International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 5, N° 1, June 2011, pp. 1458–1471.

Rejection of Periodic Disturbances with Unknown Frequency for Multivariable Systems

DEHRI Khadija, LTAIEF Majda,
NOURI Ahmed Said and BEN ABDENNOUR Ridha

Unité CONPRI - Ecole Nationale d' Ingénieurs de Gabès
Route de Medenine, 6029 Gabès, Tunisie
khadija.dehri@gmail.com majda.ltaief@enig.rnu.tn
ahmedsaid.nouri@enig.rnu.tn ridha@ieee.org

Abstract. *Reference trajectory asymptotic tracking and disturbance rejection are an important field of control theory. In this paper, we are interested on the tracking of reference trajectories and the rejection of periodic disturbances with unknown frequency for multivariable systems. If the frequency is known, a classical solution is to use a multivariable repetitive generalized predictive control law. If the frequency is unknown, the solution is to combine the multivariable repetitive generalized predictive control with an on-line frequency estimator. The method based on the magnitude/phase locked loop is used to solve the estimation frequency problem. The simulation results are satisfactory and show a good performances in terms of rejecting periodic disturbances and reference trajectories tracking.*

Keywords: *Multivariable control systems, Generalized predictive control, Repetitive control, Rejection of periodic disturbances, Frequency estimator.*

1 Introduction

The rejection of disturbances is a problem frequently encountered in control engineering. The disturbances are, often, periodic and can be with known or unknown frequency [1, 2].
In the literature, there are several main approaches dealing with periodic disturbance cancellation [3–5]. The most common approach is based on the internal model principle (IMP) [6]. This approach consists in simply incorporating the poles of the disturbance model into the regulator pole configuration. Another approach is the adaptive feedforward cancellation method (AFC) [7] where a disturbance at the input of the plant can be canceled by constantly adding the negative of their values. Other researchers have proposed different approaches to deal with periodic disturbances such as repetitive control (RC), iterative learning control (ILC) and Q-parametrization. The repetitive control is based on the IMP and it is a useful technique for asymptotic tracking and rejection of periodic

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

signals [8, 9]. Periodic signals with a known period can be generated by a time delay block with a positive feedback loop. The main idea behind the RC is to use such a time delay block in the controller to ensure tracking of periodic reference trajectories or rejection of cyclic disturbances. Iterative learning control is an approach for improving the transient performance of systems that operate repetitively over a fixed time interval [10–12]. The ILC refers to the problem of finding an appropriate control input that causes the output to track a desired trajectory or to reject periodic disturbances defined in a finite time interval by iterative trials.

However, the IMC, AFC, RC and ILC are confronted with several problems: such as the problem of stability and the inability to take into account certain characteristics of processes. In order to resolve these problems, several proposals have been made to use predictive control algorithms with a repetitive control [13, 14]. The multivariable generalized predictive control (MGPC) has been mainly investigated in [15–17]. It has proved to be efficient, flexible and successful for industrial applications. However, some problems may appear trying to deteriorate the performances when the system is subject to periodic disturbances. This problem is solved by using a multivariable repetitive generalized predictive control (MRGPC).

This paper focuses on the rejection of periodic disturbances. The approach consists in the synthesis of a multivariable repetitive generalized predictive control law to reject a periodic disturbances. In practice, the frequency of the disturbances is usually unknown. In this case, the multivariable repetitive generalized predictive control with an on-line frequency estimation may be applied. To estimate the frequency, we use a method based on a magnitude/phase locked loop approach [18].

The paper is organized as follows; the next section presents the formulation of the system input-output model in the ARIMAX form with the inclusion of a repetitive noise model, as well as, the synthesis of MRGPC law. The MRGPC law is the result of the combination of two control laws. The first one has the objective of tracking the reference trajectories. Whereas, the second one deals with cancellation of periodic disturbances. The third section describes the magnitude/phase locked loop approach for the frequency estimation. Then the multivariable repetitive generalized predictive control with an on-line frequency estimator is proposed to reject a periodic disturbances. A simulation example illustrating the interest of the proposed methods is then presented.

2 The multivariable repetitive generalized predictive control

2.1 Multivariable repetitive ARIMAX model

Consider a square linear multivariable discrete-time system with p inputs and p outputs. The system can be modeled in the form ARIMAX (Auto-Regressive

Integrated Moving Average with eXternal inputs) as follows:

$$A(q^{-1})Y(k) = B(q^{-1})U(k-1) + \frac{I_p}{\Delta_1(q^{-1})}E(k) \tag{1}$$

where $Y(k) = \begin{bmatrix} y_1(k) \dots y_p(k) \end{bmatrix}^T$ and $U(k) = \begin{bmatrix} u_1(k) \dots u_p(k) \end{bmatrix}^T$ are respectively the output and input vectors, $E(k) = \begin{bmatrix} e_1(k) \dots e_p(k) \end{bmatrix}^T$ is a vector sequence of independent random variables.
$\Delta_1(q^{-1}) = \Delta(q^{-1}) = (1 - q^{-1})$ is the differential operator.
$A(q^{-1})$ and $B(q^{-1})$ are two polynomial matrices defined as follows:

$$\begin{cases} A(q^{-1}) = I_p + A_1 q^{-1} + A_2 q^{-2} + \ldots + A_{n_A} q^{-n_A}; \\ \qquad \dim A_{\tau_1} = (p,\, p);\ \tau_1 \in [1,\, n_A] \\ B(q^{-1}) = B_0 + B_1 q^{-1} + B_2 q^{-2} + \ldots + B_{n_B} q^{-n_B}; \\ \qquad \dim B_{\tau_2} = (p,\, p);\ \tau_2 \in [0,\, n_B] \end{cases}$$

