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Comparison between Polya's theorem and Sum-of Squares Approaches for relaxing conditions in fuzzy control

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Abstract. *Stability, performance requirements in fuzzy control of Takagi–Sugeno systems are usually stated as multiple sums of terms involving degrees of freedom, such as the gains of the controllers, and matrices of the model, such as the state transition matrices. These summations of complex terms are weighted by non linear terms, called membership functions, which have to be removed to obtain standard conditions, such as Linear matrix inequalities (LMIs). Such a step is called a relaxation. Many researches have been performed on this topic, leading to only sufficient conditions. Recently two similar results have applied the Polya's theorem to obtain sufficient and necessary conditions through a set of bigger and bigger LMIs. However problems become quickly intractable. This paper proposes to apply Sum of Squares (SOS) approaches, and show that the conditions are in fact more effective than those based on the Polya's theorem.*

1 Introduction

Since the work of Takagi and Sugeno [16], fuzzy control and fuzzy observation of non linear model have known an increasing success. This is due partly to the ease of the modelling, and partly to the linear

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conclusion rules, which makes possible the use of the powerful tools of the linear control theory such as Linear Matrix Inequalities (LMIs), when the stability is related to a Lyapunov function.

Usually the almost final step is checking the positivity or negativity of a multiple sum of the type:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij} \tag{1}$$

Where r is the number of rules, h_i are nonlinear functions called membership functions which respect a convex sum property, z is a vector of usually measurable variables. Finally Υ_{ij} are terms that are more or less complex, depending on the problem (stability, or H_∞ control, and so on…) and depending on many matrix terms such as gains of the controllers, matrices representing the model, slack variables…

The last step is the so called relaxation, and is the removal of h_i terms from the equation (1) to obtain a finite number of LMIs that can be solved with standard software, such as Lmilab from Mathworks.

For a very long time it was sought that the relaxation step could only deliver sufficient conditions, because the information about the h_i functions was lost, expect their convex sum property. So many papers tried to obtain less and less conservative sets of LMIs for a given sets of Υ_{ij}. In this paper we do not tackle the definition of these terms and the possibility to obtain also less conservative conditions by introducing more degree of freedom in the Υ_{ij}.

Recently two papers [13], [9] prove that the relaxation could propose finite sets of LMIs that were sufficient but also necessary. The proof was based on the Polya's theorem, because it is a result concerning homogenous polynomials. Indeed (1) can be seen as a homogenous polynomial of the variables h_i. The two papers are similar, the second one [9] proposing an improvement based on a modified Υ_{ij} with the Finsler lemma. Indeed it has been proved that the Finsler lemma can help reducing the conservativeness of the LMI problems associated to given Υ_{ij} [2], [3].

It is worth mentioning also a recent paper from [6] that proposes sets of finite LMIs that are necessary and sufficient, by increasing progressively the number of rules.

These three approaches suffer from a problem of size: the LMIs problems become bigger and bigger when trying to obtain necessary conditions. At some point the LMIs problems are so big that they cannot be solve.

In this paper we propose another different approach to try to reduce the conservativeness of the relaxation step. Contrarily to the three previous approaches, we do not obtain necessary conditions. However we prove numerically that the associated problems, that are no longer written as a set of LMIs, can be solve much more efficiently and that outperform the LMIs based on the Polya's theorem or the ones based on an increased number of rules.

The approach we propose in based on Sum-of-Squares decompositions (SOS). SOS and LMIs are both semidefinite problems. SOS problems were introduced and begun to spread ten years ago, with the works of [10], [12].

Basically SOS problems try to decompose a polynomial as a sum of squares to check its positivity. To see how SOS problems intervene in the relaxation step, simply recall that (1), as for the use of the Polya's theorem, is a polynomial of the variables h_i. Instead of obtained a set of LMIs condition with approaches based on the Polya's theorem, we will simply have one – but rather complex- SOS condition.

We will see on a numerical example that this approach is much more efficient that previous ones. This example concerns the stabilization of a continuous fuzzy model, but the approach can be applied also on discrete model, and on any classical control or observation problems, provided the Υ_{ij} are given.

2 Fuzzy models, relaxation and SOS problems

2.1 Fuzzy models

Consider a nonlinear model

$$\begin{cases} \dot{x}(t) = f(z(t))x(t) + g(z(t))u(t) \\ y(t) = k(z(t))x(t) \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ the output, $u(t) \in \mathbb{R}^m$ the input, and $z(t) \in \mathbb{R}^q$ the vector of the premises which involves parts of $y(t)$ and $u(t)$. The functions f, g, k are assumed to be smooth.

