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Benchmarking for tuning Predictive Controllers, with demonstration on a neutralization process

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Abstract. *This paper presents a solution, using benchmarking ideas, to tuning linear predictive controllers for non-linear time varying systems. The technique is based on estimating analytically the expected total cost for different tuning parameters, so that the control engineer can then select the parameters that give lower cost. A case study involving the tuning of LQGPC controller for a neutralization plant is presented.*

Keywords. *LQGPC, Non-linear Systems, Benchmarking, Predictive Controllers*

1. Introduction

Benchmarking techniques were first applied by Rank Xerox in the late 1970s for business processes, although the interest of the control community comes later, from the paper by Harris (1989), that was a turning point in the field. He used minimum variance control as a benchmark for controller loop assessment. Since then, many papers have been published: For a Benchmarking techniques see Joe (1998), Harris *et al*. (1996), Cinar and Undey (1999), Huang and Shah (1999), Huang and Tamayo (2000), Ko and Edgar (2000, 2001) and Grimble (2002, 2004).

Benchmarking ideas can be applied to any type of predictive controller, in fact, to any controller. We have selected Predictive Controllers, as it is an advanced control technique equally regarded by industry and academia, with several parameters to tune and not many clear rules for tuning them, so they are frequently tuned based on the experience of the engineer. Our objective is to show that from the careful study of specific cost functions, the tuning parameters can be set according to pre-specified criteria. We have selected the LQGPC (Linear Quadratic Gaussian Predictive Controller) in state space form because of its good stability properties and the fact that the proposed costs can be easily expressed in terms of matrices solution of simple

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Ricatti equations (Grimble, 2003). The proposed methodology was first presented by the author in Alvarez et al. (2002), for the particular case of the prediction horizon. The general approach is presented here.

The paper is structured as follows: Section 2 presents the LQGPC controller, Section 3 the selected benchmarking indexes and Section 4 the application of the technique to a neutralization plant. Finally, some conclusions are given.

2. Predictive Control

Predictive Control is a control strategy based on the explicit use of a model to predict the process output over a period of time (Maciejowski, 2002). At each sampling time the future control signals are calculated by minimization of a cost function, which is usually defined as a weighted combination of tracking errors and control variations. Most of the times, a receding control horizon technique is applied: the calculations are repeated every sampling time, to take into account the difference between the predicted state and the measured state. The controller strategy considered in this paper is a multivariable predictive controller based on the Linear Quadratic Gaussian Predictive Controller (LQGPC: Grimble, 1997). The basic ideas of LQGPC are now presented.

The main advantage of LQGPC is that, compared with other techniques predictive control techniques (such as the popular GPC, Clarke *et al*., 1987), LQGPC ensures guaranteed stability properties for all cost weightings (Grimble, 1997; Grimble, 2001a). This come from the fact that the control at time t is not affected by the future control computations, so a controller equivalent to an LQG design is obtained, and the LQG stability properties are recovered (Grimble, 1992).

The predicted values of the input signal u[t+j] are obtained in a statistical way, by minimizing the expected value of the cost function J in (1):

$$J = E\left\{ \lim \frac{1}{2T} \sum_{t=-T}^{T} \left(\begin{array}{l} \sum_{j=1}^{N_y} \left(r[t+j] - \tilde{y}[t+j]\right)^T Q_j \left(r[t+j] - \tilde{y}[t+j]\right) + \\ y_u^T(t) y_u(t) + \sum_{j=1}^{N_u} u[t+j]^T R_j u[t+j] \end{array} \right) \right\} . \tag{1}$$

This J is called the dynamic performance index and it is taken as the nominal benchmark cost. It must be pointed out that the error weightings Q_j and the control weightings R_j are normally selected to be time dependent. Grimble (1997) showed that based on some mild assumptions, selection of an adequate state-space representation makes possible to transform this problem into an equivalent LQG problem:

$$J = E\left\{\lim \frac{1}{2T}\sum_{t=-T}^{T}\left(x^T[t]Q_c x[t] + u^T[t]R_c u[t] + 2x^T[t]G_c u[t]\right)\right\}, \tag{2}$$

Where Q_c is a positive semi-definite Hermitian matrix (evaluated from the Q_i) and R_c is a positive-definite Hermitian matrix (evaluated from the R_i).

