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Weighted minimum-variance self-tuning regulation of stochastic time-varying systems: application to a heat transfer process

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Abstract. *This paper deals with the weighted minimum-variance self-tuning regulation of stochastic time-varying systems, which can be described by linear input-output mathematical models. We consider the input-output ARMAX mathematical models with unknown time-varying parameters. The recursive extended least squares RELS algorithm, which can be applied to the stochastic time-varying systems, is presented. Self-tuning regulators are developed on the basis upon the weighted minimum-variance control strategy. The developed theoretical results are applied to a heat transfer process. The obtained practical results show the good performances of the developed weighted minimum-variance self-tuning regulators.*

Key words: *Stochastic time-varying systems; Input-output ARMAX mathematical models; Weighted minimum-variance self-tuning regulation; Heat transfer process*.

1. Introduction

The object of this paper is the development of weighted minimum-variance self-tuning regulators, which can be applied to the stochastic time-varying systems. We consider the dynamic systems, which can be described by an input-output mathematical model, linear, stochastic, monovariable, with unknown time-varying parameters.

The traditional controllers with fixed parameters are often unsuited to industrial processes because of the change of their parameters resulting on their stochastic character. One possible alternative for improving the quality of control for such processes is the use of adaptive control systems Bobál *et al*. (2005).

The study of the problems relating to the regulation of the dynamic systems operating in a stochastic environment has worried several control engineers in various research teams, and this, for numerous decades. In this context, several types

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of regulators were developed and published in the literature, while being based on various control strategies (minimum-variance control, PID, self-tuning PID controller, self-tuning control etc.), see for example, Åström and Wittenmark (1973), Åström *et al.* (1977), Ben Abdennour et *al*. (2001), Wieslander and Zarrop (1991), Isermann *et al.* (1992), Li and Evans (2002), Kamoun (2003), Kharrat *et al*. (2005), Petete *et al*. (2008) and Zulfatman and Rahmat (2009).

Many researchers have proved that it's possible, for stationary processes, to determine the unknown parameters through identification. However, the experiments and the evaluation can be rather time consuming. It's thus desirable to design self-tuning regulators which tune their parameters on-line. Indeed, the Self-Tuning Control STC is an area which has attracted, and continues to attract, extensive interest from academia and the literature is littered with references to leading figures such as: Åström (1983), Clarke and Gawthrop (1979), Grimble (1982) and Love (2007). The motivation for the work outlined here has been to study weighted minimum-variance self-tuning regulation of stochastic time-varying systems. Such regulation is applied for the control of a heat transfer process.

In this paper, we consider the weighted minimum-variance control strategy. Various types of self-tuning regulators are developed, while being based on the weighted minimum-variance regulation strategy. The stability conditions and the techniques of the practical implementation of these regulators are given. The validation of the developed analytical results carried out on a heat transfer process.

The structure of the explicit self-tuning regulator is shown in Figure 1:

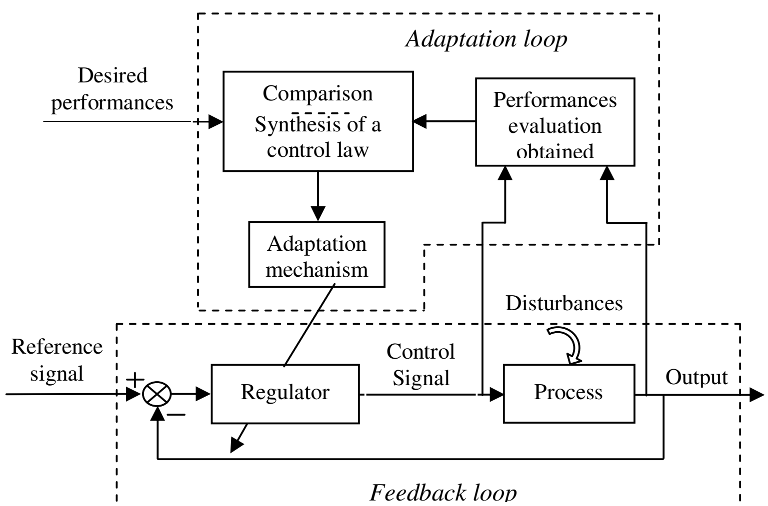

Figure 1. Structure of the explicit self-tuning regulator.

The structure, Figure 1., is essentially composed of three parts: a parameter estimator, linear regulator and a bloc which determines the regulator parameters from the estimated parameters.

2. Input-output ARMAX mathematical model

The description of the dynamic systems with time-varying parameters by input-output mathematical models became increasingly used in the last few years in the development of the diagrams of parametric estimate or adaptive control, and this, because of the simplicity of their practical implementation. Thus, one proposes here to use an input-output mathematical model of the type ARMAX (Auto-Regressive Moving Average with eXogenous) allowing the description of the dynamic systems time-varying parameters, which operate in a stochastic environment.

Let us consider a stochastic system with time-varying parameters, which can be described by the following input-output ARMAX mathematical model:

$$A(q^{-1},k)y(k) = q^{-d}B(q^{-1},k)u(k) + C(q^{-1})e(k) \tag{1}$$

where $u(k)$ and $y(k)$ represent the input and the output of the system at the discrete-time k respectively, $e(k)$ designs the noise (a set of different kinds of disturbances) which affects the system, d is the delay of the system, and $A(q^{-1},k)$, $B(q^{-1},k)$ are polynomials with unknown time-varying parameters, such as:

$$A(q^{-1},k) = 1 + a_1(k)q^{-1} + \cdots + a_{n_A}(k)q^{-n_A} \tag{2}$$

$$B(q^{-1},k) = b_1(k)q^{-1} + \cdots + b_{n_B}(k)q^{-n_B} \tag{3}$$

and $C(q^{-1})$ is a polynomial with constant but unknown parameters, which is defined by:

$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{n_C} q^{-n_C} \tag{4}$$

n_A, n_B and n_C being orders of the polynomials $A(q^{-1},k)$, $B(q^{-1},k)$ and $C(q^{-1})$, respectively.