The MGPC scheme contains a pure integrator, which cancel the effect of constant step disturbances. Whereas, when disturbances are periodic, the MGPC cannot compense them. Therefore, we propose to modify the ARIMAX model by including repetitive properties and writing $\Delta_1(q^{-1})$ as follows [5, 19]:

$$\Delta_1(q^{-1}) = \Delta(q^{-1})\Delta_R(q^{-1}) \tag{2}$$

with:

$$\Delta_R(q^{-1}) = 1 - \alpha q^{-N} \tag{3}$$

The coefficient α, $0 < \alpha < 1$, is the forget factor of the repetitive operator $\Delta_R(q^{-1})$. This operator allows to model periodic disturbances. N is the number of sampling periods in one period of the disturbances.

2.2 Control separation

The output vector $Y(k)$ can be written as follows:

$$Y(k) = Y_{th}(k) + \Xi(k)$$

where $Y_{th}(k)$ is the theoretical output vector, $\Xi(k)$ is the variation caused by errors design and the presence of noise or disturbances.
Then, The equation (1) is equivalent to the two following equations:

$$A(q^{-1})Y_{th}(k) = B(q^{-1})U^{(1)}(k-1) \tag{4}$$

$$A(q^{-1})\Xi(k) = B(q^{-1})U^{(2)}(k-1) + \frac{I_p}{\Delta_1(q^{-1})}E(k) \tag{5}$$

with: $U(k) = U^{(1)}(k) + U^{(2)}(k)$
$U^{(1)}(k)$ is the input vector to the theoretical system model and $U^{(2)}(k)$ is the input vector provided to a virtual system whose output vector is equal to the

difference $\Xi(k)$ between the real output vector $Y(k)$ and the theoretical output vector $Y_{th}(k)$.
By multiplying the equation (4) by $\Delta(q^{-1})$, we get:

$$A(q^{-1})\Delta(q^{-1})Y_{th}(k) = B(q^{-1})\Delta(q^{-1})U^{(1)}(k-1) \quad (6)$$

Multiplying the equation (5) by $\Delta_1(q^{-1})$ and replacing $A(q^{-1})\Delta_R(q^{-1})$ by $A_R(q^{-1})$ and $B(q^{-1})\Delta_R(q^{-1})$ by $B_R(q^{-1})$, the equation (5) becomes:

$$A_R(q^{-1})\Delta(q^{-1})\Xi(k) = B_R(q^{-1})\Delta(q^{-1})U^{(2)}(k-1) + E(k) \quad (7)$$

2.3 Synthesis of the law of multivariable repetitive generalized predictive control

The objective is to determine a control law $U(k)$ minimizing a quadratic criterion which can be given by the expressions (8), (9) and (10):

$$J = J^{(1)} + J^{(2)} \quad (8)$$

with:

$$J^{(1)} = \mathrm{E}\left\{ \begin{array}{l} \sum_{j=1}^{PH} \mu^{(1)} \left\| \hat{Y}_{th}\left(k+j/k\right) - Y_r(k+j) \right\|^2 \\ + \lambda^{(1)} \sum_{j=0}^{CH-1} \left\| \Delta U^{(1)}(k+j) \right\|^2 \end{array} \right\} \quad (9)$$

$$J^{(2)} = \mathrm{E}\{\sum_{j=1}^{PH} \mu^{(2)} \left\| \hat{\Xi}(k+j/k) \right\|^2 + \lambda^{(2)} \sum_{j=0}^{CH-1} \left\| \Delta U^{(2)}(k+j) \right\|^2 \} \quad (10)$$

where $\|X\|^2 = XX^T$.
PH and CH are respectively the prediction horizon and the control horizon. $\lambda^{(1)}$ and $\lambda^{(2)}$ weight the relative importance of both control energies. $\mu^{(1)}$ is the ponderation of the error vector between the prediction of theoretical model outputs $\hat{Y}_{th}(k+j/k)$ and the future references $Y_r(k+j)$. $\mu^{(2)}$ is the ponderation of the prediction errors $\hat{\Xi}(k+j/k)$.