3. Predictive Controller Tuning Using Benchmarking Ideas

The first step of the method proposed is to develop a simple analytic expression for the minimum cost function that could be achieved if the model were identical to the plant. In the case of LQGPC, assuming that the states are estimated using an Optimal Kalman filter, Grimble (2003) showed that the cost that minimizes (1), and by equivalence (2), can be calculated analytically as:

$$J_{min} = \mathrm{trace}\left(Q_c P_f + P_c K_f \tilde{R}_f K_f^T\right), \tag{3}$$

where as shown in Grimble, 2003:

- K_f corresponds to the gain of a Kalman filter with disturbances of variance Q_f and measurement noises of variance R_f . That is, $K_f = AP_f C^T \tilde{R}_f^{-1}$ with $\tilde{R}_f = R_f + CP_f C^T$.

- P_f is evaluated solving a Riccati equation:

$$P_f = AP_f A^T + DQ_f D^T - AP_f C^T \tilde{R}_f^{-1} CP_f A^T$$

- P_c is also evaluated solving a Riccati equation:

$$P_c = \tilde{Q}_c + \tilde{A}^T P_c \tilde{A} - \tilde{A}^T P_c B \tilde{R}_c^{-1} G_c^T ,$$

  where

$$\tilde{Q}_c = Q_c - G_c R_c^{-1} G_c^T$$
$$\tilde{R}_c = R_c + B^T P_c B$$
$$\tilde{A} = A - B^T R_c^{-1} G_c^T$$

As it has previously said, it would be very useful to obtain a procedure such that the tuning parameters can be set automatically. Equation (3) gives the control designer a simple method to compare the expected performance of controllers for different tuning parameters: it is only necessary to calculate the cost in (3) for each situation and compare the results. However if only the cost in (3) were considered, the parameter selection can give inadequate control signals. Thus, it is also important to check the variance of the control signal J_u, that can be calculated using (4):

$$J_u = E\{u^T[t]u[t]\}. \tag{4}$$

It can be easily seen that this covariance matrix J_u can be calculated from:

$$J_u = \mathrm{trace}\{P_u\}, \tag{5}$$

where $P_u = K_c P_{\hat{x}} K_c^T$, with K_c the gain solution of the stochastic linear discrete-time optimal output feedback control problem (see Grimble, 2003), and $P_{\hat{x}}$ the covariance matrix of the estimated states obtained from the following Lyapunov function:

$$P_{\hat{x}} = (A - BK_c)P_{\hat{x}}(A - BK_c)^T + K_f\left(R_f + CP_f C^T\right)K_f^T.$$

To select optimal tuning parameters, a weighted combination of these variances can be used, so the minimum can be evaluated. Thus, we propose to use a linear combination, with constant weight (that can be selected based on the acceptable control variance):

$$J_{total} \equiv J_{min} + \alpha J_u = \mathrm{trace}\{P_u\} + \alpha \cdot \mathrm{trace}\left(Q_c P_f + P_c K_f \tilde{R}_f K_f^T\right). \tag{6}$$

It is proposed here that this index is used for online tuning of predictive controllers: it would be only necessary to calculate the desired combination of parameters that minimise a positive combination of the Minimum Feedback Cost and the Control Efforts. Let Ω denotes the set of possible combinations of tuning parameters of the controller (For example, if N_y and N_u are the parameters to be tuned, with the prediction horizon between 1 and 10 and the control horizon between 1 and 3, with $N_y \geq N_u$, then Ω is the set of possible valid combinations of these parameters). Then it is proposed to solve the following minimization:

$$\min_{\Omega}\left(J_{total}\right). \tag{7}$$

It must be pointed out that additional considerations in the selection of the tuning parameters can be easily considered by simply selecting a subset of solutions σ that give lower values of J_{total}, which can then be compared using the algorithm proposed now, illustrated in the example in next section. In any case, for auto-tuning the minimization in (7) can be done automatically to select the combination of tuning parameters that gives the lower cost.

