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Unknown inputs observers for a class of nonlinear systems

M. Triki[1,2], M. Farza[1], T. Maatoug[2], M. M'Saad[1], Y. Koubaa[2],B. Dahhou[3,4]

[1]GREYC, UMR 6072 CNRS, Université de Caen, ENSICAEN
6 Bd Maréchal Juin, 14050 Caen Cedex, France
[2]ENIS, Département de Génie électrique, BP W, 3038 Sfax, Tunisia
[3]Universit´e de Toulouse; UPS, INSA, INP, ISAE; LAAS; F-31077 Toulouse. France
[4]LAAS-CNRS; Universit´e de Toulouse, 7 avenue du Colonel Roche, F-21077

Abstract. *A high gain observer is proposed for a class of multi-output nonlinear systems with unknown inputs in order to simultaneously estimate the whole state as well as the unknown inputs. The gain of this observer does not require the resolution of any dynamical system and is explicitly given. Moreover, its tuning is reduced to the choice of two real numbers. The performances of the proposed observer are demonstrated in simulation through an illustrative example.*

Key words. *Nonlinear system, High gain observer, Unknown inputs*.

1 Introduction

Over the last twenty years, many researches have focused on the observer design for linear systems with unknown inputs [9, 10, 8, 5, 2]. In most cases, the objective was to estimate the non measured state variables and the proposed observers do not provide any information on the unknown inputs. In [1], the authors proposed a LMI based observer in order to jointly estimate the missing states and the unknown inputs. However, strong conditions are assumed to ensure the convergence of the inputs estimates. In a relatively recent work [14], the authors proposed reduced order observers to simultaneously estimate state and the unknown inputs when the latter vary slowly. Other results on unknown observers synthesis for some particular classes of nonlinear systems can be found in [13, 4, 6, 11, 3].
In this paper, one considers a class of uniformly observable MIMO systems involving nonlinear inputs that intervene in a nonlinear manner. Then, one proposes a full order

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high gain observer for the simultaneous estimation of the non measured states and the unknown inputs. The proposed approach does not necessitate the output differentiation and it only assumes that the dynamics of these inputs are bounded without making any hypothesis on how these inputs vary.
This paper is organized as follows. In the next section, the class of nonlinear systems which is the basis of the observer design is introduced. Section 3 is devoted to the observer synthesis. Section 4 is devoted to a simulation example in order to highlight the performance of the proposed observer.

2 Problem formulation

Consider MIMO systems of the form:

$$\begin{cases} \dot{x}(t) = f(u(t), v(t), x(t)) \\ y = \bar{C}x = x^1 \end{cases} \tag{1}$$

$$\text{with } x = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \; ; \; f(u,v,x) = \begin{pmatrix} f^1(u,v,x^1,x^2) \\ f^2(u,v,x^1,x^2,x^3) \\ \vdots \\ f^{q-1}(u,v,x) \\ f^q(u,v,x) \end{pmatrix} ;$$

$$\bar{C} = [I_{n_1}, 0_{n_1\times n_2}, 0_{n_1\times n_3}, \ldots, 0_{n_1\times n_q}] \tag{2}$$

where the state $x \in \mathbb{R}^n$ with $x^k \in \mathbb{R}^{n_k}$, $k = 1, \ldots, q$ and $p = n_1 \geq n_2 \geq \ldots \geq n_q$, $\sum_{k=1}^{q} n_k = n$; the input $w = (u, v) \in \mathcal{W}$ the set of bounded absolutely continuous functions with bounded derivatives from $\mathbb{R}^+$ into W a compact subset of $\mathbb{R}^s$; the output $y \in \mathbb{R}^p$ and $f(u, v, x) \in \mathbb{R}^n$ with $f^k(u, x) \in \mathbb{R}^{n_k}$. One shall suppose that the subvector $u \in U \subset \mathbb{R}^{s-m}$ of the input w is known while the remaining subvector $v \in V \subset \mathbb{R}^m$ is unknown. The objective then consists in synthesizing an observer to simultaneously estimate the vector of unknown inputs $v(t)$ and the non measured states without assuming any model for the unknown inputs. The synthesis of such observer necessitates the adoption of some hypothesis which will be stated in due courses. At this step, one assumes the following:

