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Multiple harmonic disturbances rejection using a new supervision procedure

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Abstract. This paper presents a new method based on the multimodel approach for rejection of harmonic disturbances with time varying frequency and phase drift. This method combines a supervision procedure and a Magnitude Phase Locked Loop scheme, to design a local partial state model reference controllers that achieve an asymptotic disturbance rejection. These local controllers are synthesized for a given frequency, using the internal model principle. The effectiveness of this proposed multiple disturbances rejection is shown through a simulation examples.

Keywords: Multiple harmonic disturbances rejection, partial state model reference controllers, multimodel approach, Magnitude Phase Locked Loop scheme, supervision procedure.

1 Introduction

Considerable attention has been drawn to the problem of the rejection of harmonic disturbances with unknown and possibly time varying frequency in many engineering applications, namely process control, vibration monitoring and fault detection. Indeed the estimation and reconstruction of unknown disturbances are crucial and more particularly when their underlying dynamics are time varying as it has been pointed out in several contributions, namely ([4], [3], [8]). The available potential solutions are based on the knowledge of a suitable model of the disturbances. When the disturbances model is known, an asymptotic disturbance rejection can be performed using the internal model principle as in the indirect method proposed in ([2], [1]). Recall that the internal model principle consists in simply incorporating the poles of the disturbances model into the regulator pole configuration ([12]). When the disturbances model is not completely known or involves time varying dynamics, many approaches are developed to achieve asymptotic disturbances rejection ([13]) and further developed in ([10], [11], [7], [14]).
Recently, the multimodel approach is proposed and developed for rejection of

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multiple harmonic disturbances ([6]). This approach allows to incorporate a suitable adaptation alertness into the control design to deal with the frequency variations of the harmonic disturbance under consideration. It particularly, consists in designing a controller with a robust disturbances rejection capability for each single frequency composing the harmonic disturbances. Indeed, a set of partial state model reference controllers, which are designed according to the internal model principle with an adequate Q-parametrization, are appropriately combined to yield a robust global controller ensuring an asymptotic disturbances rejection.

In this work, we present a further multimodel development which not only overcomes the problem of time varying frequency ([5], [4], [11]), but also considers the problem of the disturbance phase drift. Seeing that, theses disturbance parameters variations can distort the performance criteria adopted in the multimodel approach. The motivation of this paper is twofold. Firstly the problem of disturbance with, simultaneously, time varying frequency and magnitude is investigated. Secondly, the proposed multimodel approach is based on modified performance criterion. This criterion depends on the harmonic disturbance estimated period generated from a Magnitude Phase Locked Loop scheme. Indeed, a set of local partial state model reference controllers are built using the internal model principle. The stabilizer controller is properly designed, for a given period, by a suitable supervision procedure.

This paper is organized as follows. Section 2 describes the multimodel proposed approach for multiple disturbance rejection using the internal model principle. Section 3 is devoted to the modified multimodel approach which combines a Magnitude Phase Locked Loop algorithm with a new supervision procedure to perform an admissible multiple harmonic disturbances rejection. Finally, the performances of the proposed method are illustrated through simulation example. A concluding remarks end the paper.

2 The multimodel approach for disturbance rejection

We consider the following system:

$$y(k) = q^{-d-1}\frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{1}{A(q^{-1})}p(k) \tag{1}$$

where:

$$p(k) = H(q^{-1})\delta(k)$$

$u(k)$ and $y(k)$ are respectively the input and the output of the system. $p(k)$ is the periodic disturbance and $\delta(k)$ is the unit Dirac impulse.

The disturbance transfer function $H(q^{-1})$ can be written as:

$$H(q^{-1}) = \frac{C(q^{-1})}{D(q^{-1})}$$

The problem of disturbance rejection is studied through the following assumptions ([10]):

1. *The polynomials $A(q^{-1})$, $D(q^{-1})$, $B(q^{-1})$ et $C(q^{-1})$ have the following forme:*

$$X(q^{-1}) = x_0 + x_1 q^{-1} + ... + x_{nx} q - nx$$

where $A(q^{-1})$ et $D(q^{-1})$ are monic.

