,u,u,u}(0,\cdots,0)/\gamma_{6,u} \end{bmatrix}$$

Using the relation (11), (13) and (14), it is obviously to deduce that the matrix *Md* is defined as follows in the case of second order Volterra model:

$$Md = h_1(0)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consequently, we can use the following algorithm to identify the order
If Md is proportional to the identity matrix, then:

- Model order=2;
- goto step 2;

Else

- Model order=3;
- goto step 3;

Step 2: Identification of memory lengths K_1 and K_2 of second order Volterra model

- Identification of the memory length K_2 of the second kernel using the following condition :

$$C_{y,u,u}(-\tau_1,-\tau_2)=0 \ \ for\ all\ \ (-\tau_1,-\tau_2)\notin\{0,1,\cdots,K_2\}$$

- Identification of the memory length K_1 of the first kernel using the following condition :

$$C_{y,u}(-\tau_1)=0 \ \ for\ all\ \ -\tau_1\notin\{0,1,\cdots,K_1\}$$

Step 3: Identification of memory lengths K_1, K_2 and K_3 of third order Volterra model

- Identification of the memory length $K = max(K_1,K_3)$ using the following condition :

$$C_{y,u}(-\tau_1)=0 \ \ for\ all\ \ -\tau_1\notin\{0,1,\cdots,K\}$$

- Identification of the memory length S the following condition :

$$C_{y,u,u,u}(0,-\tau_1,-\tau_2)=0 \ \ for\ all\ \ (-\tau_1,-\tau_2)\notin\{0,1,\cdots,S\}$$

  - If $S-1\geq max(K_1,K_3)$ then $K_1 = K_3 = max(K_1,K_3)$
  - Else $K_3 = S-1$ et $K_1 = max(K_1,K_3)$

- Identification of the memory length K_2 using the following condition :

$$C_{y,u,u}\left(-\tau_1,-\tau_2\right)=0 \ \ for\ all\ \left(-\tau_1,-\tau_2\right)\notin\left\{0,1,\cdots,K_2\right\}$$

4.2. Reformulation of the proposed method

This method gives correct results when cross-cumulants are known. However, in practice, the cross-cumulants are estimated from the data. Consequently, the exact cross-cumulants are unknown and they can not be estimated correctly. Therefore, if additional samples of cross-cumulants are used then better results can be expected. In fact, we will propose another formulation of the steps 2 and 3 using more cross-cumulant samples. This formulation is based on the following cross-cumulants matrixes $M^{(1)}$, $M^{(2)}$ and $M^{(3)}$ and the test variable $\overline{\lambda_{Mat}}\left(k\right)$:

$$M^{(j)}=\begin{bmatrix} m_{0,0}^{(j)} & m_{0,1}^{(j)} & \cdots & m_{0\tau_{2max}}^{(j)} \\ m_{1,0}^{(j)} & m_{11}^{(j)} & & \vdots \\ \vdots & & \ddots & \\ m_{\tau_{1max}0}^{(j)} & \cdots & & m_{\tau_{1max}\tau_{2max}}^{(j)} \end{bmatrix} \quad \textbf{(18)}$$

where $m_{\tau_1\tau_2}^{(j)}=C_{y,\underbrace{0,\cdots,0}_{j}}\left(\underbrace{0,\cdots,0}_{j-2},-\tau_1,-\tau_2\right)$

$$\overline{\lambda_{Mat}}\left(k\right)=\frac{\lambda_{Mat}\left(k\right)}{\max\left(\lambda_{Mat}\right)} \quad \textbf{(19)}$$

where

$$\lambda_{Mat}\left(k\right)=\sum_{i=1}^{k}\left(Mat_l\left(i\right)\right)^2 \quad , \quad Mat_l=\sqrt{\sum_t\left(M^{(j)}\left(t,l\right)\right)^2}$$

The test variable $\overline{\lambda_{Mat}}\left(k\right)$ converges to *1*. In fact, it attains *1* at the first time for $k>k_1$ where k_1 indicates the desired memory length.