(H1) The state $x(t)$, the control $u(t)$ and the unknown inputs v are bounded, i.e. $x(t) \in X$, $u(t) \in U$ and $v \in V$ where $X \subset \mathbb{R}^n$, $U \subset \mathbb{R}^{s-m}$ and $V \in \mathbb{R}^m$ are compacts sets.

(H2) There exist $\alpha_f, \beta_f > 0$ such that for all $k \in \{1, \ldots, q-1\}$, $\forall x \in \mathbb{R}^n$, $\forall (u,v) \in U \times V$, $\alpha_f^2 I_{n_{k+1}} \leq \left(\frac{\partial f^k}{\partial x^{k+1}}(u,v,x)\right)^T \frac{\partial f^k}{\partial x^{k+1}}(u,v,x) \leq \beta_f^2 I_{n_{k+1}}$

One also assumes that for $1 \leq k \leq q-1$, for all $(u,v) \in U \times V$, the map $x^{k+1} \mapsto f^k(u,v,x^1,\ldots,x^k,x^{k+1})$ from $\mathbb{R}^{n_{k+1}}$ into $\mathbb{R}^{n_k}$ is one to one.

(H3) The output x^1 can be partitioned as follows: $x^1 = \begin{pmatrix} x_1^1 \\ x_2^1 \end{pmatrix}$ with $x_1^1 \in \mathbb{R}^{m_1}$, $x_2^1 \in \mathbb{R}^{p-m_1}$ and $m \leq m_1 < p$. Such a partition induces the following one $f^1(u,v,x^1,x^2) = \begin{pmatrix} f_1^1(u,v,x^1,x^2) \\ f_2^1(u,v,x^1,x^2) \end{pmatrix}$ that has to satisfy the following two conditions:
(i) There exist $\alpha_v, \beta_v > 0$ such that
$\forall x \in \mathbb{R}^n, \forall (u,v) \in U \times V$ $\alpha_v^2 I_m \leq \left(\frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right)^T \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2) \leq \beta_v^2 I_m$

One also assumes that for all $(u,x^1,x^2) \in U \times \mathbb{R}^{n_1+n_2}$, the map $v \mapsto f_1^1(u,v,x^1,x^2)$ from $\mathbb{R}^m$ into $\mathbb{R}^{m_1}$ is one to one.

(ii) $Rank \begin{pmatrix} \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) & \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2) \\ \frac{\partial f_2^1}{\partial x^2}(u,v,x^1,x^2) & \frac{\partial f_2^1}{\partial v}(u,v,x^1,x^2) \end{pmatrix} = n_2 + m$
for all $(x^1,x^2) \in \mathbb{R}^{n_1+n_2}, \forall (u,v) \in U \times V$

(H4) The time derivative of the unknown input $v(t)$ is a completely unknown function, $\varepsilon(t)$, which is uniformly bounded that is $\sup_{t \geq 0} \|\varepsilon(t)\| \leq \beta_\varepsilon$ where $\beta_\varepsilon > 0$ is a real number.

For clarity purposes, one introduces the following notations:

$$\tilde{x} = \begin{pmatrix} \tilde{x}^1 \\ \tilde{x}^2 \end{pmatrix};\ \tilde{x}^1 = \begin{pmatrix} x_1^1 \\ v \end{pmatrix};\ \tilde{x}^2 = \begin{pmatrix} x_2^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix};\ C_1 = [I_{m_1}\ 0];\ C_2 = [I_{p-m_1}\ 0];$$