2. *The transfer function : $q^{-d-1}\frac{B(q^{-1})}{A(q^{-1})}$ is known or obtained by system identification.*

3. *The transfer function $H(q^{-1})$ is Lyapunov stable. The roots of $D(q^{-1})$ are located on the unit circle.*

4. *The polynomials $B(q^{-1})$ and $D(q^{-1})$ are coprime.*

The controller to be designed is a RS-type polynomial.
Refferring to this control structure we can write:

$$S_0(q^{-1})u(k) + R_0(q^{-1})y(k) = P_{c0}(q^{-1})\beta y^*(k+d+1) \tag{2}$$

$$\beta = \frac{1}{\sum_{i=0}^{n_B} b_i}$$

n_B is the degree of the polynomial $B(q^{-1})$.

$S_0(q^{-1})$ and $R_0(q^{-1})$ are polynomials characterizing the controller. Let $P_{c0}(q^{-1})$ the polynomial defining the desired dynamics in closed loop:

$$P_{c0}(q^{-1}) = A(q^{-1})S_0(q^{-1}) + q^{-d-1}B(q^{-1})R_0(q^{-1}) \tag{3}$$

Where: $S_0(q^{-1})$ and $R_0(q^{-1})$ are the particular solutions of the diophantien equation given by (3).
The system given by the equation (1) can be described by the behavior of its input-output tracking errors $e_u(k)$ and $e_y(k)$ given as follow:

$$e_u(k) = u(k) - A(q^{-1})\beta y^*(k+d+1) = -\frac{R_0(q^{-1})}{P_{c0}(q^{-1})}p(k) \tag{4}$$

$$e_y(k) = y(k) - B(q^{-1})\beta y^*(k) = \frac{S_0(q^{-1})}{P_{c0}(q^{-1})}p(k) \tag{5}$$

Using the "Youla-Kucera" parametrization ([10], [11], [7]), the structure of the stabilizing controller has the form :

$$S(q^{-1}) = S_0(q^{-1}) - q^{-d-1}Q(q^{-1})B(q^{-1}) \tag{6}$$

$$R(q^{-1}) = R_0(q^{-1}) + Q(q^{-1})A(q^{-1}) \tag{7}$$

with $Q(q^{-1})$ is the "Youla-Kucera" parameter written as:

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + ... + q_{n_Q} q^{-n_Q}$$

The controller given by (6) and (7) achieves asymptotic disturbance rejection provided that the polynomial $S(q^{-1})$ has the form:

$$S(q^{-1}) = M(q^{-1})D(q^{-1})$$

Using this last expression the equation(6) can be written as:

$$S_0(q^{-1}) = M(q^{-1})D(q^{-1}) + q^{-d-1}B(q^{-1})Q(q^{-1}) \tag{8}$$

To determine $Q(q^{-1})$ in order that the controller incorporates the internal model of the disturbance we need to solve the diophantien equation (8). Where $M(q^{-1})$ and $Q(q^{-1})$ correspond to the solution of the diophantien equation (8). Indeed, if a solution for the equation (8) exists, the stabilizing controller given by the equations (6) and (7) satisfies the following condition:

$$\lim_{k\to\infty} e_y(k) = 0.$$

Equation (8) has a unique solution for $M(q^{-1})$ and $Q(q^{-1})$ with a degree n_M and n_Q given as follow:

$$n_M = n_B + d \quad \text{et} \quad n_Q = n_D - 1$$

where n_D is the degree of the polynomial $D(q^{-1})$.
The structure of the parametrization of the stabilizing controller is given on figure 1.

