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Terminal sliding mode feedback linearization control

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Abstract. *This paper presents a terminal sliding model used to synthesis a feedback linearization control. Compared to the conventional sliding mode control basing on a linear surface that guarantees only an asymptotic convergence to the final state, the terminal sliding mode control is characterized by a nonlinear surface assuring fast finite time convergence. Besides, the terminal sliding mode control has the same robustness property to parameter variations and disturbances. A simple method to determine a control is to use a feedback linearization that can be easily applied to multivariable systems. This control is tested in simulation to a 3-degree-of-freedom robot manipulator to prove its effectiveness.*

keywords. *Conventional sliding mode, terminal sliding mode, feedback linearization, robot manipulator.*

1. Introduction

The sliding mode control has been studied for many decades and it is now one of the most active areas of researches on nonlinear system control theory. The sliding mode control is characterized by the choice of a sliding surface describing the desired performances and by the determination of a control law which drives the system state to reach and remain on this surface. An asymptotic convergence to the final state will be achieved in sliding mode.
In order to increase the convergence rate in the sliding mode, the design parameters must be chosen such that the poles of the sliding mode dynamics are far from the origin on the left-half plane of s-plane, bringing as a consequence a controller with a high gain. Considering the saturation of the control input signals in a practical control, this may be undesirable [11][13]. To solve this problem, one applies the terminal sliding mode control which is characterized by a nonlinear sliding surface. This control guarantees the same robustness property to disturbances and uncertainties that conven-

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tional sliding mode control. Terminal sliding mode control offers superior property such as finite time convergence to the desired state [4][5][8].
The paper is organized as follows. In section 2, the principle of the conventional sliding mode control for a second order nonlinear system is presented. After the choice of the sliding surface, the control law can be determined by the equivalent control method. In section 3, a terminal sliding mode control is presented. A nonlinear sliding surface guarantees a finite time convergence to the final state. In section 4, a feedback linearization control is elaborated. In section 5, the terminal sliding mode feedback linearization control law is applied to a 3-degree-of-freedom robot manipulator, and it is compared to the conventional sliding mode linearization control to prove its performance. The paper is finished by a conclusion.

2. Conventional sliding mode control

2.1. Principle

Consider a second order nonlinear dynamical system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + b(x)\,u \end{cases} \tag{1}$$

where $x = [x_1 \quad x_2]^T$ is the system state vector, $f(x)$ and $b(x) \neq 0$ are nonlinear functions of x, and u is the scalar input.
The design of sliding mode control is constituted by two stages: the first consist to choice a sliding surface defined in state space, the second is to determine a discontinuous control law which drives the system state to remain on the sliding surface. The system will be then in sliding regime and its behavior will be described by the surface dynamics. In this case the system is independent of uncertainties and disturbances [3].
In general, the sliding surface is

$$S = x_2 + \beta\, x_1$$

where $\beta > 0$.
To guarantee the existence of sliding mode, the control must satisfy the condition

$$S\,\dot{S} < 0$$

and which in general is given by [9]

$$u = \begin{cases} u^+ & si\ S > 0 \\ u^- & si\ S < 0 \end{cases} \tag{2}$$

To determine the control, one uses in general the equivalent control method.

2.2. Equivalent control method

The control permitting to have an ideal sliding regime in the surface $S = 0$ is named equivalent control noted u_{eq}. It is determined by the equation:

$$\dot{S} = 0$$

however $$\dot{S} = \dot{x}_2 + \beta\dot{x}_1 = f(x) + b(x)\,u_{eq} + \beta\,x_2 = 0$$

then $$u_{eq} = -b^{-1}(x)(f(x) + \beta\,x_2)$$

The equivalent control method is a technique easily applicable to monovariable and multivariable systems in order to determine the system movement in ideal sliding regime.

Replacing the expression of u_{eq} in (1), the system behavior in sliding regime is described by the following equations:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\beta\,x_2 \end{cases}$$

The control u_{eq} have a physical sense : of equations (2) one can deduce [3]

$$u_{\min} = Min\{u^+, u^-\} < u_{eq} < Max\{u^+, u^-\} = u_{\max}$$

one can interpret u_{eq} the mean value of u at the time of the fast commutation between $u_{\max}$ and $u_{\min}$.