The reformulation of steps 2 and 3 can be expressed as follows:

Step 2— Identification of memory lengths K_1 and K_2 of second order Volterra model

- Determine the smallest integer l verifying $\overline{\lambda_{M^{(1)}}}\left(l\right)=1$ so $K_1=l-1$
- Determine the smallest integer k for which $\overline{\lambda_{M^{(2)}}}\left(k\right)$ attains the unity so $K_2=k-1$

Step 3— Identification of memory lengths K_1, K_2 and K_3 of second order Volterra model

- Determine the smallest integer *k* verifying $\overline{\lambda_{M^{(1)}}}(k) = 1$ so
$max(K_1, K_3) = k - 1$
- Determine the smallest integer *s* for which the test variable $\overline{\lambda_{M^{(3)}}}(s)$ of the matrix $M^{(3)}$ attains the unity;
  - If $s - 1 \geq max(K_1, K_3)$, $K_3 = \max(K_1, K_2)$ and we suppose that $K_1 = K_3$.
  - Else $K_3 = s - 1$ & $K_1 = \max(K_1, K_3)$.
- Determine the smallest integer *l* which verifies $\overline{\lambda_{M^{(2)}}}(l) = 1$ so that
$K_2 = l - 1$

5. Parameter estimation method

This paragraph suggests an explicit solution for the identification of the kernels of second and third order Volterra models excited by a symmetric input with four levels.

5.1. Second order Volterra model

Using (11) and (12), we deduce:
- *Linear kernel Volterra model*

$$h_1(m_1) = \frac{C_{y,u}(-m_1)}{\gamma_{2,u}} \qquad for \quad 1 \leq m_1 \leq K_1$$

- *Quadratic kernel Volterra model*

$$h_2(m_1, m_2) = \frac{C_{y,u,u}(-m_1, -m_2)}{2\gamma^2_{2,u} + \gamma_{4,u}\delta(m_1 - m_2)} \qquad for \quad 0 \leq m_1, m_2 \leq K_2$$

5.2. Third order Volterra model

Using (15), (16) and (17), we deduce:

- *Cubic kernel Volterra model*

$$h_3(m_1, m_2, m_3) = \frac{C_{y,u,u,u}(-m_1, -m_2, -m_3)}{6\gamma_{2,u}^{\ 3}} \quad if \ m_1 \neq m_2 \neq m_3$$

$$h_3(m_1, m_2, m_3) = \frac{C_{y,u,u,u}(-m_1, -m_2, -m_3)}{6\gamma^3_{2,u} + 3\gamma_{2,u}\gamma_{4,u}} \quad if \quad m_1 = m_2 \neq m_3 \quad or \quad m_1 = m_3 \neq m_2 \quad or \quad m_2 = m_3 \neq m_1$$

$$h_3(m_1,m_2,m_3)=\frac{C_{y,u,u,u}(-m_1,-m_2,-m_3)-\gamma_{4,u}/\gamma_{2,u}\,c_{y(1)u(2)}(-m_1)}{6\gamma^3{}_{2u}+9\gamma_{2,u}\gamma_{4,u}+\gamma_{6,u}+\gamma_{4,u}{}^2/\gamma_{2,u}}\quad if\quad m_1=m_2=m_3$$

- *Quadratic kernel Volterra model*

$$h_2(m_1,m_2)=\frac{C_{y,u}(-m_1,-m_2)}{2\gamma^2{}_{2,u}+\gamma_{4,u}\delta(m_1-m_2)}\qquad for\quad 0\le m_1,m_2\le K_2$$

- *Linear kernel Volterra model*

$$h_1(m_1)=\frac{C_{y,u}(-m_1)}{\gamma_{2,u}}-3\gamma_{2,u}\sum_{l=0}^{K_3}h_3(l,l,m_1)+\frac{\gamma_{4,u}}{\gamma_{2,u}}h_3(m_1,m_1,m_1)\qquad for\quad 0\le m_1\le K_1$$

6. Simulation results

The objective of the simulations is to illustrate the performance of the proposed methods. The simulations are performed under the following conditions:

- The input signal *u(n)* is zero mean, independent and identically distributed sequence with four levels.
- The additive colored noise *v(n)* is simulated as the output of MA(p) model driving by a Gaussian sequence *w(n)*.
- The Signal to Noise Ratio (SNR) is defined as :

$$SNR_{(dB)}=10\log_{10}\left[\frac{E\{x^2(n)\}}{E\{v^2(n)\}}\right]$$

- The parameters were obtained from 500 Monte Carlo runs, where *N* data are used to estimate the crosscumulants.