Formulation of $U^{(1)}(k)$ From the equation (6), the optimal predictor of theoretical output vector is designed under the following form:

$$\begin{array}{l} \hat{Y}_{th}\left(k+j/k\right) = [Q_{1,\,j}(q^{-1})\Delta(q^{-1})U^{(1)}(k+j-1) \\ +R_{1,\,j}(q^{-1})\Delta(q^{-1})U^{(1)}(k-1) + G_{1,\,j}(q^{-1})Y_{th}(k)] \end{array} \quad (11)$$

where polynomial matrices $Q_{1,j}(q^{-1})$, $R_{1,j}(q^{-1})$ and $G_{1,j}(q^{-1})$ are the unique solutions derived from solving the matrix polynomial equations:

$$\left\{ \begin{array}{l} I_p = F_{1,j}(q^{-1})A(q^{-1})\Delta(q^{-1}) + q^{-j}G_{1,j}(q^{-1}) \\ F_{1,j}(q^{-1})B(q^{-1}) = Q_{1,j}(q^{-1}) + q^{-j}R_{1,j}(q^{-1}) \end{array} \right.$$

with:

$$\begin{cases} F_{1,j}(q^{-1}) = I_p + F_{1,1}^{(j)} q^{-1} + ... + F_{1,j-1}^{(j)} q^{-(j-1)} \\ G_{1,j}(q^{-1}) = G_{1,0}^{(j)} + G_{1,1}^{(j)} q^{-1} + ... + G_{1,n_A}^{(j)} q^{-n_A} \\ Q_{1,j}(q^{-1}) = Q_{1,0}^{(j)} + Q_{1,1}^{(j)} q^{-1} + ... + Q_{1,j-1}^{(j)} q^{-(j-1)} \\ R_{1,j}(q^{-1}) = R_{1,0}^{(j)} + R_{1,1}^{(j)} q^{-1} + ... + R_{1,n_B-1}^{(j)} q^{-(n_B-1)} \end{cases}$$

The equation (11) can be transcribed in the matrix form:

$$\underline{\hat{Y}}_{th} = \underline{R}^{(1)} \underline{\Delta U}^{(1)} + \underline{G}^{(1)} \underline{Y}_{th} + \underline{Q}^{(1)} \underline{\Delta U}_p^{(1)}$$

with:

$$\begin{aligned} \underline{\hat{Y}}_{th} &= \left[\hat{Y}_{th}(k+1/k)^T \cdots \hat{Y}_{th}(k+PH/k)^T \right]^T ; \\ \underline{\Delta U}^{(1)} &= \left[\Delta U^{(1)}(k-1)^T \cdots \Delta U^{(1)}(k-n_B)^T \right]^T ; \\ \underline{Y}_{th} &= \left[Y_{th}(k)^T \cdots Y_{th}(k-n_A)^T \right]^T ; \\ \underline{\Delta U}_p^{(1)} &= \left[\Delta U^{(1)}(k)^T \cdots \Delta U^{(1)}(k+CH-1)^T \right]^T ; \end{aligned}$$

$$\underline{R}^{(1)} = \begin{bmatrix} R_1^{(1)} \\ \vdots \\ R_1^{(CH)} \\ \vdots \\ R_1^{(PH)} \end{bmatrix} ; \ \underline{G}^{(1)} = \begin{bmatrix} G_1^{(1)} \\ \vdots \\ G_1^{(CH)} \\ \vdots \\ G_1^{(PH)} \end{bmatrix} ; \ \underline{Y}_r = \begin{bmatrix} Y_r(k+1) \\ \vdots \\ Y_r(k+CH) \\ \vdots \\ Y_r(k+PH) \end{bmatrix}$$

$$\begin{aligned} R_1^{(i)} &= \left[R_{1,0}^{(i)} \ldots R_{1,n_B-1}^{(i)} \right] ; \\ G_1^{(i)} &= \left[G_{1,0}^{(i)} \ldots G_{1,n_A}^{(i)} \right] ; \ i \in [1, \ PH] \end{aligned}$$

$$\underline{Q}^{(1)} = \begin{pmatrix} Q_{1,0}^{(1)} & 0 & 0 \\ \vdots & \ddots & \\ Q_{1,CH-1}^{(CH)} & \cdots & Q_{1,0}^{(CH)} \\ \vdots & & \vdots \\ Q_{1,PH-1}^{(PH)} & \cdots & Q_{1,PH-CH}^{(PH)} \end{pmatrix}$$

The future control sequence, that minimizes the criterion $J^{(1)}$, can be written as follows:

$$\underline{\Delta U}_p^{(1)} = -M^{(1)} [\underline{R}^{(1)} \underline{\Delta U}^{(1)} + \underline{G}^{(1)} \underline{Y}_{th} - \underline{Y}_r]$$

with:

$$M^{(1)} = \mu^{(1)} [\mu^{(1)} \underline{Q}^{(1)^T} \underline{Q}^{(1)} + \lambda^{(1)} I_{p.CH}]^{-1} \underline{Q}^{(1)^T} = \begin{pmatrix} \underline{M}_{1,1} \\ \vdots \\ \underline{M}_{1,CH} \end{pmatrix}$$

Only the first element of this sequence is considered and applied to the system:

$$\begin{aligned} U^{(1)}(k) &= \Delta U^{(1)}(k) + U^{(1)}(k-1) \\ &= -\underline{M}_{1,1}[\underline{R}^{(1)}\underline{\Delta U}^{(1)} + \underline{G}^{(1)}\underline{Y}_{th} - \underline{Y}_r] + U^{(1)}(k-1) \end{aligned} \tag{12}$$