In practice, the system state remains on the neighborhood of $S = 0$, the control is then composed by the component u_{eq} low-frequency and a component Δu high frequency which permits to assure the sliding regime and in consequence robustness to parameter variations.

$$u = u_{eq} + \Delta u$$

The discontinuous part of u verifies

$$\Delta u = \begin{cases} \Delta u^+ & si\ S > 0 \\ 0 & si\ S = 0 \\ \Delta u^- & si\ S < 0 \end{cases}$$

in general, Δu has the following expression :

$$\Delta u = -k\ sign(S)$$

where $k > 0$.
The control elaborated guarantees only an asymptotic convergence to the final state. To have a fast convergence, it is sufficient to modify the sliding surface.

3. Terminal sliding mode control

The terminal sliding mode control design is based on a particular choice of the sliding surface and the determination of a control law permitting to drive the system state to remain on this surface. When the representative point of the system movement slide on the surface, a terminal sliding mode is established and a fast finite convergence is guaranteed [12].
In this goal, one defines a nonlinear sliding surface [12][14]:

$$S = x_2 + \beta x_1^{q/p} \qquad (3)$$

Where $\beta > 0$, p and q are positive odd integers verifying $p > q$.
The sufficient condition for the existence of the terminal sliding mode is

$$\frac{1}{2}\frac{dS^2}{dt} < -\eta |S| \qquad (4)$$

where $\eta > 0$.
The condition (4) leads to

$$S\dot{S} < -\eta |S| \qquad (5)$$

differentiating (3) with respect to time and using in (5), one has

$$S(\dot{x}_2 + \beta \frac{q}{p} x_1^{\frac{q}{p}-1} \dot{x}_1) < -\eta |S|$$

Substituting (1) in this inequality we obtain

$$S(f(x) + b(x)u + \beta \frac{q}{p} x_1^{\frac{q}{p}-1} x_2) < -\eta |S|$$

if $S > 0$ then

$$u < b^{-1}(x)(-\eta - f(x) - \beta \frac{q}{p} x_1^{\frac{q}{p}-1} x_2)$$

if $S < 0$ then

$$u > b^{-1}(x)(\eta - f(x) - \beta \frac{q}{p} x_1^{\frac{q}{p}-1} x_2)$$

to verify these two inequalities, it is sufficient to take u in the following expression

$$u = -b^{-1}(x)(f(x) + \beta \frac{q}{p} x_1^{\frac{q}{p}-1} x_2 + (\eta + K)\,\mathrm{sgn}(S))$$

where $K > 0$.
If $S(0) \neq 0$ the state system reaches the sliding mode $S = 0$ in a finite time verifying [12]

$$t_r \leq \frac{|S(0)|}{\eta}$$

When the sliding regime is established, the system dynamic is determined by the following equation

$$\dot{x}_1 = -\beta x_1^{\frac{q}{p}} \qquad (6)$$

and the finite time taken to travel from $x_1(t_r) \neq 0$ to $x_1 = 0$, $x_2 = 0$ is given by [12]

$$t_s = -\beta^{-1} \int_{x_1(t_r)}^{0} \frac{dx_1}{x_1^{\frac{q}{p}}} = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-\frac{q}{p}} \qquad (7)$$

The expression (7) means that in terminal sliding mode (6) the system state x_1 converge to zero in finite time, the same for x_2.
The terminal sliding mode control is characterized, like the conventional sliding mode control, by a strong robustness to uncertainties and disturbances.
The terminal sliding mode technique discussed in the above has been successfully used for the control of nonlinear rigid robotic manipulators and has been tested in the case of the 2-degree-of-freedom [13][14].
The terminal sliding mode or conventional can be used to synthesis feedback linearization which can useful in the dynamic system control.

4. Feedback linearization with terminal sliding mode control

In the goal to control the system describes by (1) and drive its state to desired trajectory defined by $x_d = (x_{1d} \quad x_{2d})^T$ a sliding feedback linearization control is used [6][10].
From the system (1) and with the notation $e = x_d - x$ one get the following system:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = \dot{x}_{2d} - f(x_d - e) - b(x_d - e)\,u \end{cases}$$

To reach the terminal sliding mode, the sliding surface is chosen as:

$$S = e_2 + \beta e_1^{\frac{q}{p}} \tag{8}$$

Let's suppose that this surface is solution of the following equation:

$$\dot{S} + \eta S = -\eta\, w\, \mathrm{sgn}(S) \tag{9}$$

where $\mathrm{sgn}(S) = \begin{cases} +1 & si\ S > 0 \\ 0 & si\ S = 0 \\ -1 & si\ S < 0 \end{cases}$ and $\eta > 0$, $w > 0$