Indeed the disturbance $p(k)$ with multiple parameters: multiple frequencies (periods) and magnitudes entering the loop is decomposed into a disturbance $p_i(k)$ with single and corresponding period $T_i(k)$ and magnitude $A_{pi}(k)$. For each disturbance $p_i(k)$, an asymptotic local controller capable to perform asymptotic disturbance rejection is calculated.
Each local controller is computed using the internal model principle as follow:

$$\begin{aligned} S_i(q^{-1}) &= S_0(q^{-1}) - q^{-d-1}Q_i(q^{-1})B(q^{-1}) \\ R_i(q^{-1}) &= R_0(q^{-1}) + Q_i(q^{-1})A(q^{-1}) \end{aligned} \tag{9}$$

where the polynomial $Q_i(q^{-1})$ is computed, for a given period $T_i(k)$ by solving the following diophantien equation:

$$S_0(q^{-1}) = M(q^{-1})D_i(q^{-1}) + q^{-d-1}B(q^{-1})Q(q^{-1}) \tag{10}$$

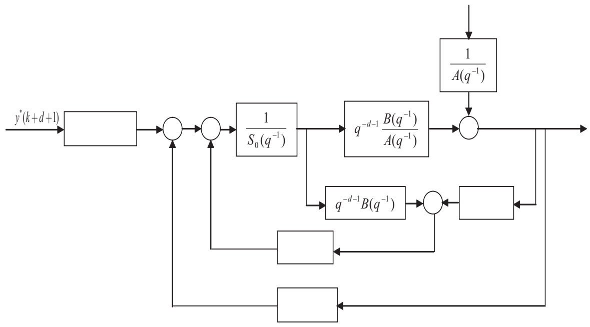

Fig. 1. Structure of the stabilizing controller parametrization.

The local partial model reference controller is given by the following equation:

$$S_i(q^{-1})u_i(k) + R_i(q^{-1})y(k) = P_{c0}(q^{-1})\beta y^*(k+d+1)$$

For each elementary period $T_i(k)$, there is one controller $(S_i(q^{-1}), R_i(q^{-1}))$ that satisfies the control objective is selected according to a supervision procedure. This last uses a switching rule based on the minimizing of the following performance criterion:

$$J_i(k) = \alpha\varepsilon_i^2(k) + \beta\sum_{j=1}^{k} e^{-\lambda(k-j)}\varepsilon_i^2(k) \qquad i = 1..N \tag{11}$$

with α, β and λ are positive tuning parameters.λ is a forgetting factor which also assures the boundedness of the criterion.
If we choose a large value for these parameters, we will obtain a very quick response to the abrupt parameter changing but a bad response with respect to disturbances. It means that, an output disturbance will generate an unwanted switching to another controller which result in a poor control. Contrary, a small value will reduce the number of unwanted switching but lead to a slow response with respect to the parameter variation ([9]).

In the proposed multimodel strategy ([6]), the error $\varepsilon_i(k)$ criterion depends on the disturbance $p(k)$ and the local disturbances for a given period T_i as following:

$$\varepsilon_i(k) = p(k) - p_i(k)$$

This method was shown to provide an asymptotic disturbance rejection in presence of time varying disturbance frequency. Otherwise, to study the effectiveness of this approach where not only the frequency of the disturbance is variable but also its magnitude and phase, a simulation example is carried out.

2.1 Simulation results

The disturbance rejection of unknown and simultaneously, time varying parameters; phase, magnitude and frequency using the multimodel approach based on the internal model principle, as synthesized in the previous section is illustrated in the case of discrete second order system described by the following equation:

$$y(k) = 0.74y_p(k-1) + 0.35y(k-2) + \\ 0.95u(k-1) + 0.75u(k-2) + p(k) \tag{12}$$

The following closed loop characteristic polynomial $P_{c0}(q^{-1})$ is considered:

$$P_{c0}(q^{-1}) = 1 + 1.339q^{-1} + 1.0635q^{-2} + 0.4987q^{-3} + 0.1298q^{-4}$$