We will use in this paper the following notation :

$Y \in \{A, B, C, H, P, ...\}$: $Y_z = \sum_{i=1}^{r} h_i(z(t))Y_i$ and

$Y_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))Y_{ij}$.

A fuzzy model is obtained if it is possible - at least locally - to rewrite (2) as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \end{cases} \tag{3}$$

The integer r denotes the number of rules; functions h_i are such that $h_i(z) \geq 0 \quad \forall\, i \in \{1, ..., r\}$ and $\sum_{i=1}^{r} h_i(z) = 1$. A_i, B_i, C_i are constant matrices, but are matrices. In this paper is assumed that all the pair of matrices (A_i, B_i) are controllable.

2.2 Conditions of stabilization

In this paragraph we detail the Υ_{ij} that will be used in the numerical example.

With a quadratic Lyapunov function $V(x) = x^T P x$ with $P > 0$, a basic condition for the stabilisation of (3) with a control $u(t) = \sum_{i=1}^{r} h_i(z(t)) F_i x(t)$ is (4)

$$\Upsilon_{zz} < 0 \tag{5}$$

with Υ_{zz} as in (1), and with Υ_{ij} given by:

$$A_i X + X A_i^T + B_i N_j + N_j^T B_i \tag{6}$$

where $X = P^{-1}$, $N_j = F_j X$ and $u(t) = F_z x(t)$

First relaxation procedures lead to the next set of LMIs conditions [17]:

$$i = 1...r \ \Upsilon_{ii} < 0$$

$$i = 1...r,\ j = i...r \ \Upsilon_{ij} + \Upsilon_{ji} < 0 \tag{7}$$

Various relaxations followed, in [5], [7], [4]. They involved either no further decision variables, as in [19], or new variables [5], [7], [4] in the right side of the inequality (5).

We present now the best previous set of LMIs conditions, coming from the application of the Polya's theorem in the relaxation step, and the use of the Finsler lemma in the Υ_{ij} of (6).

Theorem 1 [9]. Model (3) is stabilizable by a control law $u(t) = F_z x(t)$ if the following LMIs are checked:

$$\Upsilon_i = \sum_{i' \in \mathbb{k}(d), i > i'} \left(\sum_{\substack{k \in \{1,\dots,r\} \\ i_k > i'_k}} \frac{d!}{\pi(i')} \left(S_{i-i'-e_k} T_k + T_k^T S_{i-i'-e_k}^T \right) \dots \right.$$

$$\left. + \sum_{\substack{j,k \in \{1,\dots,r\} \\ i-i'-e_k-e_j}} \frac{d!}{\pi(i')} \frac{(g-1)!}{\pi(i-i'-e_k-e_j)} \begin{bmatrix} B_k N_j + N_j^T B_k^T & X \\ X & 0_n \end{bmatrix} \right) < 0_{2n}$$

$$i \in \mathbb{k}(g+d+1) \tag{8}$$

Where $T_k = \begin{bmatrix} A_k^T & -I_n \end{bmatrix}$, $X > 0 \in \mathbb{R}^{n \times n}$, $S_i \in \mathbb{R}^{2n \times n}$, $i \in \mathbb{k}(g)$, $g \geq 1 \in \mathbb{N}$, $d \geq 1 \in \mathbb{N}$, $N_j \in \mathbb{R}^{m \times n}$, $j \in \{1, ..., r\}$, $\alpha \in \mathbb{k}(\beta)$ is a multi index, such as $\alpha = \alpha_1 \alpha_2 ... \alpha_r$ with $\alpha_1 + ... + \alpha_r = \beta$ and $\mathbb{k}(\beta)$ is the set of all possible combinations of such multi indexes, e_k is a null vector of size r except for a 1 at the place k, and $\pi(i) = (i_1 !)(i_2 !)...(i_r !)$.

If these LMIs are checked, then the controller gains are defined by $F_i = N_i X^{-1}$.

Remark 1. Readers unfamiliar to the Polya's results can see the set of LMIs becoming bigger and bigger first through the parameter d that rules the size of the resulting LMIs. We know that for a finite d the problem (8) becomes necessary. The drawback is that d may be too large to be able to solve (8). Second g is a degree of freedom introduced in [9] that allow a faster convergence for a fixed d. It allows the slack variables $S_i \in \mathbb{R}^{2n \times n}$ to be a multi sum too.