Formulation of $U^{(2)}(k)$ From the equation (7), the predictor vector $\hat{\Xi}(k+j/k)$ can be expressed as follows:

$$\begin{aligned} &\hat{\Xi}(k+j/k) = [Q_{2,\,j}(q^{-1})\Delta(q^{-1})U^{(2)}(k+j-1) \\ &+R_{2,\,j}(q^{-1})\Delta(q^{-1})U^{(2)}(k-1) + G_{2,\,j}(q^{-1})\Xi(k)] \end{aligned} \tag{13}$$

where polynomial matrices $Q_{2,j}(q^{-1})$, $R_{2,j}(q^{-1})$ and $G_{2,j}(q^{-1})$ are the solutions of the diophantine matrix equations:

$$\begin{cases} I_p = F_{2,j}(q^{-1})A_R(q^{-1})\Delta(q^{-1}) + q^{-j}G_{2,j}(q^{-1}) \\ F_{2,j}(q^{-1})B_R(q^{-1}) = Q_{2,j}(q^{-1}) + q^{-j}R_{2,j}(q^{-1}) \end{cases}$$

with:

$$\begin{cases} F_{2,j}(q^{-1}) = I_p + F_{2,1}^{(j)}q^{-1} + ... + F_{2,j-1}^{(j)}q^{-(j-1)} \\ G_{2,j}(q^{-1}) = G_{2,0}^{(j)} + G_{2,1}^{(j)}q^{-1} + ... + G_{2,n_{A_R}}^{(j)}q^{-n_{A_R}} \\ n_{A_R} = n_A + N \\ Q_{2,j}(q^{-1}) = Q_{2,0}^{(j)} + Q_{2,1}^{(j)}q^{-1} + ... + Q_{2,j-1}^{(j)}q^{-(j-1)} \\ R_{2,j}(q^{-1}) = R_{2,0}^{(j)} + R_{2,1}^{(j)}q^{-1} + ... + R_{2,n_{B_R}-1}^{(j)}q^{-(n_{B_R}-1)} \\ n_{B_R} = n_B + N \end{cases}$$

From the equation (13), we can write:

$$\underline{\hat{\Xi}} = \underline{R}^{(2)}\underline{\Delta U}^{(2)} + \underline{G}^{(2)}\underline{\Xi} + \underline{Q}^{(2)}\underline{\Delta U}_p^{(2)}$$

with:

$$\begin{aligned} &\underline{\hat{\Xi}} = \left[\hat{\Xi}(k+1/k)^T \cdots \hat{\Xi}(k+PH/k)^T\right]^T; \\ &\underline{\Delta U}^{(2)} = \left[\Delta U^{(2)}(k-1)^T \cdots \Delta U^{(2)}(k-n_{B_R})^T\right]^T; \\ &\underline{\Xi} = \left[\Xi(k)^T \cdots \Xi(k-n_{A_R})^T\right]^T; \\ &\underline{\Delta U}_p^{(2)} = \left[\Delta U^{(2)}(k)^T \cdots \Delta U^{(2)}(k+CH-1)^T\right]^T; \end{aligned}$$

$$\underline{R}^{(2)} = \begin{bmatrix} R_2^{(1)} \\ \vdots \\ R_2^{(CH)} \\ \vdots \\ R_2^{(PH)} \end{bmatrix}; \ \underline{G}^{(2)} = \begin{bmatrix} G_2^{(1)} \\ \vdots \\ G_2^{(CH)} \\ \vdots \\ G_2^{(PH)} \end{bmatrix}$$

$$\begin{aligned} R_2^{(i)} &= \left[R_{2,\,0}^{(i)} \dots R_{2,\,n_{B_R}-1}^{(i)}\right]; \\ G_2^{(i)} &= \left[G_{2,\,0}^{(i)} \dots G_{2,\,n_{A_R}}^{(i)}\right]; \ i \in [1,\ PH] \end{aligned}$$

$$\underline{Q}^{(2)} = \begin{pmatrix} Q_{2,0}^{(1)} & 0 & 0 \\ \vdots & \ddots & \\ Q_{2,CH-1}^{(CH)} & \cdots & Q_{2,0}^{(CH)} \\ \vdots & & \vdots \\ Q_{2,PH-1}^{(PH)} & \cdots & Q_{2,PH-CH}^{(PH)} \end{pmatrix}$$

The minimization of the equation (10) provides the future optimal control $\underline{\Delta U}_p^{(2)}$:

$$\underline{\Delta U}_p^{(2)} = -M^{(2)}[\underline{R}^{(2)}\underline{\Delta U}^{(2)} + \underline{G}^{(2)}\underline{\Xi}]$$

with:

$$M^{(2)} = \mu^{(2)}[\mu^{(2)}\underline{Q}^{(2)^T}\underline{Q}^{(2)} + \lambda^{(2)}I_{p.CH}]^{-1}\underline{Q}^{(2)^T} = \begin{pmatrix} \underline{M}_{2,1} \\ \vdots \\ \underline{M}_{2,CH} \end{pmatrix}$$

The control vector $U^{(2)}(k)$ will be deduced from this last vector as follows:

$$\begin{aligned} U^{(2)}(k) &= \Delta U^{(2)}(k) + U^{(2)}(k-1) \\ &= -\underline{M}_{2,1}[\underline{R}^{(2)}\underline{\Delta U}^{(2)} + \underline{G}^{(2)}\underline{\Xi}] + U^{(2)}(k-1) \end{aligned} \quad (14)$$

The synthesis of the multivariable repetitive generalized predictive control law requires precise knowledge of the disturbance period. However in practice, this period may be unknown or time-varying. So it is interesting to on-line estimate the disturbance period from input output measurement. We present a magnitude/phase-locked loop approach to parameter estimation of periodic disturbances.