The considered disturbance $p(k)$ applied on the output of the system have the form given by the following equation:

$$p(k) = \begin{cases} 0.6sin(\frac{2\Pi}{60}k) \quad ; \quad k < 1000 \\ 0.8sin(\frac{2\Pi}{40}k + \frac{2\Pi}{3}) \quad ; \quad 1000 \leq k < 2000 \\ 0.5sin(\frac{2\Pi}{80}k) \quad ; \quad k \geq 2000 \end{cases} \tag{13}$$

The disturbance periods $T_i(k)$ considered in this synthesis stage are :

$$T_i(k) = 60, 40, 80 \quad , \qquad i = 1..3$$

It's clear that the disturbance between the samples 1000 and 2000, has a period $T_2(k) = 40$ but it presents a phase drift equals to $\frac{2\Pi}{3}$.
The switching parameters are choosing as following:

$$\alpha = 0.0001; \qquad \beta = 0.0001; \quad \lambda = 0.09$$

Figure 2 gives the evolution of the plant output. It can be seen, that the proposed supervision procedure was unable to achieve asymptotic disturbance rejection, namely when a phase drift or magnitude distortion in any elementary disturbance $p_i(k)$. This was proved mainly by the switching evolution between the partial controllers (figure 4), in the presence of a phase drift in any elementary disturbance, the supervision procedure failed to design the corresponding controller that ensure a perfect disturbance rejection.

To improve the multimodel approach, it is therefore, appropriate to modify the supervision procedure that considers, for the decision, the error between the partial disturbances $p_i(k)$ and the real one.

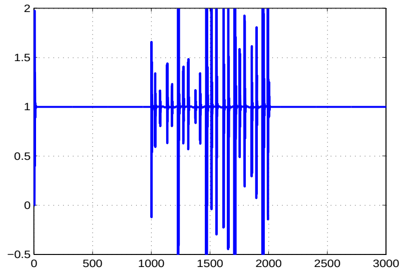

Fig. 2. Evolution of the plant output.

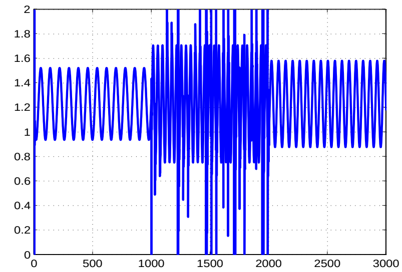

Fig. 3. Evolution of the plant input.

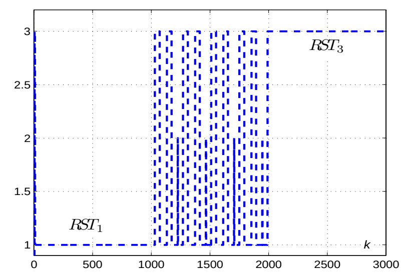

Fig. 4. Evolution of the controllers switching law.

3 Rejection of harmonic disturbance with time varying parameters using a modified supervision procedure

The designed controller is performed using a frequency estimator based on Magnitude Phase Locked Loop approach, to provide asymptotic disturbance rejection in presence of time varying disturbance frequency and magnitude. This MPLL scheme is exploited by an appropriate multimodel based new suitable supervision procedure to deduce simple and powerful local controllers that would perform well in the presence of time varying disturbance parameters. The following section emphasizes the MPLL principle.

3.1 The MPLL principle

The Magnitude Phase Locked Loop concept was proposed in ([4]). The magnitude and the phase of an incoming harmonic disturbance can, simultaneously, estimated using this algorithm. The estimated disturbance period is deduced directly from the estimated phase.
The algorithm principle can be summarized as follows :