Authors of [9] also says that non PDC control laws could be use, with N_j being also multi sums, but no results are provided for this issue. This possibility is not studied in this paper too either, the motivation of this paper is to propose another way of relaxing conditions through Sum of Squares.

2.3 Sums of Squares

A scalar polynomial $f(z)$ is a sum of squares (SOS) if there exist polynomials $f_i(z)$, for $i = 1, ..., v$ such that $f(z) = \sum_{1 \leq i \leq v} f_i^2(z)$ [10]. It is obvious that a SOS polynomial is positive, but the converse is not true in general. However, experimental results show that there is only a small difference between a positive function and its approximation obtained as a sum of squares [12]. Moreover for polynomials of order 2 there is equivalence.

Thus, in the paper, conditions such as "$f(z) \geq 0$" will be replaced by "$f(z)$ is a SOS". This last condition can be rewritten as a convex problem, and standard solvers for LMIs can be used. The free toolbox of Matlab called SOSTOOLS aims to translate SOS conditions as LMIs conditions [12]. Remark also that conditions of the type "$f(z) > 0$" are transformed into "$f(z) - \varepsilon$ SOS" with $\varepsilon > 0$.

Now assume we want to check the positivity of $f(z)$ only for z belonging to a subset of $\mathbb{R}^n$ that is a defined by polynomial constraints. Then proposition 1 allows defining a new problem to take into account the local search.

Proposition 1. (Local study). $v^T f(z) v > 0$ is positive with v and z tow variables of $\mathbb{R}^n$, when $c_k(z) \geq 0$, $k = 1 \ldots s$, with f and c_k polynomials, when $v^T f(z) v - \sum_{k=1\ldots s} \sigma_k(v,z) c_k(z) > 0$, with $\sigma_k(v,z) \geq 0$ being polynomials of order 2 with respect to v.

The proof is given in [12] and is similar to the S-procedure, but with polynomials for the multipliers.

3 Relaxation based on SOS conditions

In fact it is very easy to apply a SOS approach for a relaxation step, and only a very small proof is required. We simply have to check with SOStools if a given polynomial is positive or not. The only problem is to take into account the constraint about its variables, here the h_i terms. Proposition 1 is used.

Recall that we want to check (5) that we rewrite for clarity:

$$\Upsilon_{zz} = \sum_{i,j} h_i(z) h_j(z) \Upsilon_{ij} < 0 \tag{9}$$

With Υ_{ij} involving decision variables as well as data from the model, and.

$$h_i \geq 0, \sum h_i = 1 \tag{10}$$

Theorem 2 (SOS relaxation)

Equation (9) is verified with the h_i functions respecting (10) if the next problem is verified:

$$-v^T \Upsilon_{zz} v - \sum_{i=1}^{r} \sigma_i (v, h_i)(1-h_i)(h_i) - \varepsilon v^T v \text{ is SOS} \tag{11}$$

With v symbolic variables of the same order than Υ_{zz}, h_i, $i = 1 ... r-1$ symbolic scalars, $h_r = 1 - \sum_{i=1}^{r-1} h_i$ a symbolic constant, and $\sigma_i (v, h_i)$ positive polynomials of order 2 with respects to v.

The proof is trivial when the issue of the constraints is solved. We have r symbolic variables h_i which respect (10). By substitution, we can delete one variable with $h_r = 1 - \sum_{i=1}^{r-1} h_i$. Now remains the fact that all functions $h_i(z)$ must belong to $[0,1]$, which is replaced by constraints of the type $h_i (1-h_i) \geq 0$.

For the numerical comparison of section 4, we provide conditions with detailed Υ_{ij}:

Theorem 3

Fuzzy model (3) is stable in closed loop with a control law $u(t) = F_z x(t)$ if there exist $X > 0 \in \mathbb{R}^{n \times n}$, $S_i \in \mathbb{R}^{2n \times n}$, $i \in \{1,...,r\}$, $N_j \in \mathbb{R}^{m \times n}$, $j \in \{1,...,r\}$, such as theorem 2 is true with:

$$\Upsilon_{ij} = S_i T_j + T_j^T S_i^T + \begin{bmatrix} B_k N_j + N_j^T B_k^T & X \\ X & 0_n \end{bmatrix}, \text{ in where } T_j = \begin{bmatrix} A_j^T & -I_n \end{bmatrix} \tag{12}$$

If these conditions are satisfied, controller gains are given by $F_i = N_i X^{-1}$.