Without loss of general information, but for reasons of simplicity, one supposes that the various polynomials $A(q^{-1},k)$, $B(q^{-1},k)$ and $C(q^{-1})$ have the same order n (i.e.: $n_A = n_B = n_C = n$). In the following study, the delay d and the order n of the selected mathematical model are supposed to be known.

The development of an ARMAX mathematical model, as described by (1), can be carried out while being based on the knowledge of measurements of the input $u(k)$ and the output $y(k)$, which result from the considered system. In such a situation, and for a number M of measurements, one must have two sequences of measurements, namely:

a. the sequence of the input $I_{u(k)}$, such as: $I_{u(k)} = \{u(k); k = 1, \ldots, M\}$;

b. the sequence of the output $I_{y(k)}$, such as: $I_{y(k)} = \{y(k); k = 1, \ldots, M\}$.

It is supposed here that $\{e(k)\}$ is a sequence of independent random variables, of zero mean and variance σ^2. Moreover, one supposes that this sequence of noise is independent of the sequences of the input $I_{u(k)}$ and the output $I_{y(k)}$. These two assumptions are often allowed in the majority of the industrial applications, since they can well represent the reality of the characteristics of the random disturbances which act on the system. Moreover, they can give appropriate solutions to the identification or control problems, and consequently, to simplify the practical implementation concerning the elaboration of the schemes of estimate or control.

3. Recursive algorithm of parametric estimate of wide least squares

One proposes to work out a recursive algorithm which considers the parameters of stochastic systems to time-varying parameters. These systems can be described by an ARMAX input-output mathematical model, as given by (1). The problem formulation of parametric estimate will be carried out starting from the use of the recursive method of extended least squares, by including a forgetting factor of exponential while being based on the knowledge of several measurements (couples of input-output) resulting from the considered system.

The output $y(k)$ of the ARMAX mathematical model (1) can be expressed as follows:

$$\begin{aligned} y(k) = &-a_1(k)y(k-1) - \cdots - a_n(k)y(k-n) + b_1(k)u(k-d-1) + \cdots \\ &+ b_n(k)u(k-d-n) + e(k) + c_1e(k-1) + \cdots + c_ne(k-n) \end{aligned} \tag{5}$$

or in an equivalent way, in a compact form:

$$y(k) = \theta^T(k)\psi(k) + e(k) \tag{6}$$

in which the vectors of parameters $\theta(k)$ and observations $\psi(k)$ are, respectively, defined by:

$$\theta^T(k) = [a_1(k) \ldots a_n(k)\; b_1(k) \ldots b_n(k)\; c_1 \ldots c_n] \tag{7}$$

and

$$\psi^T(k) = [-y(k-1) - \cdots - y(k-n)\ u(k-d-1) \cdots u(k-d-n)\ e(k-1) \cdots e(k-n)] \qquad (8)$$

The method of parametric estimate by wide least squares is an extension of that of ordinary least squares, where the vectors of parameters and observations are large. In this direction, the vector of parameters $\theta(k)$, defined by (7), contains the parameters $a_i(k)$ and $b_j(k)$, $i, j = 1,\ldots,n$, of the system dynamics, and the parameters c_i, $i = 1,\ldots,n$, which correspond to the $e(k)$ noise dynamics. Let us add that the vector of observations $\psi(k)$, as given by (8), consists in one hand of measurable sequences, which are related to the measured sizes of the output and the input of the system, and in other hand of a non-measurable sequence, which is related to the noise acting on this system. It is quite obvious that, the introduction of this observations vector $\psi(k)$ into the experiment implementation of identification method using ordinary least squares algorithm, leads to a failure. To solve this problem, one can replace the noise sequence elements $\{e(k-i); i = 1,\ldots,n\}$ by their *a priori* estimated values $\{\varepsilon(k-i);\ i = 1,\ldots,n\}$. While taking into account this solution, it will be possible to have an approximation $\hat{\psi}(k)$ of the observations vector $\psi(k)$, such as:

$$\hat{\psi}^T(k) = [-y(k-1) - \cdots - y(k-n)\ u(k-d-1) \cdots u(k-d-n)\ \varepsilon(k-1) \cdots \varepsilon(k-n)] \qquad (9)$$

Thus, one can describe the predicted output of the system $\hat{y}(k)$ by the following expression:

$$\hat{y}(k) = \hat{\theta}^T(k-1)\hat{\psi}(k) \qquad (10)$$

where $\hat{\theta}(k-1)$ is the parameters vector, estimated at the discrete sample $k-1$, such as:

$$\begin{aligned} \hat{\theta}^T(k-1) = [&\hat{a}_1(k-1) \cdots \hat{a}_n(k-1)\ \hat{b}_1(k-d-1) \cdots \hat{b}_n(k-d-1) \\ &\hat{c}_1(k-1) \cdots \hat{c}_n(k-1)] \end{aligned} \qquad (11)$$

One defines the *a priori* prediction error $\varepsilon(k)$, which corresponds to the difference between the output $y(k)$ of the system and that predicted $\hat{y}(k)$ of the tuning model, by the following expression:

$$\varepsilon(k) = y(k) - \hat{\theta}^T(k-1)\hat{\psi}(k) \qquad (12)$$

The problem arising here consists of the estimate parameters intervening in the vector $\theta(k)$, defined by (7). The study of this problem must allow the variance minimization of a certain bearing criterion on the differences between the output of the system $y(k)$ and that predicted by the tuning model $\hat{y}(k)$.