The mean (μ), the standard deviation (σ) and the normalized mean square error (*NMSE*) values are considered to study the performance of each method:

$$\mu_{\theta(i)}=\frac{1}{N_{MC}}\sum_{k=1}^{N_{MC}}\hat{\theta}(i)_k\quad,\qquad \sigma_{\theta(i)}=\sqrt{\frac{1}{N_{MC}}\sum_{k=1}^{N_{MC}}\left(\hat{\theta}(i)_k-\mu_{\theta(i)}\right)^2}$$

$$NMSE_{(dB)}=10\log_{10}\left(\frac{\sum_i\left(\hat{\theta}(i)-\theta(i)\right)^2}{\sum_i\theta^2(i)}\right)$$

where $\hat{\theta}(i)_k$ is a i^{th} coefficient of the Volterra kernel estimated in the k^{th} iteration for each simulation, $\theta(i)$ is an exact i^{th} coefficient of the Volterra model and N_{MC} is the number of Monte Carlo simulations.

Four simulation examples, presented in tables 1 and 2, are selected from literature [12], [19].

Table 1. Characteristics of selected models: order and memory

	order	K_1	K_2	K_3
Model 1	2	2	3	---------
Model 2	2	3	2	---------
Model 3	3	3	2	2
Model 4	3	2	3	2

Table 2. Model Coefficients.

	$h_1(:)$	$h_2(:,:)$	
Model 2	$\begin{bmatrix} 1 \\ 0.5 \\ -0.66 \\ -0.78 \end{bmatrix}$	$\begin{bmatrix} 0 & -0.15 & 0.1 \\ -0.15 & 0.5 & -0.5 \\ 0.1 & -0.5 & 0.4 \end{bmatrix}$	
Model 3	$\begin{bmatrix} 1 \\ 0.5 \\ -0.66 \\ -0.78 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0.43 & -0.9 \\ 0.43 & 0.12 & 0.7 \\ -0.9 & 0.7 & 0.1 \end{bmatrix}$	
	$h_3(:,:,0)$	$h_3(:,:,1)$	$h_3(:,:,2)$
Model 2	------------------------	------------------	------------------
Model 3	$\begin{bmatrix} 0.5 & 0.2/3 & 0.1/3 \\ 0.2/3 & 0 & -0.5/6 \\ 0.1/3 & -0.5/6 & -0.1 \end{bmatrix}$	$\begin{bmatrix} 0.2/3 & 0 & -0.5/6 \\ 0 & 1 & 0.4/3 \\ -0.5/6 & 0.4/3 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.1/3 & -0.5/6 & -0.1 \\ -0.5/6 & 0.4/3 & 0 \\ -0.1 & 0 & -0.7 \end{bmatrix}$

6.1. Model structure identification

The first step of the second method consists in estimating the order of the Volterra model. Therefore, we must estimate $C_{y,u}(0)/\gamma_{2,u}$, $C_{y,u,u,u}(0,0,0)/\gamma_{4,u}$ and $C_{y,u,u,u,u,u}/\gamma_{6,u}$ for each model. The estimation of these statistics information are given in table 3.

Table 3. Estimation values of $Md1$, Symmetric Input, SNR=10dB, N=1000.

	Model1	Model2	Model3	Model4
diag(Md1)	1.0000 ± 0.1073	0.9970 ± 0.0919	3.5939 ± 0.3608	3.6187 ± 0.3079
	0.9982 ± 0.1037	0.9985 ± 0.0955	3.1760 ± 0.3650	3.1946 ± 0.3163
	0.9976 ± 0.1053	0.9989 ± 0.0988	3.0474 ± 0.3760	3.0641 ± 0.3291

For the two first models, we remark that the matrix $Md1$ is proportional to the unity matrix and consequently model 1 and model 2 are second order Volterra model. However, the elements of $diag(Md1)$ are different for model 3 & model 4. We can deduce that these models are third order Volterra models.