Remark 2. In theorem 3 the slack variables S_i have been simplified compared to theorem 1, since there are only r of them, leading to only a double sum Υ_{zz}. Of course we could also introduce more variables, with S_i replaced by S_{ij}, leading to multiple sum indexes $\Upsilon_{z^{n_0}}$ with n_0 of any order. Simply theorem 3 should be adapted to handle polynoms in h_i of order n_0+1. Also better results could be expected when considering higher order variables control laws, for example with $u(t)=F_{zz}x(t)=\sum_{i,j} h_i h_j F_{ij} x(t)$. In the comparison section we have tested a problem with a triple sum Υ_{zzz} corresponding to S_{ij} and F_{ij}. In this case the answer is negative.

Remark 3. As said in the previous section, for polynoms of order 2 there is an equivalence between $f(z)\geq 0$ and $f(z)$ SOS. (9) is a polynom of order 2 with respect to the h_i, but the equivalence is lost, due to the constraints that are included in the SOS condition.

Remark 4. Of course theorem 3 concerns only a feasibility problem. Any optimization step can be added to it. For example in the numerical comparison we have added constraints about the controller gains, as in [2], which is very easy. In this case, the constraints to add to (11) are:

$$i \in \{1,...,r\} \quad \begin{bmatrix} X & N_i^T \\ N_i & \mu \end{bmatrix} > 0 \tag{13}$$

Where μ is a given constant, or a variable to minimize.

Remark 5. With a SOS formulation, it is very easy to include further relations about the h_i, if they can be written as polynoms.

4 Numerical comparison

4.1 The model

This is a 3 rules fuzzy model, defined by the next matrices :

$$A_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix},\ A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} -b+6 \\ -1 \end{bmatrix}$$

4.2 The solvers

LMIs associated to theorem 1 are build with the Matlab code. They use YALMIP as the parser.

SOS conditions of theorem 3 are built with the SOS module of YALMIP.

In both cases the solver is SEDUMI.

4.3 Comparisons.

We take [9] for the comparison, since it slightly improves [13] and is more robust numerically. For reference about the computation time, an athlon x2 4000+ is used with 2 Gigabytes of memory. Results are given in table 1.

	$a=2$	$a=4$	$a=6$	$a=9$
[9]	$b=6.5$	$b=7.5$	$b=9$	$b=10$
SOS approach	$b=6.5$	$b=7.5$	$b=9$	$b=10.5$

Table 1. Comparison with results obtained with [9], for $n=5$ for the Polya's theorem, and the SOS approach. Solutions are obtained in less than 1 second in both cases. There is no improvement for $n=15$ for reference [9], which takes 7 seconds to be solved.

As can be seen from table 1, results are similar for both approaches, and this shows that the SOS approach can provide accurate results quickly while being able to conveniently add information about the h_i functions.

For reference [9], there are 33 decision variables and 147 LMIs rows. For the SOS approach without remark 4, there are 90 decision variables and 29 LMIs rows, and with the remark 4 enforced, there are 108 decision variables and 38 LMIs rows. Rising n with [9], does not increase the number of decision variables, but the number of LMIs rows raises quickly.

Figure 1 shows a simulation of the model, for $a=2$, when the premises are given by:

$w_1 = 1-\cos^2(x_1)$, $w_2 = 1-\sin^2(x_2)$, $h_1 = w_1 w_2$, $h_2 = w_1(1-w_2)$, $h_3 = 1-h_1-h_2$

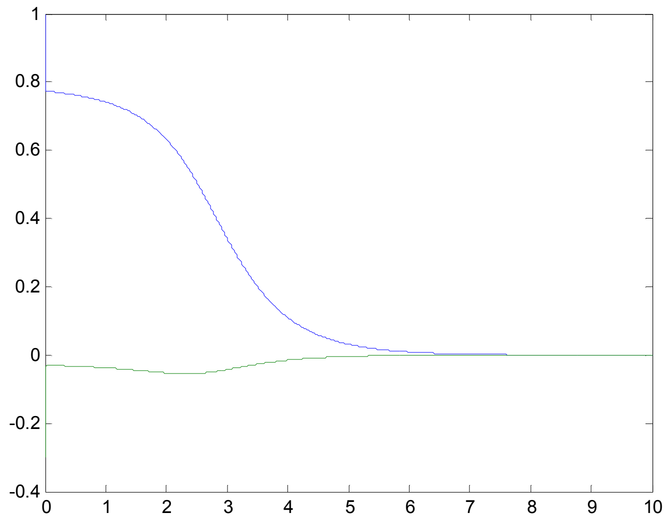

Figure. 1. $x_1(t)$ (in blue) and $x_2(t)$ (in green) for the example, when $a=2$, $b=6.75$. $F(1)=[-86 \quad -45]$, $F2=[-657 \quad -399]$, $F(3)=[113 \quad 823]$ ($\mu=10^3$ for the remark 4).