It is well-known that the parametric estimation algorithm of the Recursive Extended Least Squares RELS, which can be applied to a stochastic system with unknown parameters, where $\theta(k) = \theta$ is a constant $\forall\, k$, can be described by:

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) + P(k)\hat{\psi}(k)\varepsilon(k) \\ P(k) &= P(k-1) - \frac{P(k-1)\hat{\psi}(k)\hat{\psi}^T(k)P(k-1)}{1+\hat{\psi}^T(k)P(k-1)\hat{\psi}(k)} \\ \varepsilon(k) &= y(k) - \hat{\theta}^T(k-1)\hat{\psi}(k) \end{aligned} \tag{13}$$

It must be emphasized that the use of the recursive algorithm of parametric estimate RELS, as given by (13), in order to consider the time-varying parameters in the vector $\theta(k)$, described by (7), leads to a failure (bad quality of estimate, etc). To overcome this difficulty, one can choose various procedures allowing the parameters calculation of the adaptation gain matrix $P(k)$, which is involved in this algorithm, in such way that the tracking of parametric variations is achievable and can be ensured in the course of time (Kamoun, 2003). Among these procedures, the one that includes a forgetting factor of an exponential kind is considered. The procedure, which consists in introducing a forgetting factor into the adaptation gain matrix of a recursive parametric estimation algorithm, allows improving its capacity to the benefit of adaptation, while ensuring best tracking of the time-varying parameters of the system considered. This procedure is largely discussed and analyzed in several publications (see, e.g., Ljung and Gunnarsson, 1990; Kamoun, 2003). In such a procedure, the forgetting factor prevents that the parameters of this matrix to the benefit of adaptation do not become too small so that all new data (measured values of the input and the output), in the observation vector, affect the quality performances of identification. The forgetting factor thus allows to introduce an exponential factor which influences the old values measured to the benefit of the new measured values; this permits thus to balance the old observations less and less.

One can show easily that the estimate of the parameters intervening in the vector $\theta(k)$, defined by (7), can be achieved by using the following recursive algorithm of parametric estimate RELS, by including a forgetting factor of an exponential kind:

$$\begin{aligned} \hat{\theta}(k) &= \hat{\theta}(k-1) + P(k)\hat{\psi}(k)\varepsilon(k) \\ P(k) &= \frac{1}{\lambda(k)}[P(k-1) - \frac{P(k-1)\hat{\psi}(k)\hat{\psi}^T(k)P(k-1)}{\lambda(k)+\hat{\psi}^T(k)P(k-1)\hat{\psi}(k)}] \\ \varepsilon(k) &= y(k) - \hat{\theta}^T(k-1)\hat{\psi}(k) \end{aligned} \tag{14}$$

where $\lambda(k)$ is a forgetting factor, which can be calculated starting from the recurrent equation, as follow:

$$\lambda(k) = \lambda_{\circ}\lambda(k-1) + \lambda^{\circ}(1-\lambda_{\circ}) \tag{15}$$

with: $0 < \lambda_{\circ} < 1$, $0 < \lambda^{\circ} < 1$.

One can go up to:

$$\lim_{k \to \infty} \lambda(k) = \lambda^{\circ} \tag{16}$$

Let us emphasize that the range of variation of forgetting factor $\lambda(k)$ must satisfy the following condition: $0 < \lambda(k) < 1$. Let us add that the purpose of the choice of this time-varying forgetting factor $\lambda(k)$ is to more refine the estimation quality of the time-varying parameters. However, in certain types of industrial applications, one can choose a constant forgetting factor (i.e.: $\lambda(k) = \lambda$, $\forall k$).

4. Weighted minimum-variance self-tuning regulation

The object of this section is the study of certain problems relating to the self-tuning regulation of stochastic time-varying systems. We will more particularly putting the focus on the class of the systems to time-varying parameters, which can be described by mathematical models input-output of the ARMAX type, with unknown and time-varying parameters.

The use of the structure of self-tuning regulation of the systems, while being based on various approaches (pole placement, PID, with variance of minimal output, etc), became increasingly widespread in the industrial applications. We will be interested here in the synthesis of a weighted minimum-variance self-tuning regulator. This type of regulator allows limiting the control signal magnitude to a certain desired value. The development of the weighted minimum-variance self-tuning regulator can be made starting from the minimization of a behaviour criterion on the variance of the output and the input of the system to be controlled, such as definite by the mathematical model input-output ARMAX (1). Two schemes of self-tuning regulation are used, namely: the explicit scheme and the implicit one.

The output $y(k)$ of the stochastic system, described by the ARMAX mathematical model input-output (1), can be written at the discrete sample $k+d+1$:

$$y(k+d+1) = \frac{qB(q^{-1},k)}{A(q^{-1},k)}u(k) + \frac{C(q^{-1})}{A(q^{-1},k)}e(k+d+1) \tag{17}$$

or in an equivalent way:

$$y(k+d+1) = \frac{qB(q^{-1},k)}{A(q^{-1},k)}u(k) + \frac{G(q^{-1},k)}{A(q^{-1},k)}e(k) + F(q^{-1},k)e(k+d+1) \tag{18}$$

where the polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$, are solution of the following polynomial equation:

$$C(q^{-1}) = A(q^{-1},k)F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{19}$$

They are given by:

$$F(q^{-1},k) = 1 + f_1(k)q^{-1} + \cdots + f_d(k)q^{-d} \tag{20}$$

and

$$G(q^{-1},k) = g_0(k) + g_1(k)q^{-1} + \cdots + g_{n-1}(k)q^{-n+1} \tag{21}$$

The problem of the self-tuning regulation of the stochastic system considered, which is described by mathematical model ARMAX (1), consists on determining a control law $u(k)$ to reduce ″ as well as possible ″ the effect of the noise $e(k)$ on the output of the system $y(k)$. The study of this problem of regulation can be carried out starting from the minimization of a quadratic criterion $J(k+d+1)$ on the variance of the output $y(k+d+1)$ and a weighting α on the variance of the control law $u(k)$, such as:

$$J(k+d+1) = \mathcal{E}[y^2(k+d+1) + \alpha u^2(k)] \tag{22}$$

where $\mathcal{E}$ indicates the expectation and α is a weighting, which must be selected in a suitable way ($\alpha > 0$).