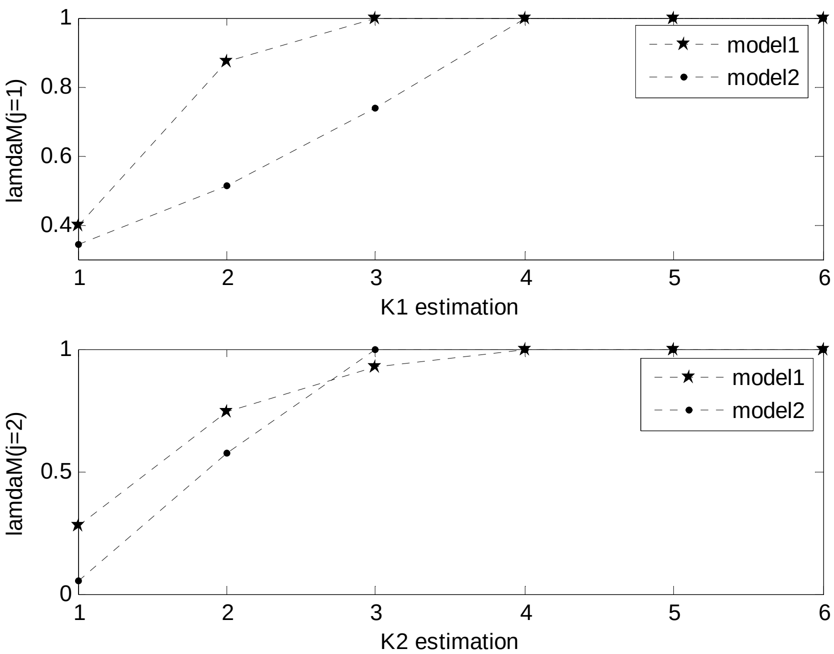

Fig 1. Estimation values of $\overline{\lambda_{M^{(j)}}}(\cdot)$, Symmetric Input, SNR=10dB, N=1000, Models 1&2.

Applying the second step of algorithm 2, we can estimate the values of K_1 and K_2 for models 1 and 2. Analyzing the simulation results given in figure 1, we deduce that:

$$K_1^{model\ 1} = 2,\ K_2^{model\ 1} = 3,\ K_1^{model\ 2} = 3 \text{ and } K_2^{model\ 2} = 2$$

Step 3 of the second algorithm concerns the third order Volterra models: it's the case of models 3 and 4. Using $\overline{\lambda_{M^{(1)}}}(\cdot)$ and $\overline{\lambda_{M^{(2)}}}(\cdot)$, we can estimate K_2 and $K = max(K_1, K_3)$:

$$K_2^{model\ 3} = 2\,,\ K^{model\ 3} = 3\,,\ K_2^{model\ 4} = 3 \text{ and } K^{model\ 4} = 2$$

In this case we must estimate $M^{(3)}$ (figure 2):

For model 3, $\overline{\lambda_{M^{(3)}}}(k)$ attains the unity for $k = 3 < K^{model\ 3} + 1$ which implies that $K_1^{model\ 3} = 3$ and $K_3^{model\ 3} = 2$.

For model 4, $\overline{\lambda_{M^{(3)}}}(k)$ attains the unity for $k = 3 = K^{model\ 4} + 1$ which implies that $K_3^{model\ 4} = 2$ if we suppose that $K_1^{model\ 4} = 2$

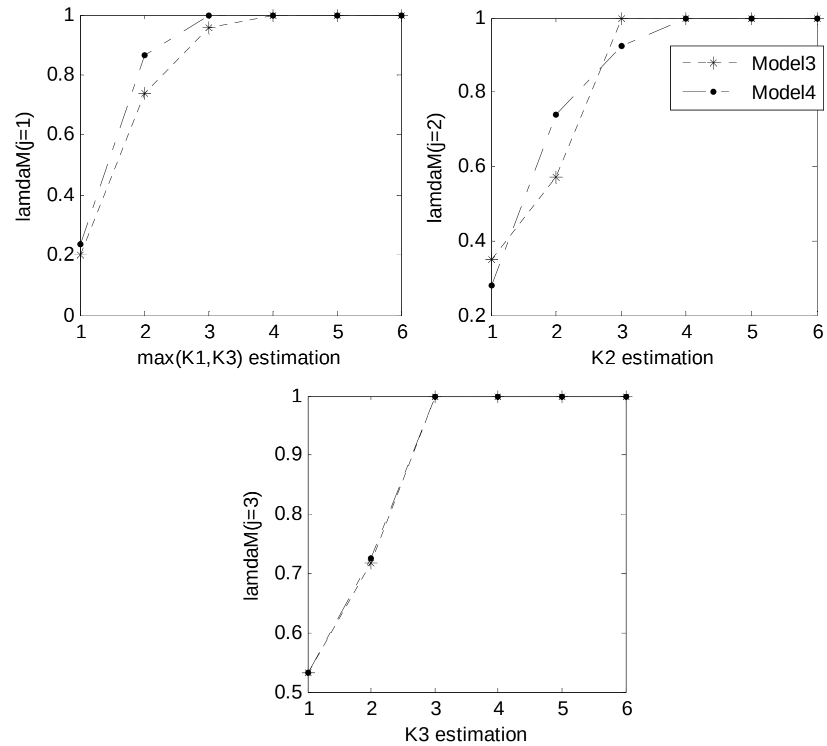

Fig 2. Estimation values of $\overline{\lambda_{M^{(j)}}}(\cdot)$, Symmetric Input, SNR=10dB, N=1000, Models 3&4.