3 A magnitude/phase-locked loop approach to parameter Estimation of periodic signals

Online estimation of the frequency of periodic signals is a classical problem in systems theory that has many practical applications. There exist several algorithms to on-line estimate the frequency of periodic signals: adaptive notch filtering [20, 21], extended kalman filter frequency estimation [22, 23], the phase locked loop [24], and many other techniques [25].

This section presents a frequency estimator based on a magnitude/phase locked loop (MPLL). The approach is a simple procedure which consists to estimate simultaneously the frequencies, magnitudes and phases of a periodic signals [18, 26]. This approach is able to maintain tracking of the fundamental frequency despite changes in signals characteristics.

Let $P(k)$, a vector of p periodic disturbance signals affecting the outputs of multivariable system, which can be defined as follows:

$$P(k) = \begin{bmatrix} p_1(k) \\ \vdots \\ p_p(k) \end{bmatrix} = \begin{bmatrix} a_{p_1}\cos(\phi_1(k)) \\ \vdots \\ a_{p_p}\cos(\phi_p(k)) \end{bmatrix} \quad (15)$$

where a_{p_l} and $\phi_l(k)$ ($l \in [1..p]$) are respectively the magnitude and the phase of the periodic signal $p_l(k)$.
$w_l = \frac{2\pi}{T_l}$ ($l \in [1..p]$) is the frequency of $p_l(k)$ in which T_l is the period of this signal.
The estimate signal of $P(k)$, noted $\hat{P}(k)$, is given by the following expression:

$$\hat{P}(k) = \begin{bmatrix} \hat{p}_1(k) \\ \vdots \\ \hat{p}_p(k) \end{bmatrix} = \begin{bmatrix} \hat{a}_{p_1}(k)\cos(\hat{\phi}_1(k)) \\ \vdots \\ \hat{a}_{p_p}(k)\cos(\hat{\phi}_p(k)) \end{bmatrix} \quad (16)$$

where $\hat{a}_{p_l}(k)$, $\hat{\phi}_l(k)$, $\hat{w}_l(k)$ and $\hat{T}_l(k)$ ($l \in [1..p]$) are respectively the estimates of magnitudes, phases, frequencies and periods of the signal $p_l(k)$.
The algorithm of MPLL approach is given by [18]:

$$\begin{cases} \hat{a}_{p_l}(k) = \frac{2g_{a_l}}{q-1}(p_l(k) - \hat{p}_l(k))cos(\hat{\phi}_l(k)), \quad l \in [1..p] \\ \hat{w}_l(k) = -\frac{2g_{w_l}}{q-1}(p_l(k) - \hat{p}_l(k))sin(\hat{\phi}_l(k)), \quad l \in [1..p] \\ \hat{\phi}_l(k) = \frac{K_{w_l}(q-N_{w_l})}{q-1}\hat{w}_l(k), \quad l \in [1..p] \\ N_{w_l} = 1 - \frac{1}{K_{w_l}}, \quad l \in [1..p] \end{cases} \quad (17)$$

where the design parameters g_{a_l}, g_{w_l} and K_{w_l} ($l \in [1..p]$) are adaptive parameters.

4 Multivariable Repetitive generalized predictive control with an online frequency estimator of a periodic disturbances

Periodic disturbances can occur in many different engineering control applications. Clearly, these disturbances will degrade system performance. Multivariable repetitive generalized predictive control, which combines the idea of repetitive control and multivariable generalized predictive control, results in perfect set point tracking and disturbances rejection only when the period of disturbances is known. When disturbances of unknown period are considered, the performance of MRGPC will be considerably reduced. To cancellate a periodic disturbances with unknown frequency, an on-line frequency estimator can be inserted in the control design, namely a multivariable repetitive generalized predictive control design.
To estimate the disturbance frequency, we use a magnitude/phase locked loop approach. This approach requires the knowledge of a periodic disturbances to identify its different parameters. In fact, the disturbance is known or measured, only through its effect on the output of the system.
Then, we define:

$$E_c(k) = Y(k) - Y_r(k) \quad (18)$$

This difference may be considered as an image of a frequency variation if the rejection is ensured.

We note, $\hat{E}_c(k)$ the estimate of $E_c(k)$.
Initially, we suppose that the period of the harmonic disturbances is known, $T = T^*$, with T^* is a known constant.
Then, the MRGPC control can be applied to the system. When the period of disturbances varies, we use the MPLL approach to estimate the new period of the disturbances. When convergence is ensured, the multivariable repetitive generalized predictive control is recalculated with the new estimated period.
The structure of MRGPC with an on-line frequency estimator is represented in Fig. 1. This control scheme is divided into two parts: a multivariable repetitive generalized predictive control and a magnitude phase/locked loop approach. The MRGPC is sum of two controller: MGPC1 responsible for the reference tracking and MGPC2 responsible for rejection of periodic disturbances that depends on the frequency of disturbances. Magnitude phase/locked loop approach is responsible for the determination of the frequency of disturbances.