The output $y(k+d+1)$ of the dynamic system considered, as defined by (17), can be rewritten as follows:

$$\begin{aligned} y(k+d+1) = b_1(k)u(k) + \frac{L(q^{-1},k)}{A(q^{-1},k)}u(k-1) + \frac{G(q^{-1},k)}{A(q^{-1},k)}e(k) \\ + F(q^{-1},k)e(k+d+1) \end{aligned} \tag{23}$$

where the polynomial $L(q^{-1},k)$ is defined by:

$$\begin{aligned} L(q^{-1},k) = b_2(k) - a_1(k)b_1(k) + (b_3(k) - a_2(k)b_1(k))q^{-1} + \cdots \\ + (b_n(k) - a_{n-1}(k)b_1(k))q^{-n+2} - a_n(k)b_1(k)q^{-n+1} \end{aligned} \tag{24}$$

By using the two following notations:

$$T(k) = \frac{L(q^{-1},k)}{A(q^{-1},k)}u(k-1) + \frac{G(q^{-1},k)}{A(q^{-1},k)}e(k) \tag{25}$$

and

$$T_\circ(k) = T(k) + b_1(k)u(k) \tag{26}$$

we can rewrite the quadratic criterion (22), as follows:

$$J(k+d+1) = \mathcal{E}\left[[T_\circ(k) + F(q^{-1},k)e(k+d+1)]^2 + \alpha u^2(k)\right] \tag{27}$$

Let know that the sizes intervening in $T_\circ(k)$ are measurable at the discrete sample $k+d+1$ and that the sequence $\{e(k+d+1)\}$ is not correlated with $T_\circ(k)$, we can put the quadratic criterion (27) in the following form:

$$J(k+d+1) = T_\circ^2(k) + \alpha u^2(k) + [1 + f_1^2(k) + \cdots + f_d^2(k)]\sigma^2 \tag{28}$$

The control law $u(k)$ allows minimizing this quadratic criterion (28), and can be obtained by solving the following expression:

$$\frac{\partial J(k+d+1)}{\partial u(k)} = b_1(k)T_\circ(k) + \alpha u(k) = 0 \tag{29}$$

It results that the expression of the control law $u(k)$ can be given by:

$$u(k) = -\frac{G(q^{-1},k)}{Z(q^{-1},k)}\, y(k) \tag{30}$$

where the polynomial $Z(q^{-1},k)$ is given by:

$$\begin{aligned} Z(q^{-1},k) &= qB(q^{-1},k)F(q^{-1},k) + (\alpha / b_1(k))C(q^{-1}) \\ &= z_1(k) + z_2(k)q^{-1} + \cdots + z_{n+d}(k)q^{-n-d+1} \end{aligned} \tag{31}$$

with: $z_1(k) = b_1(k) + (\alpha / b_1(k))$.

The stability of the control law $u(k)$ described by (30), is related to the roots of the polynomial $Z(q^{-1},k)$. Of course, a suitable choice of the α weighting allows applying this control law for a stochastic minimum phase system.

4.1. Weighted minimum-variance explicit self-tuning regulator

The weighted minimum-variance explicit self-tuning regulator can be held by considering the three following stages:

Stage 1: estimate the parameters intervening in mathematical model ARMAX (5), by using the recursive algorithm of parametric estimate RELS (14);

Stage 2: determine the parameters intervening in the polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$, by solving the polynomial equation (19);

Stage 3: calculate the control law $u(k)$ from the following equation:

$$u(k) = -\frac{\sum_{r=2}^{n+d} z_r(k)}{z_1(k)} u(k-r+1) - \frac{\sum_{t=0}^{n-1} g_t(k)}{z_1(k)} y(k-t) \tag{32}$$

We notice that during the calculation of the control law $u(k)$, defined by (32), we can have, at a given k_0 sample: $z_1(k_0) = 0$. In such a situation, the boundedness of the control signal $u(k_0)$ is not ensured, which can cause instability of the system. To overcome this problem, we can test the value of the parameter $z_1(k)$; so: if $z_1(k_0) = 0$, then we take $z_1(k_0) = \xi$, with ξ a rather weak parameter which must be chosen in an adequate way.

4.2. Weighted minimum-variance implicit self-tuning regulator

We can show that the minimization of a quadratic criterion $J(k+d+1)$ of the type (22) compared to the control law $u(k)$ is equivalent to the minimization of the following quadratic criterion:

$$J(k+d+1) = \mathcal{E}[s^2(k+d+1)] \tag{33}$$

where $s(k+d+1)$ is the output of an implicit model, such as:

$$s(k+d+1) = y(k+d+1) + (\alpha / b_1(k))u(k) \tag{34}$$

We can express this output $s(k+d+1)$ as follows:

$$\begin{aligned} s(k+d+1) = {} & Z(q^{-1},k)u(k) + G(q^{-1},k)y(k) + [1 - C(q^{-1})]s^*(k+d+1) \\ & + \upsilon(k+d+1) \end{aligned} \tag{35}$$

in which $s^*(k+d+1)$ is the prediction with $d+1$ step of the implicit model output $s(k)$.