$$C = diag(C_1, C_2)\tilde{f}(u,\tilde{x}) = \begin{pmatrix} \tilde{f}^1(u,\tilde{x}) \\ \tilde{f}^2(u,\tilde{x}) \end{pmatrix};\ \tilde{f}^1(u,\tilde{x}) = \begin{pmatrix} f_1^1(u,v,x^1,x^2) \\ 0 \end{pmatrix};$$

$$\tilde{f}^2(u,\tilde{x}) = \begin{pmatrix} f_2^1(u,v,x^1,x^2) \\ f^2(u,v,x^1,x^2,x^3 \\ \vdots \\ f^q(u,v,x)) \end{pmatrix} ; \underline{\varepsilon} = \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix} ; \ \bar{\varepsilon} = \begin{pmatrix} \underline{\varepsilon} \\ 0 \end{pmatrix}$$

Using these notations, system (1) can be written as follows:

$$\begin{cases} \dot{\tilde{x}}^1(t) = \tilde{f}^1(u(t),\tilde{x}(t)) + \underline{\varepsilon}(t) \\ \dot{\tilde{x}}^2(t) = \tilde{f}^2(u(t),\tilde{x}(t)) \\ y = C\tilde{x} = \begin{pmatrix} x_1^1 \\ x_2^1 \end{pmatrix} \end{cases} \tag{3}$$

or equivalently in the more following condensed form

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{f}(u(t),\tilde{x}(t)) + \bar{\varepsilon}(t) \\ y = C\tilde{x} \end{cases} \tag{4}$$

One shall consider the following injective map
$\Phi_1 : R^{n+m} \longrightarrow R^{n+m_1}$

$$\begin{pmatrix} x_1^1 \\ v \\ x_2^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \mapsto \begin{pmatrix} z_1^1 \\ z_2^1 \\ x_2^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \text{ with } \begin{cases} z_1^1 = x_1^1 \\ z_2^1 = f_1^1(u,v,x^1,x^2) \end{cases}$$

According to Assumption (i) of (H3) and using the implicit function Theorem, the unknown input v can be considered as a function of (u,x^1,x^2). Taking the derivative with respect to x^2 of each side of the second equation of (2) gives:

$$0 = \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) + \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\frac{\partial v}{\partial x^2}(u,x^1,x^2)$$

This means that

$$\frac{\partial v}{\partial x^2}(u,x^1,x^2) = -\left(\frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right)^+ \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2)$$

To summarize, the nonlinear system which will be considered with view to observer synthesis can be written under the following form:

$$\begin{cases} \dot{z}_1^1 & = z_2^1 \\ \dot{z}_2^1 & = \frac{\partial f_1^1}{\partial x^1}(u,v,x^1,x^2) f^1(u,v,x^1,x^2) + \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) f^2(u,v,x^1,x^2,x^3) \\ & \quad + \frac{\partial f_1^1}{\partial u}(u,v,x^1,x^2)\dot{u}(t) + \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\varepsilon(t) \\ y_1 = z_1^1 \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_2^1 & = f_2^1(u, v(u,x^1,x^2), x^1, x^2) \\ \dot{x}^2 & = f^2(u, v(u,x^1,x^2), x^1, x^2, x^3) \\ \vdots \\ \dot{x}^q & = f^q(u, v(u,x^1,x^2), x^1, \ldots, x^q) \\ y_2 = x_2^1 \end{cases} \tag{6}$$

Notice that subsystem (5) can be written in the following condensed form:

$$\begin{cases} \dot{z}^1 = A_1 z^1 + \varphi^1(u, \dot{u}, z^1, x^2, x^3) + \underline{\varepsilon}_2(t) \\ y_1 = C_1 z^1 = z_1^1 \end{cases} \tag{7}$$

with $z^1 = \begin{pmatrix} z_1^1 \\ z_2^1 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & I_{m_1} \\ 0 & 0 \end{pmatrix}$ is the $2m_1 \times 2m_1$ anti-shift matrix and $\varphi^1(u, \dot{u}, z^1, x^2, x^3) = \begin{pmatrix} 0 \\ \varphi_2^1(u, \dot{u}, z^1, x^2, x^3) \end{pmatrix} \in \mathbb{R}^{2m_1}$, $\underline{\varepsilon}(t) = \begin{pmatrix} 0 \\ \varepsilon_2(t) \end{pmatrix} \in \mathbb{R}^{2m_1}$ with