6.2. Model parameter estimation

Table 4. Parameter estimation, Model 2, symmetric Input, 500 Monte Carlo runs, N=1000

SNR	0dB		10dB	
	Mean	Standard deviation	Mean	Standard deviation
$h_1(1)$= 0.5000	0.4958	0.1143	0.4967	0.0710
$h_1(2)$= -0.6600	-0.6506	0.1207	-0.6562	0.0835
$h_1(3)$= -0.7800	-0.7846	0.1368	-0.7750	0.0948
$h_2(0,0)$= 0	0.0036	0.0732	0.0020	0.0503
$h_2(0,1)$= -0.1500	-0.1490	0.0322	-0.1480	0.0272
$h_2(0,2)$= 0.1000	0.1017	0.0330	0.1000	0.0290
$h_2(1,1)$= 0.5000	0.4985	0.0708	0.4994	0.0570
$h_2(1,2)$= -0.5000	-0.4989	0.0280	-0.4971	0.0190
$h_2(2,2)$= 0.4000	0.3972	0.0746	0.3962	0.0520
NMSE(dB)	-42.8293		-45.3260	

Table 5. Parameter estimation, Model 3, 500 Monte Carlo runs, N=1000, SNR=10dB

	Symmetric Input	
	Mean	Standard deviation
$h_1(1)$= 0.5000	0.6405	1.3833
$h_1(2)$= -0.6600	-0.7071	1.7166
$h_1(3)$= -0.7800	-0.7912	0.4620
$h_2(0,0)$= 0.5000	0.4888	0.2281
$h_2(0,1)$= 0.4300	0.4233	0.0876
$h_2(0,2)$= -0.9000	-0.8924	0.1012
$h_2(1,1)$= 0.1200	0.1092	0.1718
$h_2(1,2)$= 0.7000	0.7028	0.0940
$h_2(2,2)$= 0.1000	0.0805	0.2321
$h_3(0,0,0)$= 0.5000	0.4934	0.1589
$h_3(0,0,1)$= 0.0667	0.0639	0.0355
$h_3(0,0,2)$= 0.0333	0.0355	0.0395
$h_3(0,1,1)$= 0	-0.0004	0.0501
$h_3(0,1,2)$= -0.0833	-0.0829	0.0135
$h_3(0,2,2)$= -0.1000	-0.1035	0.0399
$h_3(1,1,1)$= 1.0000	0.9937	0.1221
$h_3(1,1,2)$= 0.1333	0.1346	0.0505
$h_3(1,2,2)$= 0	-0.0044	0.0382
$h_3(2,2,2)$= -0.7000	-0.7010	0.1652
NMSE(dB)	-23.2352	

We can remark the following observations from tables 4 and 5 :

- A large number of data N is necessary to improve the performance of the proposed methods because they use high order cumulants and cross-cumulants.

- The explicit expression of $h_2(m_1, m_2)$ if $m_1 = m_2$ contains an additional term as compared to that with $m_1 \neq m_2$. Therefore, diagonal kernels estimation tends to have a large standard deviation.
- The 'best' performances are obtained for the cubic kernel if $m_1 \neq m_2 \neq m_3$. The estimation of $h_3(m_1, m_2, m_3)$ for $m_1 = m_2 = m_3$, necessitate the use of more additional terms, leading to a large standard deviation.
- The estimation of $h_1(m_1)$ for the symmetric input depends on the estimation of the cubic kernel. In fact, the explicit relationship of the linear kernel for the third order Volterra contains $h_3(m_1, m_2, m_3)$ terms for $m_i = m_j$ ($\{i, j\}_{i \neq j} \in \{1, 2, 3\}$), which confirm the large value of standard deviation for the estimation of the linear kernel.

7. Conclusion

In this paper, we have addressed the problem of structure and parameter identification of Volterra models driven by symmetric input with four levels. Obviously, this sequence insures the identifiability of the parameters of second and third order Volterra models. The proposed methods are based on the crosscumulants between the input and the output and the statistics proprieties of the input sequence. The proposed structure identification method consists in estimating the order of the Volterra model that will be used to identify the length of each kernel. Our method can be used to identify Volterra model structure having different kernel memory lengths. Moreover, a closed form solution has been developed to estimate the parameters of Volterra models. Simulations are presented to illustrate the performance of the proposed methods.

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