The output $s(k+d+1)$ can be given in the following compact form:

$$s(k+d+1) = \theta^T(k)\psi(k+d+1) + \upsilon(k+d+1) \tag{36}$$

where the vectors of parameters $\theta(k)$ and observations $\psi(k+d+1)$ are defined by:

$$\theta^T(k) = [z_1(k) \cdots z_{n+d}(k)\ g_0(k) \cdots g_{n-1}(k)\ c_1 \cdots c_n] \tag{37}$$

and

$$\begin{aligned} \psi^T(k+d+1) = {} & [u(k) \cdots u(k-d-n+1)\ y(k) \cdots y(k-n+1) \\ & -s^*(k+d) - \cdots - s^*(k+d+1-n)] \end{aligned} \tag{38}$$

In the vector of observations $\psi(k+d+1)$ the sequence of the prediction with $d+1$ step of the output $s(k)$ is not observable. Thus, we can choose an approximation $\widehat{\psi}(k+d+1)$ of the vector of observations $\psi(k+d+1)$, such as:

$$\begin{aligned}\widehat{\psi}^T(k+d+1) = [&u(k)\cdots u(k-d-n+1)\ y(k)\cdots y(k-n+1) \\ &-\hat{s}^*(k+d)-\cdots-\hat{s}^*(k+d+1-n)]\end{aligned} \tag{39}$$

We can define the optimal self-tuning predictor $\hat{s}^*(k+d+1)$ in $d+1$ step of the output $s(k)$ by the following expression:

$$\hat{s}^*(k+d+1) = \hat{\theta}^T(k)\widehat{\psi}(k+d+1) \tag{40}$$

The weighted minimum-variance implicit self-tuning regulator can be held by considering the two following stages:

Stage 1: estimate the parameters intervening in the implicit mathematical model (36), by using the recursive algorithm of parametric estimate RELS (14);

Stage 2: calculate the control law $u(k)$, which allows cancelling the optimal self-tuning predictor $\hat{s}^*(k+d+1)$, as follows:

$$\begin{aligned}u(k) = &-\frac{\sum_{r=2}^{n+d}\hat{z}_r(k)}{\hat{z}_1(k)}u(k-r+1) - \frac{\sum_{t=0}^{n-1}\hat{g}_t(k)}{\hat{z}_1(k)}y(k-t) \\ &+\frac{\sum_{h=0}^{n}\hat{c}_h(k)}{\hat{z}_1(k)}\hat{s}^*(k+d+1-h)\end{aligned} \tag{41}$$

Let us notice that during the calculation of the control law $u(k)$, defined by (41), we can have, at a given k_0 sample: $\hat{z}_1(k_0)=0$. In this case, the boundedness of the control signal $u(k_0)$ is not ensured, which can cause instability of the system. To overcome this problem, we can test the value of the estimated parameter $\hat{z}_1(k)$; so: if $\hat{z}_1(k_0)=0$, then we take $\hat{z}_1(k_0)=\xi$, with ξ a rather weak parameter which must be chosen in an adequate way.

4.3. Simplified weighted minimum-variance implicit self-tuning regulator

The control law $u(k)$, defined by (41), is elaborate to cancel the self-tuning optimal predictor $\hat{s}^*(k+d+1)$. This enables us to choose a simplified version of the implicit model (4.47), and this, by neglecting the sequence $\{s^*(k+d-h+1); h=1,\ldots,n\}$. The simplified implicit model which results is given by:

$$s(k+d+1) = \theta^T(k)\psi(k+d+1) + \upsilon(k+d+1) \tag{42}$$

where the vectors of parameters $\theta(k)$ and observations $\psi(k+d+1)$ are defined by:

$$\theta^T(k) = [z_1(k) \cdots z_{n+d}(k)\ g_0(k) \cdots g_{n-1}(k)] \tag{43}$$

$$\psi^T(k+d+1) = [u(k) \cdots u(k-d-n+1)\ y(k) \cdots y(k-n+1)] \tag{44}$$

The various sizes intervening in this vector of observations $\psi(k+d+1)$ are measurable at the discrete sample $k+d+1$. It thus results that the estimate of the vector of parameters $\theta(k)$, as defined by (4.54), can be achieved by a recursive algorithm of parametric estimate based on the techniques of least squares (e.g., the recursive ordinary least squares (RLS) parametric estimation algorithm with a forgetting factor).

The simplified weighted minimum-variance implicit self-tuning regulator can be held by considering the two following stages:

Stage 1: estimate the parameters intervening in the simplified implicit mathematical model (42), by using a recursive ordinary least squares (RLS) parametric estimation algorithm with a forgetting factor (see, e.g, Kamoun, 2003);

Stage 2: calculate the control law $u(k)$ starting from the following expression:

$$\hat{\theta}^T(k)\psi(k+d+1) = 0 \tag{45}$$

we deduce:

$$u(k) = -\frac{\sum_{r=2}^{n+d} \hat{z}_r(k)}{\hat{z}_1(k)} u(k-r+1) - \frac{\sum_{t=0}^{n-1} \hat{g}_t(k)}{\hat{z}_1(k)} y(k-t) \tag{46}$$

We must exclude the situation which gives $\hat{z}_1(k_0) = 0$ at a given k_0 sample, to guarantee the boundedness of the control signal $u(k_0)$.

5. Application to a heat transfer process

We will test here the effectiveness of the developed weighted minimum-variance explicit self-tuning regulator, by the experiments on a heat transfer process. The problem arising thus consists of the regulation of this process, while supposing that it operates in a stochastic environment. Before modelling the considered process, we will present it, briefly, with its environment in the following section.

5.1. Description of the process

The Heat Transfer Process is represented by a tube of conical form, controlled by a variable heating power amplifier from 0 to 3 kw, Figure 2.a. The picture of the process is shown in Figure 2.b.

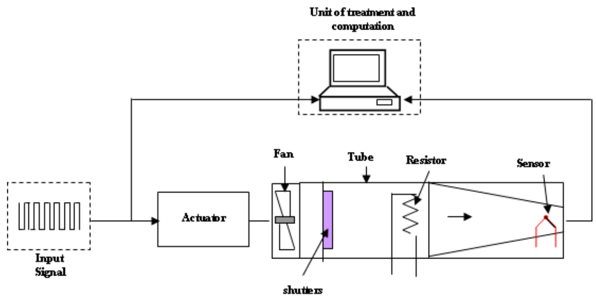

Figure 2.a Experimental setup of the heat transfer process

Figure 2.b Picture of the process

The air conditioning is puffed up at the other end of the tube by using a Fan rotating at a constant speed. The air is heated in contact with all the heating resistances. They are associated in one phase load connected to the power amplifier circuit containing a triac, controlled by an electronic circuit around the integrated circuit "TCA 785", which is governed between 0 and 10 V. The voltage variation acts in order to control the angle of beginning of the triac and consequently to change the power of the heating resistances, it forms the Actuator, Figure 3.