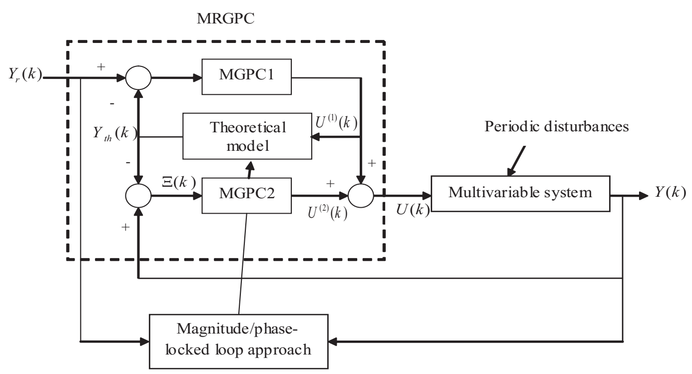

Fig. 1. The structure of MRGPC with an on-line frequency estimator.

5 Simulation example

Consider a two inputs/two outputs discrete-time linear time invariant system described by the following equation:

$$A(q^{-1})Y(k) = B(q^{-1})U(k-1) + \frac{I_2}{\Delta_1(q^{-1})}E(k)$$

with:

$$\begin{cases} A(q^{-1}) = I_2 + A_1 q^{-1} + A_2 q^{-2} \\ B(q^{-1}) = B_0 + B_1 q^{-1} \\ P(k) = \frac{I_2}{\Delta_1(q^{-1})} E(k) \end{cases}$$

and

$$\Delta_1(q^{-1}) = \Delta(q^{-1})\Delta_R(q^{-1}) = (1-q^{-1})(1-\alpha q^{-N})$$

$$A_1 = \begin{pmatrix} -1.856 & 0 \\ 0 & -1.9252 \end{pmatrix}; \qquad A_2 = \begin{pmatrix} 0.8606 & 0 \\ 0 & 0.9265 \end{pmatrix}$$

$$B_0 = \begin{pmatrix} 0.09516 & 0.04877 \\ 0.04877 & 0.2597 \end{pmatrix}; \qquad B_1 = \begin{pmatrix} -0.0905 & -0.0441 \\ -0.0475 & -0.247 \end{pmatrix}$$

$$Y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}; \qquad U(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}; \qquad P(k) = \begin{bmatrix} p_1(k) \\ p_2(k) \end{bmatrix}$$

We suppose that the disturbances are time varying and are given by:

$$\begin{cases} p_1(k) = \begin{cases} 0.4\cos(\frac{2\pi}{30}k); & k \leq 500 \\ 0.2\cos(\frac{2\pi}{20}k); & k > 500 \end{cases} \\ \\ p_2(k) = \begin{cases} 0.3\cos(\frac{2\pi}{30}k); & k \leq 500 \\ 0.15\cos(\frac{2\pi}{20}k); & k > 500 \end{cases} \end{cases}$$

The fixed reference trajectories is given by the following equation:

$$Y_r(k) = \begin{bmatrix} y_{r1}(k) \\ y_{r2}(k) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

To track the reference trajectories and reject the periodic disturbances, we use the multivariable repetitive generalized predictive control. The retained synthesis parameters are:

$$PH = 10,\ \ CH = 5,\ \ \mu^{(1)} = 10,\ \ \lambda^{(1)} = \lambda^{(2)} = 0.0001$$
$$\mu^{(2)} = 20,\ \ \alpha = 0.99.$$

The MRGPC requires knowledge of the frequency of disturbances. Then, we suppose that the frequency of disturbances is known and $N = 30$.
The evolution of periodic disturbance $p_1(k)$ and $p_2(k)$ is given in Fig. 2. The Fig. 3 shows the evolutions of the desired reference trajectory $Y_r(k)$ and the output of the system $Y(k)$. The Fig. 4 presents the evolutions of the controller $U(k)$. For $k \leq 500$, it can be seen that the considered control allows a good disturbances rejection. After $k = 500$, the disturbances are not rejected completely because the frequency of disturbances change their value.

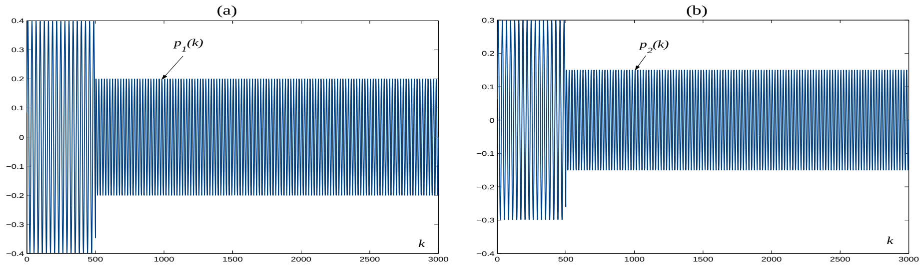

Fig. 2. (a). The evolution of periodic disturbance $p_1(k)$. -(b). The evolution of periodic disturbance $p_2(k)$.