$$\begin{aligned} \varphi_2^1(u, \dot{u}, z^1, x^2, x^3) &= \frac{\partial f_1^1}{\partial x^1}(u,v,x^1,x^2) f^1(u,v,x^1,x^2) \\ &\quad + \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) f^2(u,v,x^1,x^2,x^3) + \frac{\partial f_1^1}{\partial u}(u,v,x^1,x^2)\dot{u}(t) \\ \varepsilon_2(t) &= \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\varepsilon(t) \end{aligned} \tag{8}$$

Now, one shall introduce a coordinate transformation which shall put subsystem (6) under an appropriate form for the observer synthesis. Before the presentation of such transformation one shall point out some properties satisfied by this subsystem.

According to Hypothesis (H2), one has

$$\begin{aligned} Rank \frac{\partial \dot{x}^k}{\partial x^{k+1}}(u,v,x) &= Rank \frac{\partial f^k}{\partial x^{k+1}}(u,v,x) \\ &= n_{k+1} \text{ for } k = 2, \ldots, q-1 \end{aligned} \tag{9}$$

Let us show that

$$Rank\frac{\partial \dot{x}_2^1}{\partial x^2}(u,v,x) = n_2 \tag{10}$$

Indeed, on one hand, one has

$$\begin{aligned}
\frac{\partial \dot{x}_2^1}{\partial x^2}(u,v,x) &= \frac{\partial f_2^1}{\partial x^2}(u,v,x^1,x^2) + \frac{\partial f_2^1}{\partial v}(u,v,x^1,x^2)\frac{\partial v}{\partial x^2}(u,x^1,x^2) \\
&= \frac{\partial f_2^1}{\partial x^2}(u,v,x^1,x^2) \\
&\quad -\frac{\partial f_2^1}{\partial v}(u,v,x^1,x^2)\left(\frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right)^+ \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2)
\end{aligned} \tag{11}$$

On the other hand, consider the following full rank matrix,

$$P(u,x) = \begin{pmatrix} I_{m_1} & 0 \\ -\frac{\partial f_2^1}{\partial v}(u,v,x^1,x^2)\left(\frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right)^+ & I_{p-m_1} \end{pmatrix} \tag{12}$$

Since $rank(P(u,x)) = p$ for all $(u,x) \in U \times X$ and $p \geq n_2 + m$ (according to (ii) of (H3)), one has:

$$\begin{aligned}
n_2 + m &= Rank\left(P(u,x)\cdot\begin{pmatrix} \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) & \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2) \\ \frac{\partial f_2^1}{\partial x^2}(u,v,x^1,x^2) & \frac{\partial f_2^1}{\partial v}(u,v,x^1,x^2)\end{pmatrix}\right) \\
&= Rank\begin{pmatrix} \frac{\partial f_1^1}{\partial x^2}(u,v,x^1,x^2) & \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2) \\ \frac{\partial \dot{x}_2^1}{\partial x^2}(u,v,x) & 0\end{pmatrix} \\
&= Rank\left(\frac{\partial \dot{x}_2^1}{\partial x^2}(u,v,x)\right) + Rank\left(\frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right) \\
&= Rank\left(\frac{\partial \dot{x}_2^1}{\partial x^2}(u,v,x)\right) + m
\end{aligned}$$

This leads to (10).

Now, since properties (9) and (10) are satisfied by subsystem (6), one can consider the transformation introduced in [7, 4] which puts (6) under a canonical form that characterized a subclass of uniformly observable systems.