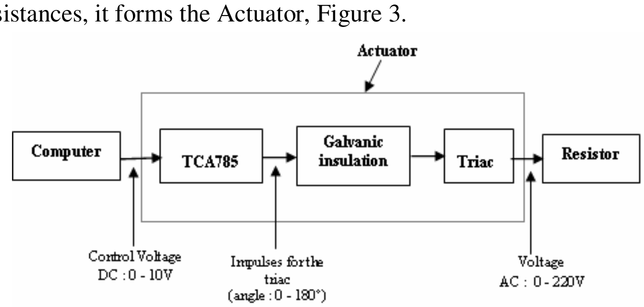

Figure 3. The actuator synoptic

On the other side of the heat transfer process, the temperature sensor, type LM 335, permitted the temperature measurement of the air blown at the end of the tube. The measures acquirement of the process is done through an unit of treatment and computation that is constituted of a computer incorporating an acquirement card of APCI-ADADIO type including the blocks of numerical analogical conversion (sensor-calculator) and the analogical numerical conversion (calculator-actuator or its control circuit). The card of acquirement is constituted of:

- a digital analogical converter for the analogical inputs: 8 differential inputs or 16 simple, all multiplexed;
- two independent analogical digital converters for two analogical outputs;
- 16 numeric inputs/outputs;
- three channels of counter/timer.

5.2. Experiment results

The description of the heat transfer process considered can be made by a mathematical model input-output of the type ARMAX, such as:

$$y(k) = -a_1(k)y(k-1) + b_1(k)u(k-2) + e(k) + c_1 e(k-1) \tag{47}$$

where $u(k)$ is the voltage applied to the heat process at the discrete sample k, $y(k)$ represents the value of the measured temperature of the sensor, $e(k)$ indicates the noise affecting the process, $a_1(k)$ is an unknown time-varying parameter, and b_1 and c_1 are unknown parameters, but presumed constant. We suppose that the sequence $\{e(k); k = 1, \ldots, M\}$ consists of random variables independent of zero mean and variance σ^2.

From the ARMAX mathematical model (4.48), we can rewrite the output $y(k)$ in the following matrix form:

$$y(k) = \theta^T(k)\psi(k) + e(k) \tag{48}$$

in which the vectors of parameters $\theta(k)$ and observations $\psi(k)$ are defined by:

$$\theta^T(k) = \begin{bmatrix} a_1(k) & b_1(k) & c_1 \end{bmatrix} \tag{49}$$

$$\psi^T(k) = [-y(k-1) \quad u(k-2) \quad e(k-1)] \tag{50}$$

The problem here is related to the self-tuning regulation of the considered process temperature, in the sense of generalized minimum variance of the temperature, which corresponds to a reference temperature: $y_r(k) = 38°C$. For this, we propose to use a self-tuning regulator explicit to minimum generalized output variance. Data related to the experimental implementation of this self-tuning regulator are:

- the identification of the parameters involved in the vector $\theta(k)$, given by (49), is done while using the recursive identification algorithm RELS (14). The initial conditions of this identification algorithm are selected as: $\hat{\theta}(0)=0$ and $P(0)=1000I$, where I is the identity matrix. The forgetting factor involved in the recursive identification algorithm is chosen as: $\lambda(k)=0.99, \forall\ k$;
- the α weighted coefficient is chosen as : $\alpha=0.5$;
- the control horizon is taken in the following range: $k=0,1,\cdots,150$.

The curves of the output $y(k)$ and the reference signal $y_r(k)$, the variance $\sigma_y^2(k)$ of the output $y(k)$, the control law $u(k)$ and the variance of the control law $u(k)$ are shown in Figure 4.

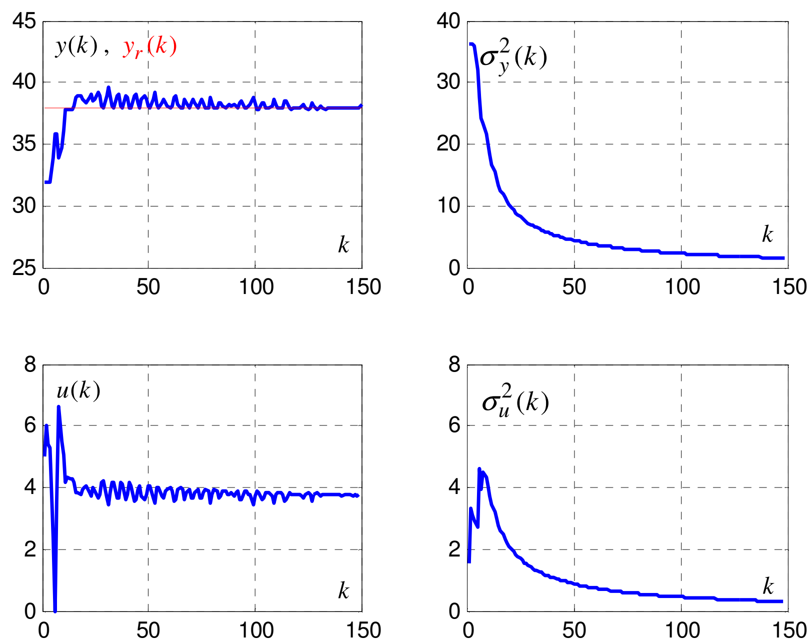

Figure 4. Curves of the output $y(k)$ and $y_r(k)$, the variance $\sigma_y^2(k)$ of the output $y(k)$, the control law $u(k)$ and its variance $\sigma_u^2(k)$.