The harmonic disturbance $p(k)$ is given by the following equation:

$$p(k) = A_p(k)cos(\phi(k)) \tag{14}$$

where $A_p(k)$ and $\phi(k)$ are respectively the magnitude and the phase of the disturbance. The estimated disturbance is designed as following:

$$\hat{p}(k) = \hat{A}_p(k)cos(\hat{\phi}(k)) \tag{15}$$

with $\hat{A}_p(k)$ and $\hat{\phi}(k)$ are respectively the estimated magnitude and phase of the disturbance.
The estimated frequency, can be built using the estimated phase as:

$$\hat{w}(k) = \hat{\phi}(k+1) - \hat{\phi}(k) = (q-1)\hat{\phi}(k) \tag{16}$$

The estimated period of the harmonic disturbance is obtained by:

$$\hat{T}(k) = \frac{2\Pi}{\hat{w}(k)} \tag{17}$$

The equation (16) can be written as:

$$\frac{\hat{w}(k)}{\hat{\phi}(k)} = (q-1) = \frac{1-q^{-1}}{q^{-1}} \tag{18}$$

This equation presents the transfer function of a closed loop system having as input $\hat{\phi}(k)$ and as output $\hat{w}(k)$. This transfer function is modified to insure system stability and a good performance:

$$\frac{\hat{w}(k)}{\hat{\phi}(k)} = \frac{1-q^{-1}}{K_w(1-N_w q^{-1})} \tag{19}$$

with:

$$N_w = 1 - \frac{1}{K_w}$$

The suitable choice of the parameter K_w leads to:

$$|N_w| < 1$$

The estimation of the harmonic disturbance is achieved by the minimizing of the following criteria:

$$J = E[(p(k) - \hat{p}(k))^2] = E[(e_p(k))^2] \tag{20}$$

where $e_p(k)$ is the estimation error defined as:

$$e_p(k) = p(k) - \hat{p}(k) = A_p(k)cos(\phi(k)) - \hat{A}_p(k)cos(\hat{\phi}(k)) \tag{21}$$

The estimated disturbance magnitude and frequency are carried out using the gradient algorithm, given by the following equations:

$$\hat{A}_p(k) = \hat{A}_p(k-1) + 2g_a\ e_p(k-1)cos(\hat{\phi}(k)) \tag{22}$$

$$\hat{w}(k) = \hat{w}(k-1) - 2g_w\ e_p(k-1)sin(\hat{\phi}(k)) \tag{23}$$

g_a and g_w are the adaptation parameters.

Using equations (18) and (23), the estimated phase is given by:

$$\hat{\phi}(k) = \hat{\phi}(k-1) + K_w\hat{w}(k) - K_wN_w\hat{w}(k-1) \tag{24}$$

The basis structure of the MPLL is shown on figure 5.

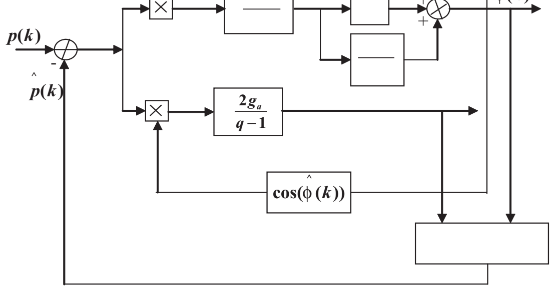

Fig. 5. Structure of the Magnitude Phase Locked Loop MPLL.

3.2 The new multimodel supervision procedure

The identification error $\varepsilon_i(k)$ depends only on the elementary disturbance periods $T_i(k)$ and the disturbance estimated period $\hat{T}(k)$ generated from the MPLL mechanism. This error is given as:

$$\varepsilon_i(k) = \hat{T}(k) - T_i(k)$$

The structure of the multiple disturbance rejection with switching control is given on the figure 6.

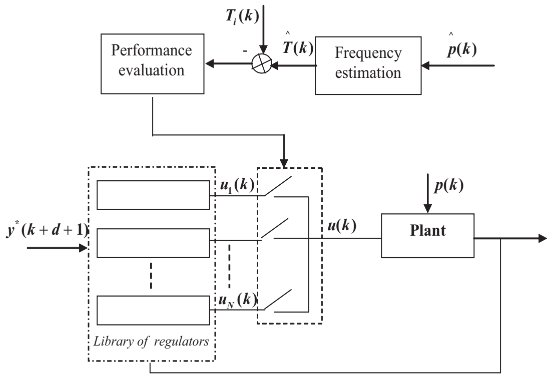

Fig. 6. Structure of the multiple disturbance rejection.