Before giving this transformation and for clarity purposes, one introduces the following notation:

$$\bar{f}^1(u,x^1,x^2) = f_2^1(u,v(u,x^1,x^2),x^1,x^2) \tag{13}$$

Now, consider the following injective map

$$\Phi_2 : \mathbb{R}^{n+m} \to \mathbb{R}^{2m_1+q(p-m_1)}$$
$$\begin{pmatrix} z_1^1 \\ z_2^1 \\ x_2^1 \\ x^2 \\ \vdots \\ x^q \end{pmatrix} \mapsto \begin{pmatrix} z_1^1 \\ z_2^1 \\ z_1^2 \\ z_2^2 \\ \vdots \\ z_q^2 \end{pmatrix}$$

with

$$\begin{cases} z_1^2 = x_2^1 \\ z_2^2 = \bar{f}^1(u, x^1, x^2) \\ z_k^2 = \dfrac{\partial \bar{f}^1}{\partial x^2}(u, x^1, x^2) \displaystyle\prod_{i=1}^{k-2} \dfrac{\partial f^i}{\partial x^{i+1}}(u, v, x^1, \ldots, x^{i+1}) f^{k-1}(u, v, x^1, \ldots, x^{k+1}) \\ \qquad \text{for } k = 3, \ldots, q \end{cases} \tag{14}$$

One can show (see [7, 4] for more details) that the transformation Φ_2 puts subsystem (6) under the following form:

$$\begin{cases} \dot{z}^2 = A_2 z^2 + \varphi^2(u, \dot{u}, z^1, z^2) \\ y_2 = C_2 z^2 = z_1^2 \end{cases} \tag{15}$$

with $z^2 = \begin{pmatrix} z_1^2 \\ \vdots \\ z_q^2 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & I_{q(p-m_1)} \\ 0 & 0 \end{pmatrix}$ is the $q(p-m_1) \times q(p-m_1)$ anti-shift matrix, $C_2 = [I_{q(p-m_1)}\ 0]$ is $(p - m_1) \times q(p - m_1)$ rectangular matrix and $\varphi^2(u, \dot{u}, z^1, z^2)$ is triangular with respect to z^2, i.e. $\varphi^2(u, \dot{u}, z^1, z^2) = \begin{pmatrix} \varphi_1^2(u, \dot{u}, z^1, z_1^2) \\ \varphi_2^2(u, \dot{u}, z^1, z_1^2, z_2^2) \\ \vdots \\ \varphi_{q-1}^2(u, \dot{u}, z^1, z_1^2, \ldots, z_{q-1}^2) \\ \varphi_q^2(u, \dot{u}, z^1, z^2) \end{pmatrix}$.

3 Observer synthesis

Consider again the overall system constituted by system (7) and (15), i.e.

$$\begin{cases} \dot{z}^1 = A_1 z^1 + \varphi^1(u, \dot{u}, z^1) + \underline{\varepsilon}(t) \\ y_1 = C_1 z^1 = z_1^1 \end{cases}$$
$$\begin{cases} \dot{z}^2 = A_2 z^2 + \varphi^2(u, \dot{u}, z^1, z^2) \\ y_2 = C_2 z^2 = z_1^2 \end{cases} \tag{16}$$

which can also be written in the more following condensed form:

$$\dot{z} = Az + \varphi(u, \dot{u}, z) + \bar{\varepsilon}(t) \tag{17}$$

where

$$z = \begin{pmatrix} z^1 \\ z^2 \end{pmatrix};\ \varphi(u, \dot{u}, z) = \begin{pmatrix} \varphi^1(u, \dot{u}, z) \\ \varphi^2(u, \dot{u}, z) \end{pmatrix};\ \bar{\varepsilon}(t) = \begin{pmatrix} 0 \\ \underline{\varepsilon}(t) \end{pmatrix};$$