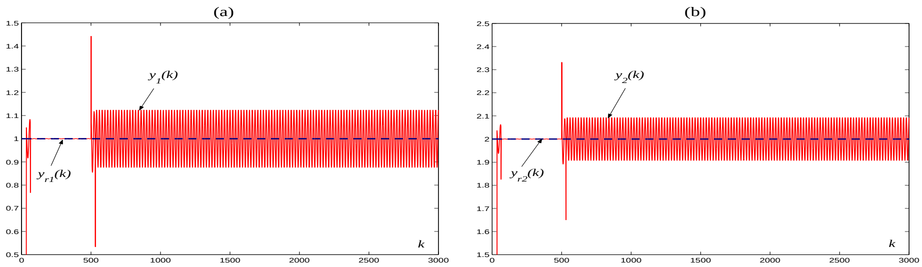

Fig. 3. (a). The evolutions of the desired reference trajectory $y_{r1}(k)$ and the output system $y_1(k)$. -(b). The evolutions of the desired reference trajectory $y_{r2}(k)$ and the output system $y_2(k)$.

To ameliorate the performance, particularly in term of the rejection of disturbances, we use a multivariable repetitive generalized predictive control with an on-line frequency estimator. The frequency of disturbances was estimated using the MPLL method.

The evolution of the estimated frequency is given in Fig. 5. It is clear refering to this figure that the estimate reproduces the frequency of disturbances satisfactorily. The Fig. 6 shows the evolution of the desired reference trajectory $Y_r(k)$ and the output of the system $Y(k)$. The evolutions of the controller $U(k)$ is given in Fig. 7. These figures proves that a very satisfactory performances are recorded in terms of tracking reference trajectories and rejecting periodic disturbances with variable frequency. A comparison between Fig. 3 and Fig. 6 reveals that the use of the on-line estimated disturbance frequency significantly reduces the tracking error.

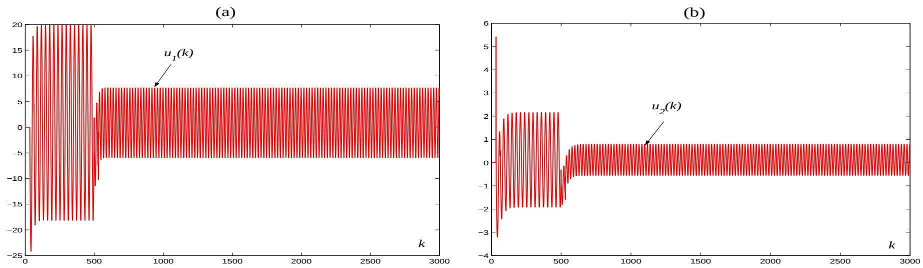

Fig. 4. (a). The evolution of the control $u_1(k)$. -(b). The evolution of the control $u_2(k)$.

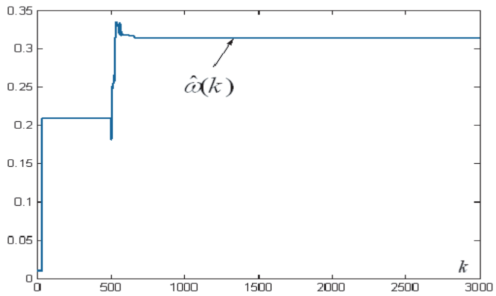

Fig. 5. The evolution of the estimated frequency.

6 Conclusion

This paper deals with control and rejection of periodic disturbances problems for multivariable systems. We have proposed a multivariable repetitive generalized predictive control with an on-line frequency estimator. A magnitude phase/locked loop approach for disturbances characteristic estimation is used. The use of the strategy shows a very satisfactory results in terms of tracking of the reference trajectories and rejection of periodic disturbances. When the frequency is unknown or time varying, the application of this approach is better than the case where the multivariable repetitive generalized predictive control is applied without frequency estimator.

The obtained simulation results show a perfect tracking of reference trajectories and a good rejection of periodic disturbances in spite of the presence of disturbance frequency variation.

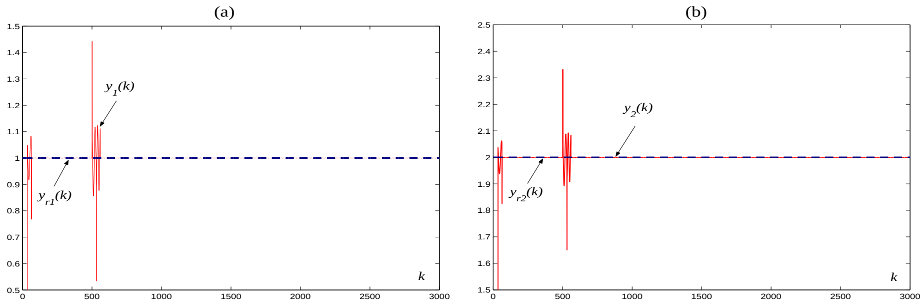

Fig. 6. (a). The evolutions of the desired reference trajectory $y_{r1}(k)$ and the output system $y_1(k)$. -(b). The evolutions of the desired reference trajectory $y_{r2}(k)$ and the output system $y_2(k)$.

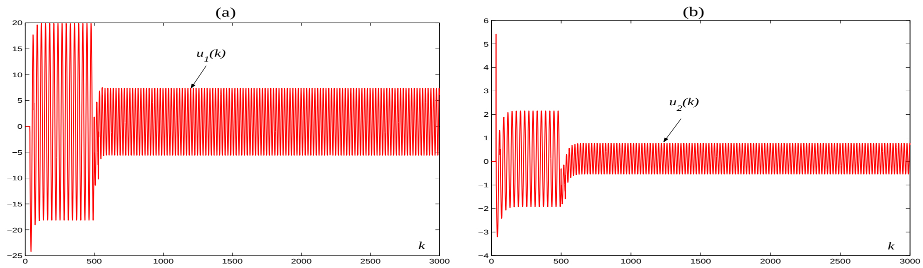

Fig. 7. (a). The evolution of the control $u_1(k)$. -(b). The evolution of the control $u_2(k)$.