On figure 2 we have plotted the highest eigenvalue of Υ_{zz} when varying h_1 and h_2 on a grid (of course it should be negative).

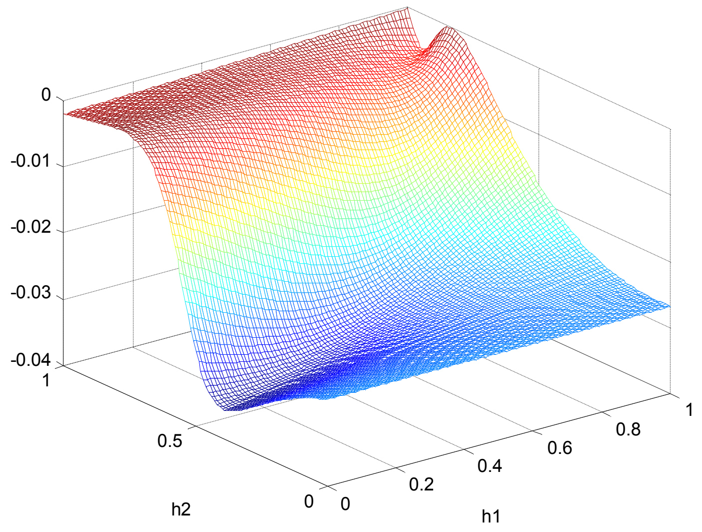

Figure 2. Highest eigenvalue of Υ_{zz} when varying h_1 and h_2 (the maximum is -5e-5)

As said in the remark 2 we can try to improve results of theorem 3 by adding more degrees of freedom in the slack variables and the controllers gains. We have tested in particular the case of double indexes for S_{ij} and F_{ij} to obtain a triple sum Υ_{zzz}. We did not notice any improvements over table 1.

5 H_∞ Control

Let be the following TS model :

$$\begin{bmatrix} \dot{x} \\ z \end{bmatrix} = \begin{bmatrix} A_z & B_{wz} & B_{uz} \\ C_z & D_{wz} & D_{uz} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \tag{14}$$

with $x \in \mathbb{R}^n$ the state of the system, $z \in \mathbb{R}^q$ the output, $u \in \mathbb{R}^m$ the control, and w an exogenous input.

The H_∞ control problem consists in finding a control law u which minimizes γ such that, with $x_0 = 0$:

$$\forall w,\ \forall T > 0:\ \int_0^T \|z\|^2\, dt < \gamma^2 \int_0^T \|w\|^2\, dt$$

Lemma 1. [20]
There exists a linearly parameter-dependent state feedback control gain $F_z \in \mathbb{R}^{m\times n}$ (4) that quadratically stabilizes the T-S fuzzy system (14) with an H_∞ guaranteed cost $\gamma > 0$ if and only if there exist a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$, linearly parameter-dependent matrices $N_i \in \mathbb{R}^{m\times n}$ and $S_i \in \mathbb{R}^{2n+p+q\times n+q}$ such that

$$\Upsilon_{zz} = Q_{zz} + S_z T_z + T_z' S_z' < 0_{2n+p+q} \tag{15}$$

With

$$Q_{zz} = \begin{bmatrix} B_z N_z + N_z' B_z' & W & 0_{n\times q} & N_z' D_z' \\ * & 0_n & 0_{n\times q} & 0_{n\times p} \\ * & * & I_q & 0_{q\times p} \\ * & * & * & -\gamma^2 I_p \end{bmatrix}$$

$$T_z = \begin{bmatrix} A_z' & -I_n & 0_{n\times q} & C_z' \\ B_z' & 0_{q\times n} & -I_q & D_z' \end{bmatrix}$$

We applied for this equation the method for theorem 3 LMI the SoS approach of theorem 3 to obtain :

Min Υ with respects to :

$$-v^T \Upsilon_{zz} v - \sum_{i=1}^{r} \sigma_i(v, h_i)(1-h_i)(h_i) - \varepsilon v^T v \text{ is SOS}$$

With v symbolic variables of the same order than Υ_{zz}, h_i, $i = 1 \ldots r-1$ symbolic scalars, $h_r = 1 - \sum_{i=1}^{r-1} h_i$ a symbolic constant, and $\sigma_i(v, h_i)$ positive polynomials of order 2 with respects to v.