In the case where $\bar{\varepsilon} = 0$, system (16) is under a form introduced in [12] that characterized a large class of uniformly observable systems. As a result, one can use the observer proposed in [12] in order to estimate z^1 and z^2. However, due to the presence of ε, one can show that the estimation error does not converge to zero but can be made as small as desired. The equation of the observer specialize as follows:

$$\begin{cases} \dot{\hat{z}}^1 = A_1\hat{z}^1 + \varphi^1(u, \dot{u}, \hat{z}^1) - \Delta_1^{-1}(\theta)K_1(C_1\hat{z}^1 - y_1) \\ \dot{\hat{z}}^2 = A_2\hat{z}^2 + \varphi^2(u, \dot{u}, \hat{z}^1, \hat{z}^2) - \Delta_2^{-1}(\theta)K_2(C_2\hat{z}^2 - y_2) \end{cases} \tag{18}$$

where

- $\hat{z}^1 = \begin{pmatrix} \hat{z}_1^1 \\ \hat{z}_2^1 \end{pmatrix} \in \mathbb{R}^{2m_1}$, $\hat{z}_1^1, \hat{z}_2^1 \in \mathbb{R}^{m_1}$, $\hat{z}^2 = \begin{pmatrix} \hat{z}_1^2 \\ \hat{z}_2^2 \\ \vdots \\ \hat{z}_q^2 \end{pmatrix} \in \mathbb{R}^{q(p-m_1)}$, $\hat{z}_i^2 \in \mathbb{R}^{p-m_1}$
- $\hat{z}_1^k = z_1^k$ (output injection) for $k = 1, 2$.
- $K_1 = \begin{pmatrix} 2I_{m_1} \\ I_{m_1} \end{pmatrix}$, $K_2 = \begin{pmatrix} C_q^1 I_{(p-m_1)} \\ C_q^2 I_{(p-m_1)} \\ \vdots \\ C_q^q I_{(p-m_1)} \end{pmatrix}$ with $C_q^k = \frac{q!}{k!(q-k)!}$, $k = 1, \ldots, q$.
- $\Delta_1(\theta) = diag(\dfrac{1}{\theta^{q-1}} I_{m_1}, \dfrac{1}{\theta^{2(q-1)}} I_{m_1})$ and $\Delta_2(\theta) = diag(\dfrac{1}{\theta} I_{p-m_1}, \dfrac{1}{\theta^2}, \ldots, \dfrac{1}{\theta^q} I_{p-m_1})$ where $\theta > 0$ is the observer design parameter.

Observer (18) can be written in the following more condensed form:

$$\dot{\hat{z}} = A\hat{z} + \varphi(u, \dot{u}, \hat{z}) - \Delta^{-1}(\theta)K(C\hat{z} - y) \tag{19}$$

where

$$z = \begin{pmatrix} \hat{z}^1 \\ \hat{z}^2 \end{pmatrix};\ \varphi(u, \dot{u}, \hat{z}) = \begin{pmatrix} \varphi^1(u, \dot{u}, \hat{z}) \\ \varphi^2(u, \dot{u}, \hat{z}) \end{pmatrix}$$

$A = diag(A_1, A_2);\ K = diag(K_1, K_2);\ C = diag(C_1, C_2);\ \Delta(\theta) = diag(\Delta_1(\theta), \Delta_2(\theta));$

Since, the injective map $\Phi_2 \circ \Phi_1$ brought system (4) under form form (17), observer (19) can be written in the original coordinates as follows:

$$\dot{\hat{x}}(t) = \tilde{f}(u(t), \hat{x}(t)) - \left(\frac{\partial \Phi_2}{\partial x}(u,v,x).\frac{\partial \Phi_1}{\partial x}(u,v,x)\right)^+ \Delta^{-1}(\theta)K(C\hat{z}-y) \quad (20)$$