4 Simulation results

To demonstrate the interest and the contribution in performance of the proposed performance multimodel criterion, we maintain the same simulation conditions given in the previous section.

Figure 7 gives the evolution of the system output. This figure shows that the proposed multimodel strategy was, effective at rejecting harmonic disturbance in the presence of multiple frequencies and magnitudes. It can be seen a perfect and relatively rapid harmonic disturbance rejection, even in the presence of a phase shift that appears at the middle of the harmonic disturbance evolution. This proves the effectiveness of the proposed supervision procedure which combines an adequate switching criterion with a MPLL scheme. The evolution of the control signal is depicted in figure 8.

The figure 9 shows the switching evolution between the partial controllers. Indeed, for a corresponding period $T_i(k)$ one local controller which achieves

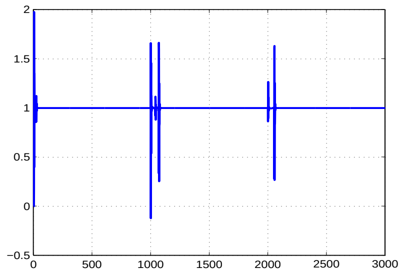

Fig. 7. Evolution of the plant output (new supervision procedure).

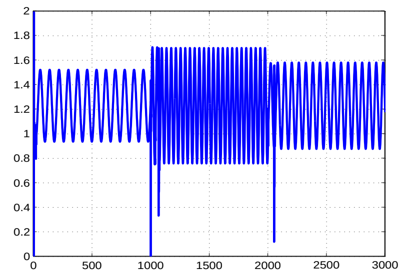

Fig. 8. Evolution of the plant input (new supervision procedure).

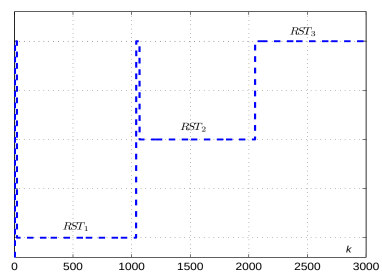

Fig. 9. Evolution of the controllers switching law (new supervision procedure).

disturbance rejection, is designed. Figure 10 illustrates the evolutions of the estimated and the real period of the harmonic disturbance. This figure shows clearly the effectiveness of the used estimation algorithm to provide disturbance parameters evolutions. The estimated multiple harmonic disturbances applied to the system output is given in figure 11.

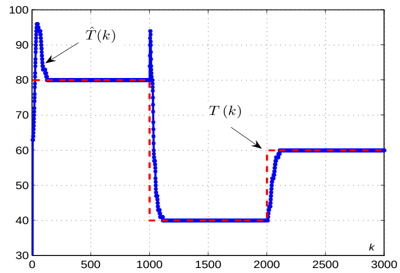

Fig. 10. Evolutions of the estimated and the real period disturbance.

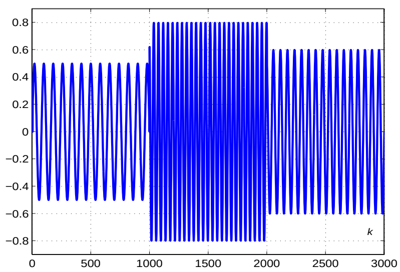

Fig. 11. Evolution of the estimated harmonic disturbance.

5 Conclusion

This paper investigates the problem of harmonic disturbances rejection with simultaneously time varying frequency and phase drift. This problem was appropriately handled using the multimodel approach. The proposed approach uses an intelligent supervision that combines a suitable switching criterion with Magnitude Phase Locked Loop algorithm. This proposed procedure depends only on the elementary periods including in the harmonic disturbance and its estimated period generated by the MPLL. For a given harmonic disturbance period, a local controller based on the internal model principle is selected to satisfy the control objective. The simulation results show that the proposed strategy achieves asymptotic multiple disturbances rejection even in the presence of disturbance phase drift.

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