The surface derivative is expressed from (8) by

$$\dot{S} = \dot{e}_2 + \beta \frac{q}{p} e_1^{\frac{q}{p}-1}\, \dot{e}_1$$

let's replace the derivative of S by its expression in equation (9)

$$\dot{e}_2 + \beta \frac{q}{p} e_1^{\frac{q}{p}}\, e_2 = -\eta S - \eta\, w\, \mathrm{sgn}(S)$$

then

$$\dot{e}_2 = -\beta \frac{q}{p} e_1^{\frac{q}{p}}\, e_2 - \eta S - \eta\, w\, \mathrm{sgn}(S)$$

We deduce the terminal sliding mode feedback linearization

$$f(x_d - e) + b(x_d - e)u = \dot{x}_{2d} + \beta \frac{q}{p} e_1^{\frac{q}{p}-1}\, e_2 + \eta(e_2 + \beta e_1^{\frac{q}{p}} + w\,\mathrm{sgn}(e_2 + \beta e_1^{\frac{q}{p}})) \tag{10}$$

Then the control is the solution of the equation (10).

In the case of multivariable systems (m inputs, m outputs), we decouple the system which will be considered like the association of m subsystems. The previous results can be applied to every subsystem. We define then m sliding surfaces

$$S_i = e_{2i} + \beta_i e_{1i}^{\frac{q}{p}} \qquad i = 1,\ldots,m$$

5. Application to a 3 degree-of-freedom robot manipulator

To compare the conventional sliding mode control to terminal sliding mode control, we applied these tow controls to a 3 degree-of-freedom robot.

5.1. Dynamic model of the robot

The dynamic model of the robot, given from the Lagrange equation system, is described by the following matrix equation [2][7]:

$$A(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = U \qquad (11)$$

Where

$q = (q_1 \;\; q_2 \;\; q_3)^T$, $\dot{q} = (\dot{q}_1 \;\; \dot{q}_2 \;\; \dot{q}_3)^T$, $\ddot{q} = (\ddot{q}_1 \;\; \ddot{q}_2 \;\; \ddot{q}_3)^T$ are the angular position vector, the angular velocity vector and the angular acceleration, respectively, $A(q) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$ is symmetric positive definite inertia matrix,

$C(q,\dot{q}) = (C_1 \quad C_2 \quad C_3)^T$ is the vector containing Coriolis and centrifugal forces,

$G(q) = (g_1 \quad g_2 \quad g_3)^T$ is the gravitational torque vector,

and $U = (U_1 \;\; U_2 \;\; U_3)^T$ is the vector of applied joint torque.

The components of the matrix $A(q)$ are:

$$A_{11} = 2b_1 \cos q_2 + 2b_2 \cos(q_2 + q_3) + 2b_3 \cos q_3 + a_1$$
$$A_{12} = A_{21} = b_1 \cos q_2 + b_2 \cos(q_2 + q_3) + 2b_3 \cos q_3 + a_2$$
$$A_{22} = a_2 + 2b_3 \cos q_3$$
$$A_{13} = A_{31} = b_2 \cos(q_2 + q_3) + b_3 \cos q_3 + a_3$$
$$A_{23} = A_{32} = a_3 + b_3 \cos q_3$$
$$A_{33} = a_3$$

where

$$a_1 = J_1 + m_1 L_{c1}^2 + J_2 + m_2(L_1^2 + L_{c2}^2) + J_3 + m_3(L_1^2 + L_2^2 + L_{c3}^2)$$
$$a_2 = J_2 + m_2 L_{c2}^2 + J_3 + m_3(L_2^2 + L_{c3}^2)$$

$$a_3 = J_3 + m_3 L_{c3}^2$$

the components of the vector $C(q,\dot{q})\dot{q}$ are

$$C_1 = -b_1 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2)\sin q_2 - b_2 (2\dot{q}_1 + \dot{q}_2 + \dot{q}_3)(\dot{q}_2 + \dot{q}_3)\sin(q_2 + q_3) - b_3 \dot{q}_3 (2\dot{q}_1 + 2\dot{q}_2 + \dot{q}_3)\sin q_3$$

$$C_2 = b_1 \dot{q}_1^2 \sin q_2 + b_2 \dot{q}_1^2 \sin(q_2 + q_3) - b_3 (2\dot{q}_1 + 2\dot{q}_2 + \dot{q}_3)\dot{q}_3 \sin q_3$$