In Figure 5., we represent the evolution curves of the error $h(k)=y_r(k)-y(k)$ and its variance $\sigma_h^2(k)$.

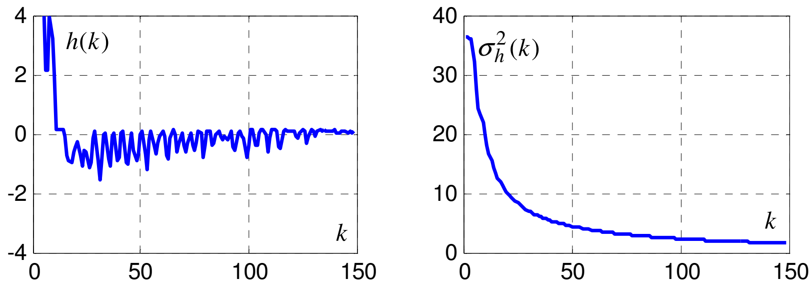

Figure 5. Curves of the error $h(k)$ and its variance $\sigma_h^2(k)$.

The evolution curves of the estimated parameters $\hat{a}_1(k)$, $\hat{b}_1(k)$, $\hat{c}_1(k)$ and the prediction error $\varepsilon(k)$ are illustrated in Figure 6.

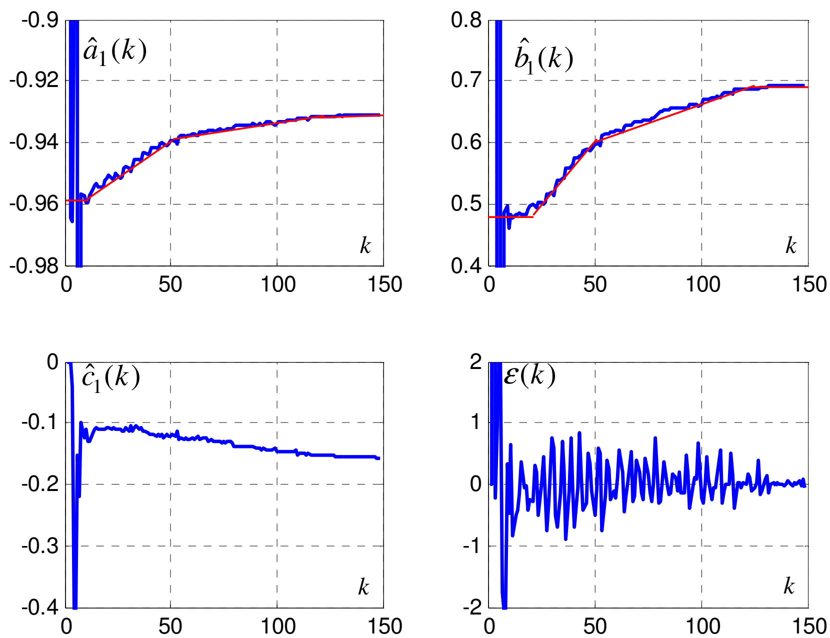

Figure 6. Evolution curves of the estimated parameters $\hat{a}_1(k)$, $\hat{b}_1(k)$, $\hat{c}_1(k)$ and the prediction error $\varepsilon(k)$.

According to experimental results, we note that the following numerical values for the parameters $a_1(k)$ and $b_1(k)$ involved in the mathematical models (47) can be smoothed linearly as follows:

for $k = 1, \cdots, 11$, $a_1(k) = -0.9590$,

for $k = 12, \cdots, 50$, $a_1(k) = 4.974.10^{-4}(k-11) - 0.959$,

for $k = 51, \cdots, 125$, $a_1(k) = 1.08.10^{-4}(k-125) - 0.9396$,

for $k = 126, \cdots, 150$, $a_1(k) = -0.9313$.

for $k = 1, \cdots, 18$, $b_1(k) = 0.4811$,

for $k = 19, \cdots, 50$, $b_1(k) = 3.603.10^{-3}(k-18) + 0.4811$,

for $k = 51, \cdots, 125$, $b_1(k) = 4.048.10^{-3}(k-125) + 0.5964$,

for $k = 126, \cdots, 150$, $b_1(k) = 0.6904$.

Let us add that the noise sequence $e(k)$ corresponds to the Gaussian distribution, with zero mean and variance $\sigma^2 = 0.0288$.

The interpretation of the evolution curves of the different signals (input and output) and their variances as well as the model parameters which are represented in Figs. 4, 5 and 6 shows well the good quality of the regulation of the heat transfer process, which is obtained by the weighted minimum variance self-tuning regulation of stochastic time-varying systems. Thus, the evolution curves of the variances $\sigma_u^2(k)$ and $\sigma_y^2(k)$ converge towards a constant minimum values.

Table 1 presents the values of the statistical averages: $\overline{m}_{\hat{c}_1}$ of the estimated parameters, $\overline{m}_{\varepsilon}$ of the prediction error and $\overline{m}_{\sigma_\varepsilon^2}$ of its variance, $\overline{m}_h$ of the error and $\overline{m}_{\sigma_h^2}$ of its variance.

Table 1 *Statistical averages:* $\overline{m}_{\hat{c}_1}$, $\overline{m}_{\varepsilon}$, $\overline{m}_{\sigma_\varepsilon^2}$, $\overline{m}_h$ and $\overline{m}_{\sigma_h^2}$

$\overline{m}_{\hat{c}_1}$	$\overline{m}_{\varepsilon}$	$\overline{m}_{\sigma_\varepsilon^2}$	$\overline{m}_h$	$\overline{m}_{\sigma_h^2}$
-0.1541	0.0240	0.0288	0.0317	0.0226

Table 2 gives the values of the statistical averages: $\overline{m}_u$ of the control law $u(k)$ and $\overline{m}_{\sigma_u^2}$ of its variance, $\overline{m}_y$ of the output $y(k)$ and $\overline{m}_{\sigma_y^2}$ of its variance, and the values of the statistical averages: $\overline{m}_{f_1}$, $\overline{m}_{g_0}$, $\overline{m}_{z_1}$ and $\overline{m}_{z_2}$ of the estimated parameters. The computation of the values is taken in the following range: k=111, …, 150.