Example:

We used the following matrices :

$$A_1 = \begin{bmatrix} -5 & 5 & 0 \\ 30 & -1 & 20 \\ 0 & -20 & -2 \end{bmatrix},\ A_2 = \begin{bmatrix} -5 & 5 & 0 \\ 30 & -1 & 30 \\ 0 & 30 & -2 \end{bmatrix},\ B_1 = \begin{bmatrix} 5 & 0 \\ 0 & 30 \\ 2 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ -30 & 0 \\ 0 & 2 \end{bmatrix},\ C_1 = 5*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ C_2 = 30*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ D_2 = \begin{bmatrix} -3 & -0.1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ Bw_1 = 5*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$Bw_2 = -30*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ Bw_1 = 5*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$Dw = F1 = F2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We obtain exactly the same results. This shows that on this example the two problems provide the same results, but the SOS approach is simpler to write, since we formulate only one SOS constraint instead of many with an LMI formulation.

The upper bound γ for problem is 55. The exogenous input was chosen as

$$w(t) = \begin{bmatrix} w_1(t) & w_2(t) & w_3(t) \end{bmatrix}^T$$

with

$$w_1(t)=\begin{cases}3 & t<5\\ 0 & t\geq 5\end{cases},\quad w_2(t)=\exp(-0.1*t)*\begin{cases}1 & t<2\\ 2 & t\geq 2\end{cases},$$

$$w_3(t)=\exp(-0.1*t)*\sin(t)$$

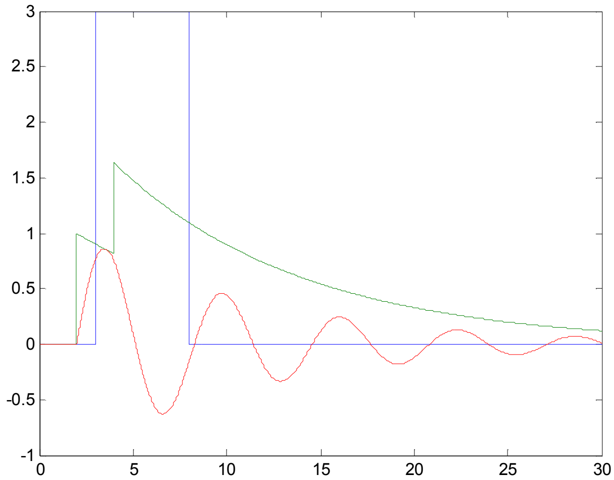

Figure 3. Evolutions of *w*.

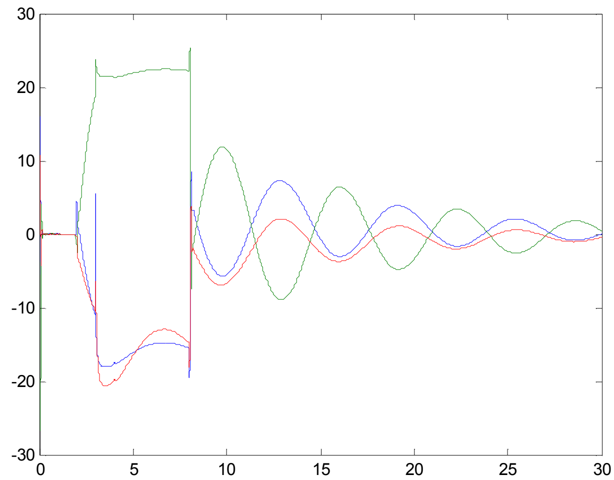

Figure 4. Evolutions of *y*.

6 Conclusion

Relaxation is an important problem with fuzzy approaches. With results based on Polya's theorem, necessary and sufficient conditions can be obtained. However the LMIs conditions can become too numerous and big to be solved.

In this paper we propose to use SOS approaches to directly try to solve the original condition including the h_i functions into one and big SOS condition.

We analyzed the H_∞ control problem for both SOS and LMI approach.

Results show that this is an interesting solution. Furthermore, this paper has studied a continuous model, but, of course, results can be obtained straightforwardly for discrete models, and any other control problems.

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