$$C_3 = b_2 \dot{q}_1^2 \sin(q_2 + q_3) + b_3 (\dot{q}_1 + \dot{q}_2)^2 \sin q_3$$

where

$$b_1 = m_2 L_1 L_{c2} + m_3 L_1 L_2$$

$$b_2 = m_3 L_1 L_{c3}$$

$$b_3 = m_3 L_2 L_{c3}$$

the components of the vector $G(q)$ are

$$G_1 = k_1 \cos(q_1) + k_2 \cos(q_1 + q_2) + k_3 \cos(q_1 + q_2 + q_3)$$

$$G_2 = k_2 \cos(q_1 + q_2) + k_3 \cos(q_1 + q_2 + q_3)$$

$$G_3 = k_3 \cos(q_1 + q_2 + q_3)$$

where

$$k_1 = (m_1 L_{c1} + m_2 L_1 + m_3 L_1) g$$

$$k_2 = (m_2 L_{c2} + m_3 L_2) g$$

$$k_3 = m_3 L_{c3} g$$

5.2. Control law

We desire drive the angular position of the three joints, vector q , to reach the desired positions, vector q_d , in a finite time with zero error.
We decouple the dynamic model robot (11) by writing U in the following expression [1][7] :

$$U = U_{e1} + U_{e2}$$

we assign to U_{e1} the complex part which represents the gravitational Coriolis and centrifugal torque:

$$U_{e1} = C(q,\dot{q})\dot{q} + G(q)$$

conducing to

$$U_{e2} = A(q)\ddot{q}$$

let $$U_{e2} = A(q)V$$

where $V = \begin{pmatrix} V_1 & V_2 & V_3 \end{pmatrix}^T$

we have $$V = \ddot{q}$$

We use terminal sliding mode feedback linearization to synthesize V according to the following diagram [7]:

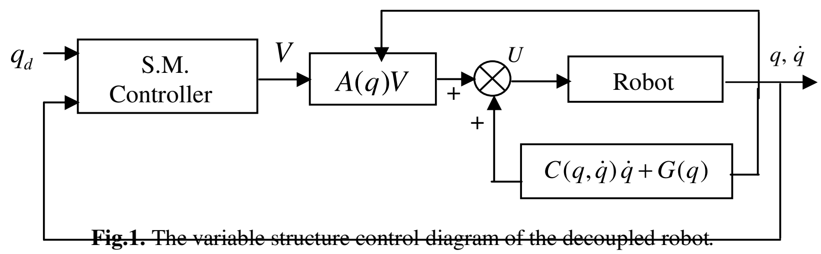

Fig.1. ~~The variable structure control diagram of the decoupled robot.~~

To determine the terminal sliding mode controller, we choice a sliding surface for every joint

$$S_i = \dot{e}_i + \beta_i \, e_i^{\frac{q}{p}} \qquad i = 1, 2, 3$$

where e_i , $i = 1,2,3$ is the error between the desired position and the output of the robot:

$$e = (e_1 \ e_2 \ e_3)^T = q_d - q$$

we impose to S_i the following dynamic

$$\dot{S}_i = -n_i \, (S_i + w_i \, \mathrm{sgn}(S_i)) \qquad i = 1, 2, 3$$

where $n_i > 0$, $w_i > 0$

from the expression of $\dot{S}$ we determine the expression of the components of the vector V :

$$V_i = \ddot{q}_{d\,i} + p_i \frac{q}{p} e_i^{\frac{q}{p}-1} \dot{e}_i + n_i (S_i + w_i \, \mathrm{sgn}(S_i)) \quad i = 1,2,3$$

5.3. Simulation results

The robot is defined by the following parameters [7]:

Table 1. Robot parameter values

i	$J_i\ (Kgm^2)$	$m_i\ (Kg)$	$l_i\ (m)$	$l_{ci}\ (m)$
1	0.12	0.5	0.5	0.25
2	0.25	1	0.5	0.35
3	0.3	0.2	-----	0.15

$g = 9.81\, m / s^2$

For the conventional sliding mode control we take $q = p = 1$ and for the terminal sliding mode control we take $q = 3$ and $p = 5$.