However, according to the particular structure of the jacobian transformation, $\frac{\partial \Phi_2}{\partial x}(u,v,x).\frac{\partial \Phi_1}{\partial x}(u,v,x)$, one shall exhibit in the sequel another observer which does not need to compute the inverse of the overall jacobian transformation but only the inverse of the diagonal of this jacobian. Indeed, let us consider the following two bloc diagonal matrices:

$$\Lambda_1 = diag\left(I_{m_1}, \frac{\partial f_1^1}{\partial v}(u,v,x^1,x^2)\right)$$

$$\Lambda_2 = diag\left(I_{p-m_1}, \frac{\partial \bar{f}^1}{\partial x^2}(u,x), \frac{\partial \bar{f}^1}{\partial x^2}(u,x)\frac{\partial f^2}{\partial x^2}(u,v,x), \ldots, \frac{\partial \bar{f}^1}{\partial x^2}(u,x)\prod_{k=1}^{q-1}\frac{\partial f^k}{\partial x^{k+1}}(u,v,x)\right)$$

$$\Lambda = diag(\Lambda_1, \Lambda_2)$$

One has :

$$\begin{aligned}\frac{\partial \Phi_2}{\partial x}(u,v,x).\frac{\partial \Phi_1}{\partial x}(u,v,x) &= \Lambda(u,v,x) + \left(\frac{\partial \Phi_2}{\partial x}(u,v,x).\frac{\partial \Phi_1}{\partial x}(u,v,x) - \Lambda(u,v,x)\right) \\ &\triangleq \Lambda(u,v,x) + \Gamma(u,v,x) \qquad (21)\end{aligned}$$

with $\Gamma(u,v,x) \triangleq \left(\frac{\partial \Phi_2}{\partial x}(u,v,x).\frac{\partial \Phi_1}{\partial x}(u,v,x) - \Lambda(u,v,x)\right)$.

One can show that the matrix Γ is lower triangular matrix with a main diagonal equal to zero. As a result, one can use the following observer in the original coordinates [4]:

$$\dot{\hat{x}}(t) = \tilde{f}(u(t), \hat{x}(t)) - (\Lambda(u,v,x))^+ \Delta^{-1}(\theta)K(C\hat{z}-y) \quad (22)$$

Observer (22) can be written in the following developed form:

$$\begin{cases} \dot{\hat{x}}_1^1(t) = f(u(t), \hat{v}(t), x^1(t), \hat{x^2}) - 2\theta^{q-1}(\hat{x}_1^1 - x_1^1) \\ \dot{\hat{v}} \quad = -\theta^{2(q-1)}\frac{\partial f_1^1}{\partial v}(u, \hat{v}, x^1, \hat{x}^2)(\hat{x}_1^1 - x_1^1) \end{cases} \quad (23)$$

$$\begin{cases} \begin{pmatrix} \dot{\hat{x}}_2^1(t) \\ \dot{\hat{x}}^2(t) \\ \vdots \\ \dot{\hat{x}}^q(t) \end{pmatrix} = \begin{pmatrix} f_2^1(u, \hat{v}, x^1, \hat{x}^2) \\ f^2(u, \hat{v}, x^1, \hat{x}^2, \hat{x}^3) \\ \vdots \\ f^q(u, \hat{v}, x^1, \hat{x}^2, \ldots, \hat{x}^q) \end{pmatrix} \\ \qquad -\Lambda_2(u, \hat{v}, x^1, \hat{x^2}, \ldots, \hat{x}^q)\Delta_2^{-1}(\theta)K_2(C_2\hat{x}_2^1 - y_2) \end{cases} \quad (24)$$

4 Example

Consider the following dynamical system:

$$\begin{cases} \dot{x}_1 = x_2x_3 + v^3 + v - tanh(x_1) - x_1^3 \\ \dot{x}_2 = -x_3 + x_2v - x_2 \\ \dot{x}_3 = x_4 - x_3 + \frac{v}{1+v^2} \\ \dot{x}_4 = -cos(x_4) + vsin(v) \\ y \ = [x_1 \ x_2]^T \end{cases} \tag{25}$$

where $x = [x_1 \ x_2 \ x_3 \ x_4]^T \in \mathbb{R}^4$ with $x_i \in \mathbb{R}$, $v(t)$ is the unknown input. To simplify, no known input has been considered. For simulation purposes, the following expression (unknown by the observer) has been used for the unknown input:

$$v(t) = cos(t) \tag{26}$$

It is easy to see that system (25) is under form (1) with:

$$x^1 = [x_1 \ x_2]^T; \ x^2 = x_3; \ x^3 = x_4;$$

Concerning the partition of x^1 needed in hypothesis (H4), one can consider the following one (the only possible partition in this example): $x_1^1 = x_1$ and $x_2^1 = x_2$. Now, one can easily check hypotheses (H1) to (H4) and an observer under form (23-24) can be used in order to achieve the required estimations.