Algorithm

1. Define the set Ω of valid combinations of tuning parameters of the controller.
2. For each element in Ω calculate its associated cost using (6).
3. For the subset σ⊂Ω of combinations of tuning parameters that give the lower costs compare them in terms of cost, controller complexity, calculation time, etc.
4. Based on this comparison, select the tuning parameters from σ.

4. Case Study: Neutralization Plant

4.1. System Description

The process under study is the neutralization of an aqueous solution of sodium acetate (CH3COONa) with hydrochloric acid (HCl) in a Continuous Stirred Tank Reactor (CSTR). This project has been previously used to test other control approaches (see Tadeo et al., 2000, Fuente et al., 2006, Syafiie et al., 2007). The experimental setup, shown in Figure 1, consists of a CSTR where a liquid of variable pH is mixed with a solution of high concentration of HCl. pH variations can be realized within the setup by adding varying amounts of Sodium Acetate to the tank liquid. This liquid is fed from the tank using a pump, which produces a variable flow depending on the level of liquid in the tank. The liquid in the mixing tank overflows (outlet not shown), so the volume of liquid in the tank can be considered constant. The output variable y is the hydrogen ion concentration in the effluent stream. The mixture pH is measured using an Ag-AgCl electrode and transmitted using a pH-meter. The electrode dynamic response presents appreciable and asymmetric inertia. The measured pH and the control signals are transmitted through an A/D interface. The plant is controlled and monitored from a PC computer, using Matlab and the Real-Time Toolbox for control. The control action calculated using the proposed algorithm is then applied to the plant by manipulating the titrating pump (every second). At the same time, the pH is recorded every second.

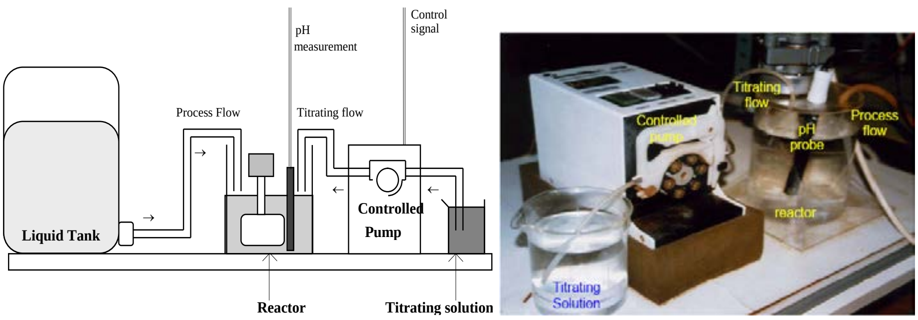

Fig. 1: Laboratory Plant

Different experiments were carried out in the real system to obtain input-output data to estimate a model to design the LQGPC controller. Using identification techniques, the estimated plant model was:

$$G(z) = \frac{-0.0000304z^{-1}}{1-3.61688z^{-1}+5.16236z^{-2}-3.4599z^{-3}+0.91453z^{-4}},$$

which corresponds to an stable 4^{th}-order system with negative gain.

The difficulty of tuning predictive controllers in this system comes from the small sample time, which must be selected to ensure good controllability at every working point: Due to the varying dynamics, the time constant changes at different working points, so the sample time is selected based on the fastest dynamic. Figure 2 shows a typical open-loop experiment, where the varying dynamics can be observed. This plant is frequently used as a benchmark for novel control techniques (Tadeo et al., 2000, Fuente et al., 2006; Syafiie et al., 2007).