References

1. S. Valentinotti, *Adaptive rejection of unstable disturbances : Application to a fed batch fermentation process*, Thèse, Ecole Polytechnique Fédérale de Lausanne, 2001.
2. K. W. Eure, *Adaptive predictive feedback techniques for vibration control*, Thèse, Faculty of the Virginia Polytechnic Institute, 1998.
3. M. Gupta, J. H. Lee, *Period-robust repetitive model predictive control*, Journal of Process Control, Vol. 16, No. 545-555, 2006.
4. Y. Q. Chen, K. L. Moore, J. Yu, and T. Zhang, *Iterative learning control and repetitive control in hard disk drive industry - a tutorial*, IEEE Trans. on Systems, Man and Cybernetics, Part-C, 2006.
5. R. Ginhoux, *Active filtering of physiological motion in robotized surgery using predictive control*, IEEE Transactions on Robotics, pp. 67-79, November 2003.
6. B. Francis, W. Wonham, *The internal model principle of control theory*, Automatica, Vol. 12, No. 5, pp. 457-465, 1976.
7. B. Wu, M. Bodson, *Multi-channel active noise control for periodic sources-indirect approach*, Automatica, Vol. 40, No. 2, pp. 203-212, 2004.
8. S. Arimoto, S. Kawamura, and F. Miyazaki, *Bettering operation of dynamic systems by learning : A new control theory for servomechanism or mechatronics systems*, Proceedings of 23rd Conference on Decision and Control, pp. 1064-1069, 1984.

9. S. Hara, Y. Yamamoto, T. Omata, M. Nakano, *Repetitive control system: A new type servo system for periodic exogenous signals*, IEEE Transactions on Automatic Control, Vol. 33, No. 7, pp. 659-668, 1988.
10. B. G. Dijkstra, *Iterative Learning Control, with applications to a wafer stage*, Systems and Control Group, Delft University of Technology, 2003.
11. J. R. Ryoo, T.-Y. Doh, *Feedback based iterative learning control for mimo LTI systems*, International Journal of Control Automation and Systems, Vol. 6, No. 2, pp. 269-277, 2008.
12. H.-S. Ahn, Y. Q. Chen, K. L. Moore, *Iterative learning control: brief survey and categorization*, IEEE Transactions on Systems, Vol. 37, No. 6, pp. 1099-1121, 2007.
13. D. W. Clarke, C. Mohtadi, and P. S. Tuffs, *Generalized Predictive Control: Part I: The basic algorithm, Part II: Extensions and interpretations*, Automatica, Vol. 23, pp. 137-160, 1987.
14. P. Boucher, D. Dumur, *La commande prédictive*, Editions Technip-Paris, 1996.
15. J. M. Dion, L. Dugard, A. Franco, N. M. Tri, D. Rey, *Mimo adaptive constrained predictive control case study: An environmental test chamber*, Automatica, Vol. 27, No. 4, pp. 611-626, 1991.
16. G. A. Munoz-Hernandez, D. Jones, *Mimo generalized predictive control for a hydroelectric power station*, IEEE Transactions on Energy Conversion, Vol. 21, No. 4, pp. 921-929, 2006.
17. M. M'Saad, G. Sanchez, *Multivariable generalized predictive adaptive control with a suitable tracking capability*, J. Proc. Cont, Vol. 4, No. 1, pp. 45-52, 1994.
18. B. Wu, M. Bodson, *A magnitude/phase-locked loop approach to parameter estimation of periodic signals*, IEEE Transactions on Automatic Control, Vol. 48, No. 4, pp. 612-618, 2003.
19. R. Ginhoux, *Compensation des mouvements physiologiques en chirurgie robotisée par commande prédictive*, Thèse, Université Louis Pasteur Strasbourg, 2003.
20. L. Hsu, R. Ortega, G. Damm, *A globally convergent frequency estimator*, IEEE Transactions on Automatic Control, Vol. 44, No. 4, pp. 698-713, 1999.
21. G. Li, *A stable and efficient adaptive notch filter for direct frequency estimation*, IEEE Transactions on Signal Processing, Vol. 45, No. 8, pp. 2001-2009, 1997.
22. B. F. L. Scala, R. R. Bitmead, *Design of an extended kalman filter frequency tracker*, IEEE, pp. 525-527, 1994.
23. S. Bittanti, S. M. Savaresi, *Frequency tracking via extended kalman filter: parameter design*, Proceedings of the American Control Conference, pp. 2225-2229, 2000.
24. J. Stensby, *Phase Locked Loops : Theory and Applications*, Boca Raton, FL : CRC, 1997.
25. P. J. Kootsookos, *A review of the frequency estimation and tracking problems*, 1999.
26. X. Guo, M. Bodson, *Adaptive rejection of multiple sinusoids of unknown frequency*, Proceeding of the European Control Conference, 2007.