The parameters p_i , n_i and w_i for $i = 1, 2, 3$ are the followings:

Table 2. Controller parameters

i	p_i	n_i	w_i
1	2	3	1
2	2	3	2.3
3	2	10	12

The obtained results in regulation mode for the conventional sliding mode control (CSMC) and the terminal sliding mode control (TSMC) are shown in figures 2 to 7.
These results show that the TSMC presents a faster convergence to the desired state then the CSMC. Indeed the position error reaches zero in a time nearly equal to 3 s for the three joints by CSMC whereas by TSMC this error reach zero in a time of the order of 1 s.
In order to test the robustness of the two control laws, we add a payload to the robot of mass $0.5Kg$ ($2.5m_3$) at the instant 4 s, we obtain the results shown in figures 8 to 13 for the two controls CSMC and TSMC.

The obtained results show that the two controls laws verify the robustness property to disturbances; indeed the presence of the load didn't change the robot angular positions.

The obtained control signals present an elevated frequency and high amplitude's commutation, which is undesirable in practice. To improve these signals of the point of view commutation, we replace the sign functions by saturation functions. The obtained results are shown in figures 14 to 16 in the case of the TSMC. It can be seen that negligible chattering occurs in the control signals.

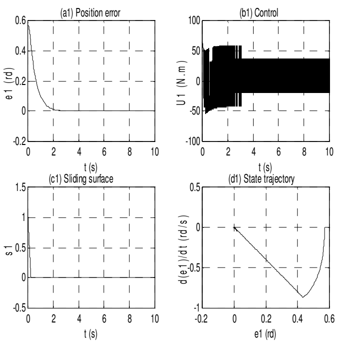

Fig 2. The regulation of the first joint by CSMC

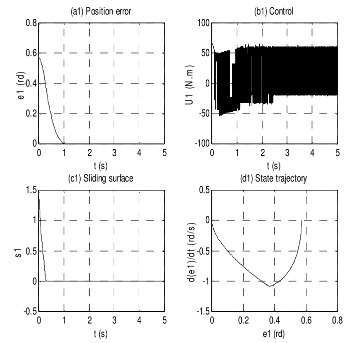

Fig 5. The regulation of the first joint by TSMC

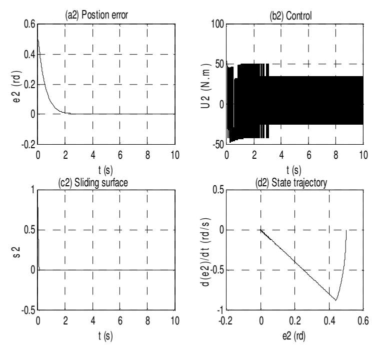

Fig 3. Regulation of *the second joint by CSMC*

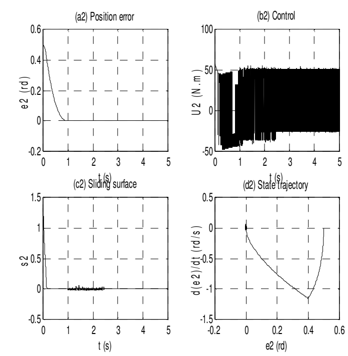

Fig 6. Regulation of the second joint by TSMC

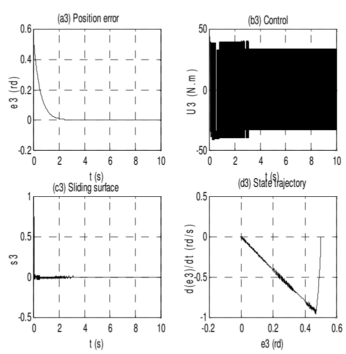

Fig 4. The regulation of the third joint by CSMC

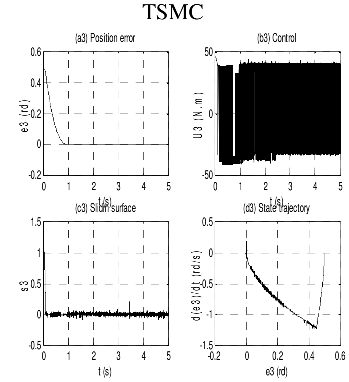