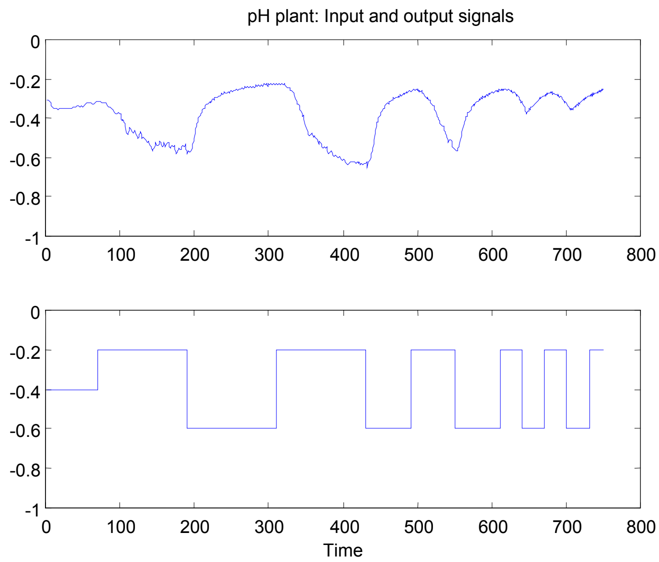

Fig. 2: Input/Output Experiments in the Laboratory Plant

At the nominal working point the stabilization time is about 800 secs, and the selection recommended in the literature for the prediction horizon would be about 800 samples, which would make the predictive controller time-consuming. Also, due to the fact that the linear model used in the predictive controller is only an approximation to the non-linear system, most of the predictions will be useless if the prediction horizon is so large.

4.2. Tuning of the Neutralization Controller Using Benchmarking Ideas

The tuning technique previously discussed was applied to the model of the neutralization plant:

- First the Minimum Feedback Cost and the corresponding Control Covariance were estimated for different values of the prediction horizon, from 1 to 30, using (3) and (5), and the identified model.

- Then the seven values of Ny that gave the lower costs were selected: they corresponded to values from 3 to 9. For these values the costs were studied: Figure 3 (left) shows the variation of the Feedback Cost with the prediction horizon, whereas Figure 3 (right) shows the variation of the Control Covariance. Based on these plots the control engineer could select N_y=4 as an adequate prediction: It can be seen that for N_y>4 there is no improvement in performance when increasing N_y: the Feedback Cost increases, without reducing the control variance.

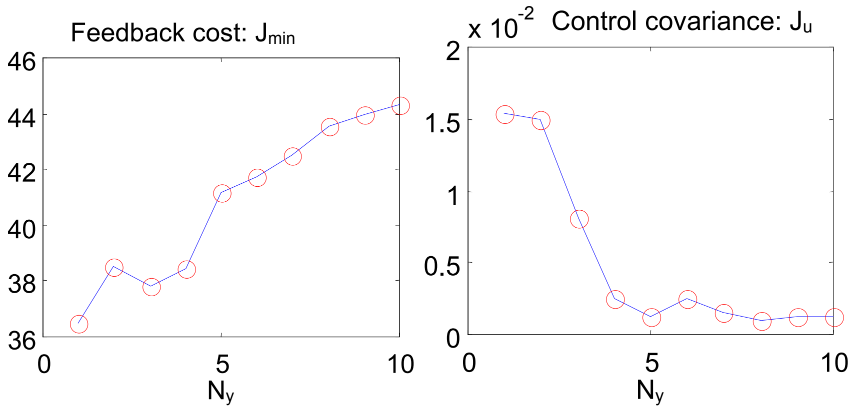

Fig. 3: Variations with the prediction horizon of the Estimated Feedback Cost and Control Covariance

- From the plant steady state gain, we can select $\alpha = 10^3$. Figure 4 plots the variation with the prediction horizon of the Total Weighted Cost. It can be seen that when N_y=4, the minimum value is attained, which also corresponds to the lower controller complexity from all the parameters in the set σ, so it is the one selected