Table 2 *Statistical averages:* $\bar{m}_u$, $\bar{m}_{\sigma_u^2}$, $\bar{m}_y$, $\bar{m}_{\sigma_y^2}$, $\bar{m}_{f_1}$, $\bar{m}_{g_0}$, $\bar{m}_{z_1}$ and $\bar{m}_{z_2}$

$\bar{m}_u$	$\bar{m}_{\sigma_u^2}$	$\bar{m}_y$	$\bar{m}_{\sigma_y^2}$	$\bar{m}_{f_1}$	$\bar{m}_{g_0}$	$\bar{m}_{z_1}$	$\bar{m}_{z_2}$
3.7501	0.0056	37.9683	0.0226	0.7774	0.7242	1.4148	0.4224

According to experimental results not presented, we note that the level of the control law $u(k)$ and the output signal $y(k)$ is related to the value of the α weighting. Indeed, if α increases, the control law signal $u(k)$ decreases and that of $y(k)$ increases.

From the experiment results, we can notice that the control performance quality obtained by this type of self-tuning controller is satisfactory.

6. Conclusion

This paper was reserved to the study of problems relating to the self-tuning regulation of the dynamic systems operating in a stochastic environment. We considered more particularly the dynamic systems with time-varying parameters, which can be described by the class of the input-output mathematical models, linear, stochastic, monovariable, to known structure (order, delay), and to unknown parameters and time-varying.

Three types of self-tuning regulators are developed, while based on the strategy of weighted minimum-variance regulation. They are usually known, respectively, as: the weighted minimum-variance explicit self-tuning regulator, the weighted minimum-variance implicit self-tuning regulator and the simplified weighted minimum-variance implicit self-tuning regulator. The convergence, the stability conditions as well as the practice method of implementation of these regulators were given.

The evolution curves, carried out from the experiment tests on a heat transfer process, have shown well the good quality of the regulation performances of the weighted minimum-variance explicit self-tuning regulator. The experimental results showed also that $a_1(k)$ and $b_1(k)$ are parameters slowly time-varying. However, we can conclude that the experimental results obtained are satisfactory.

References

[1]. Åström, K. J. "Theory and applications of adaptive control—A survey". *Automatica*, Volume 19, Issue 5, pp. 471-486, September 1983.

[2]. Åström, K.J.; V. Borisson; L. Ljung and B. Wittenmark. "Theory and application of self-tuning regulators". *Automatica*, vol. 13, pp. 457–476, 1977.

[3]. Åström, K.J. and B. Wittenmark. "On self-tuning regulators". *Automatica*, vol. 9, pp. 185–189, 1973.

[4]. Ben Abdennour, R.; P. Borne; M. Ksouri et F. M'Sahli. "Identification et Commande Numérique des Procédés Industriels". Editions Technip, Paris, 2001.

[5]. Bobál, V.; J. Böhm; J. Fessl and J. Macháček. "Digital Self-tuning Controllers Algorithms, Implementation and Applications". Advanced Textbooks in Control and Signal Processing, ISBN 978-1-85233-980-7. Springer London 2005.

[6]. Clarke, D.W. and P.J. Gawthrop. "Self-tuning control". *Proceedings of IEE*, vol. 126, pp. 633–640, 1979.

[7]. Grimble, M.J. "Weighted minimum-variance self-tuning control. *International Journal of Control*, vol. 36, pp. 597–609, 1982.

[8]. Isermann, R.; K.-H. Lachmann and D. Matko. "Adaptive Control Systems". *Prentice-Hall*, New York, 1992.

[9]. Love, J. "Process Automation Handbook a Guide to Theory and Practice". ISBN 978-1-84628-281-2, Springer London 2007.

[10]. Kharrat, M. ; K. Elleuch ; A. Chaari et M. Kamoun. "Identification et commande auto-ajustable d'une soufflerie électrique". *CD-Rom de la Sixième conférence internationale des Sciences et des Techniques de l'Automatique STA'*2005*, Réf.: STA*05-*CM*-53*,* 19-21 *décembre* 2005, Sousse, 2005.

[11]. Kamoun, S. "Contribution to identification and adaptive control of complex systems" (in French). *PhD Thesis in Automatic Control and Computer Engineering, University of Sfax,* 2003.

[12]. Ljung L., S. Gunnarsson. "Adaptive and Tracking in. System Identification - A Survey". *Automatica*, Vol. 26, No. 1, pp. : 7-21, 1990.

[13]. Li, Z. and R.J. Evans. "Generalized minimum variance control of linear time-varying systems". *IEE Proceedings – Control Theory and Applications*, vol. 149, pp. 111–116, 2002.

[14]. Patete, A.; K. Furuta and M. Tomizuka. "Self-tuning control based on generalized minimum variance criterion for auto-regressive models". *Automatica*, Vol. 44, Issue 8, pp. 1970-1975, August 2008.

[15]. Wieslander, J. and M.B. Zarrop. "Self-Tuning Systems". *Willey & Sons*, New York, 1991.

[16]. Zulfatman and M. F. Rahmat. "Application of Self-Tuning Fuzzy PID Controller on Industrial Hydraulic Actuator Using System Identification Approach". *International Journal On Smart Sensing and Intelligent Systems*. Vol. 2, N°. 2, June 2009.

[17]. Zhou, M.; F. Han. "Adaptive control for EDM process with a self-tuning regulator". *International Journal of Machine Tools and Manufacture*, Vol. 49, Issue 6, pp. 462-469, May 2009.