Fig 7. The regulation of the third joint by TSMC

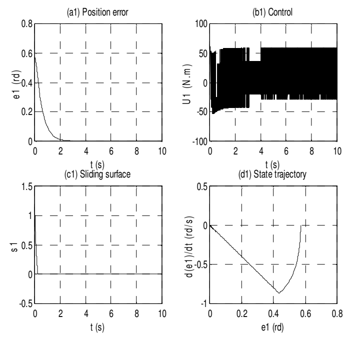

Fig 8. The regulation of the first joint by CSMC with payload

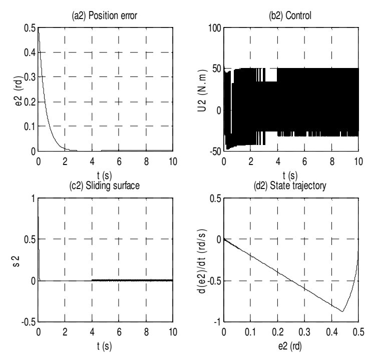

Fig 9. The regulation of he second joint by CSMC with payload

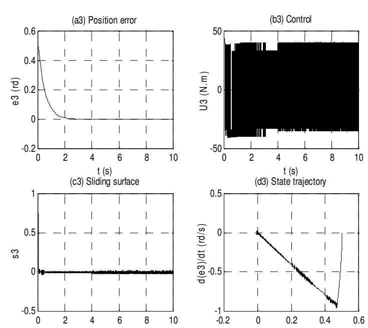

Fig 10. The regulation of the third joint by CSMC with payload

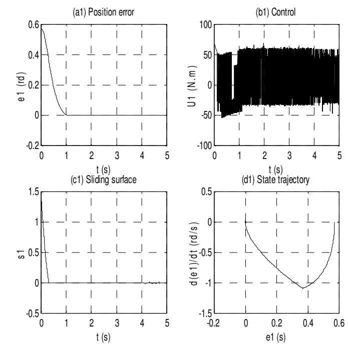

Fig 11. The regulation of the first joint by TSMC with payload

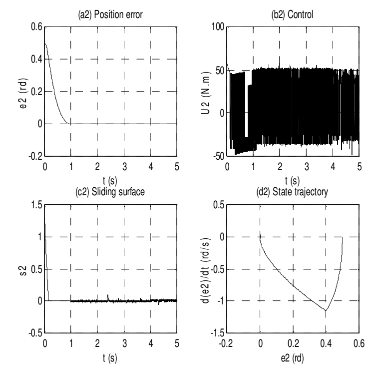

Fig 12. The regulation of the second joint by CSMC with payload

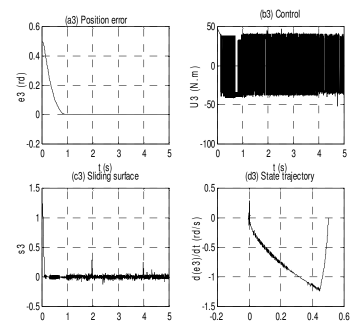

Fig 13. The regulation of the third joint by CSMC with payload

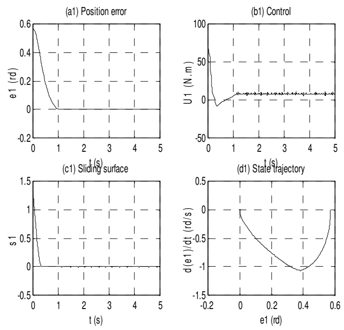

Fig 14. The regulation of the first joint by TSMC with saturation

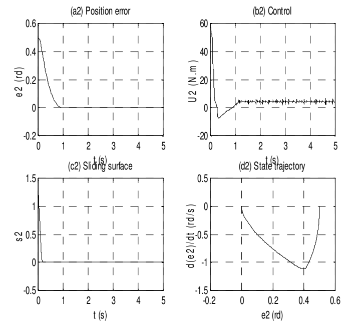

Fig 16. The regulation of the second joint by TSMC with saturation

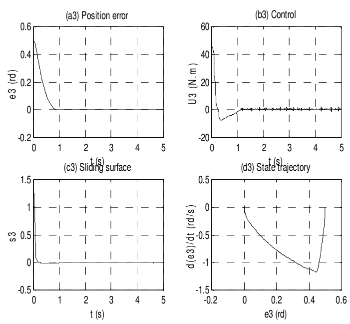

Fig 15. The regulation of the third joint by TSMC with saturation

6. Conclusion

In this paper we have presented the conventional sliding mode control and terminal sliding mode control principles. The terminal sliding mode control is characterized by a nonlinear surface. The terminal sliding mode control can be useful for synthesize of the feedback linearization. The advantage of the terminal sliding mode control compared to the conventional sliding mode is the fast finite time convergence to the final state. In addition, it guarantees the same robustness property to disturbance. These controls are applied to a 3 degree-of-freedom robot manipulator for comparison. We have showed that the terminal sliding mode control guarantees a finite time convergence to the desired state whereas the conventional sliding mode control is slower and the two controls present robustness to disturbances.

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