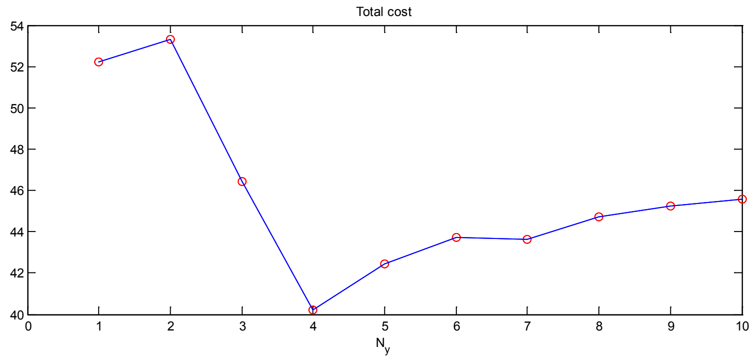

Fig. 4: Total Weighted cost

Finally, to check the behaviour predicted by the Benchmarking technique, we calculated the optimal controller for different prediction horizons and checked the feedback response. For example, Fig. 5 shows the closed-loop step responses with the LQGPC controller tuned for different parameters in **Ω**: it can be seen that there is not clear improvement when increasing the prediction horizon from 4 to 9. However, reducing the number of degrees of freedom (N_y=2) makes the control signal more active, and worsen the overall performance.

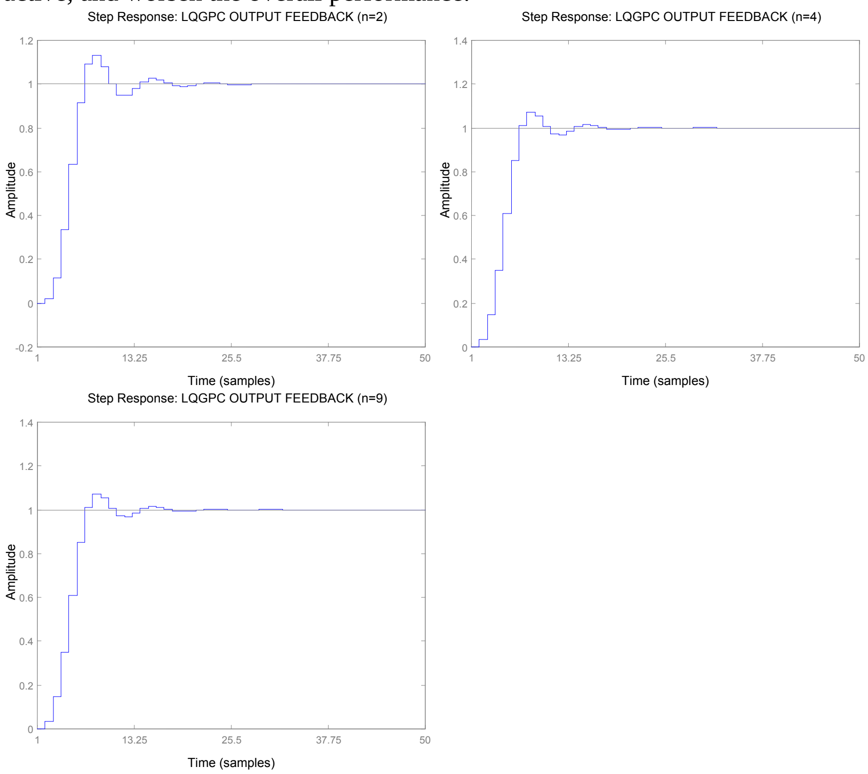

Fig. 5: Step Responses for different tuning parameters (N_y=2,4,9)

Conclusions

Predictive control is widely used in industry, but one disadvantage is that it is not possible to be sure on the ‘optimality’ of the tuned parameters. Of course, there are some rules of thumb for tuning them, but can we rely on them? The method presented in this paper offers a mathematical alternative. It is shown how it is possible to select tuning parameters based on the evaluation of simple expressions for cost functions. In particular, it is shown by way of an example (a neutralization process) how the favoured prediction horizon is the one that gives the smallest cost functions.

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