4.1 simulation Results

An observer of the form (23-24) has been used in order to estimate x_3, x_4 and v. This observer has been simulated using data issued from simulation. In figure 1, the true time evolutions of x_3, x_4 and v (issued from model simulation) are compared with their respective estimates provided by the observer. Notice that corresponding curves are almost superimposed. The employed values of θ is equal to 20. The initial conditions for the model and the observer are: $x_1(0) = \hat{x}_1(0) = -1$; $x_2(0) = \hat{x}_2(0) = 2$; $x_3(0) = -1$; $x_4(0) = -1$; $\hat{x}_3(0) = \hat{x}_4(0) = \hat{v}(0) = 0$.

The obtained results clearly show the good agreement between the estimated and simulated variables. Recall that the expression of the unknown input (equation (26)) introduced for simulation purposes is ignored by the observer.

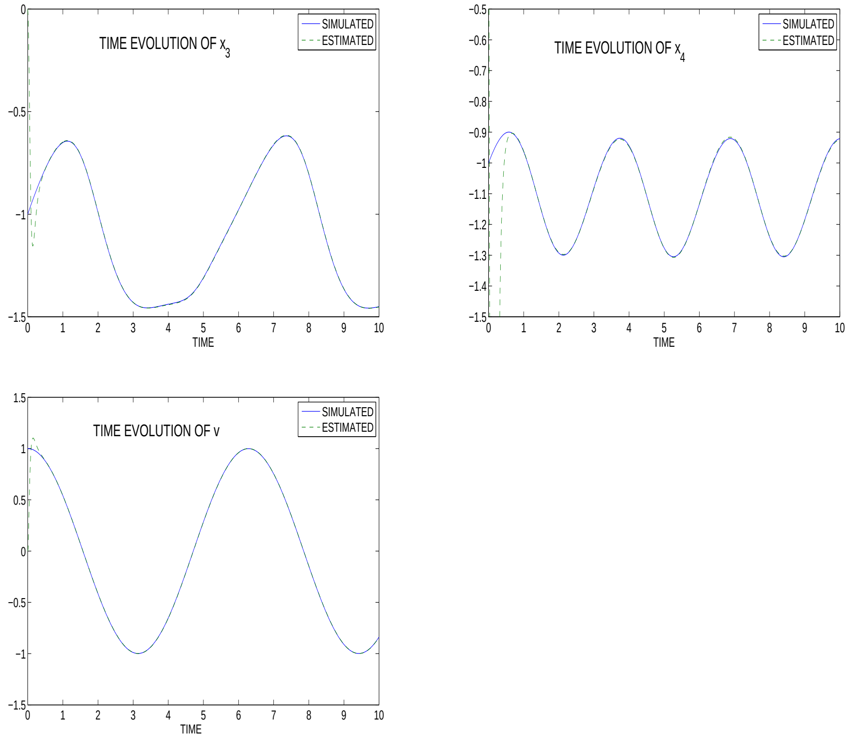

Fig. 1. Estimation of x_3, x_4 and v

Conclusion: A class of nonlinear systems which is nonlinearly parameterized with respect to unknown inputs has been considered with view to observer design. Under appropriate sufficient conditions that ensure the observability of all the state as well as that of the unknown inputs, a high gain observer was synthesized in order to simultaneously estimate the whole state and the the unknown inputs. Simulation results was given in order to highlighted the the theory.

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