International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 5, N° 1, December 2011, pp. 1486–1499.

Lyapunov Based Second Order Slinding Mode Control For MIMO Nonlinear Systems

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Abstract. *In this paper, a new second order sliding mode control (SOSMC) for coupled (multi-input multi-output) MIMO nonlinear system is presented, the proposed method can be applied to a large class of nonlinear coupled MIMO processes affected by parameters uncertainties and disturbances. This algorithm are implemented on the three tanks test-bed system and the exprimental results confirm the effectiveness of our control design.*

Keyword. *Second order sliding mode , nonlinear systems, coupled, MIMO, uncertain, Liquid level control.*

1 Introduction

Sliding mode control techniques design simple control laws which constrain the system motion on suitably chosen manifolds. The sliding motion is guaranteed despite uncertainties and strong nonlinearities, and is characterized by good properties (invariance, perfect tracking)[1-2]. But the major drawback of the conventional classic sliding mode control is the existence of chattering phenomenon [3]. To avoid this problem second order sliding mode control (SOSMC) has been proposed.

The multivariable case has been extensively studied in the framework of the

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

Classic SMC. However there are very few results in the literature about the generation of second order sliding modes for MIMO systems. This is mainly due to the coupling between the various outputs of the system and the considerate surfaces. In [4-10] authors have considered the multivariable system as a collection of mono-variable (SISO) systems and they have applied the sliding mode control algorithms to SISO systems without considering the coupling.

In this paper, we propose a new second order sliding mode control for MIMO uncertain coupled systems which permit the taking into account Coupling, parameters uncertainties and disturbances. Exprimental results are presented to illustrate the effectiveness of the proposed controllers.

The paper is organized as follows. In section 2 second order sliding mode control of coupled MIMO nonlinear systems is proposed. Its robustness to parametric uncertainties and external disturbances is studied in section 3. The model of the coupled three tanks system and its controls by SMC of this system are developed in section 4. The exprimental resultants are presented in section 5. Finally the conclusion is given in section 6.

2 Second order sliding mode control

Consider a MIMO nonlinear system which has p inputs and m outputs defined by the following state representation:

$$\begin{cases} \dot{x} = f(t,x) + g(t,x)u \\ y = c(t,x) \end{cases} \tag{1}$$

with:
x the n-dimensional state vector.

$$x = [x_1 \cdots x_n]^T \tag{2}$$

y the m-dimensional output vector.

$$y = [y_1 \cdots y_m]^T \tag{3}$$

$c(t,x)$: is a vector of dimension m whose coefficients are nonlinear functions $c_i(x,t)$.
$f(t,x)$: is a vector of dimension n the coefficients of which are nonlinear functions $f_i(x,t)$.
$g(t,x)$ a $(n \times p)$ matrix the coefficients are the nonlinear functions $g_{ij}(x,t)$.
u : is the p dimensional control vector of coefficients u_i.

$$u = [u_1 \cdots u_p]^T \tag{4}$$

Consider the sliding surface [11] defined by:

$$s = [s_1 \cdots s_p]^T \tag{5}$$

where:

$$s_i = \sum_{k=0}^{r_i-1} \lambda_k^{(i)} e_i^{(k)}, \ \ for \ \ i = 1, \ldots, p \tag{6}$$

with:
$\lambda_{r_i-1}^{(i)} = 1$ and $k = 0, \ldots, r_i - 2$
r_i : is the relative degree of the sliding surface s_i.
$\lambda_k^{(i)}$: are constants chosen to have $\sum_{k=0}^{r_i-1} \lambda_k^{(i)} P^k$ a Hurwitz polynomial.
$e_i = y_i - y_i^d = c_i(t,x) - y_i^d$.
$e_i^{(k)}$: is the $k-th$ order derivative of the error.

The derivative of every surface s_i is:

$$\frac{ds_i}{dt} = \frac{\partial\ s_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} \dot{x}_j \tag{7}$$

$$\frac{ds_i}{dt} = \frac{\partial\ s_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} (f_j + g_{j1}u_1 + \cdots + g_{jp}u_p) \tag{8}$$

$$\frac{ds_i}{dt} = \frac{\partial\ s_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} f_j + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} g_{j1}u_1 + \cdots + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} g_{jp}u_p \tag{9}$$

$$\frac{ds_i}{dt} = h_i + b_{i1}u_1 + \cdots + b_{ip}u_p = f_i + \sum_{k=1}^{p} b_{ik}u_k \tag{10}$$

with:

$$h_i = \frac{\partial\ s_i}{\partial t} + \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} f_j \tag{11}$$

$$b_{ik} = \sum_{j=1}^{n} \frac{\partial\ s_i}{\partial x_j} g_{jk} \tag{12}$$

Then one can write the derivative of surface vector under the following shape:

$$\dot{s} = h + bu \tag{13}$$

with:

$h = [h_1 \cdots h_p]^T$ and $b = \begin{bmatrix} b_{11} & \cdots & b_{1P} \\ \vdots & \ddots & \vdots \\ b_{P1} & \cdots & b_{PP} \end{bmatrix}$.

Theorem 1. *The control law for the second order sliding mode control (SOSMC) of the system (1) so that the sliding surfaces and heir derivatives go to zero in a finite time is defined by:*

$$u = -b^{-1} \left(h + \begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ \vdots \\ k_p sign(s_p + \ddot{s}_p) \end{bmatrix} \right) \tag{14}$$

where k_i are positive constants and b an inversible matrix.

Proof. We consider the following Lyapunov function:

$$V = \frac{1}{2}(s^T s + \dot{s}^T \dot{s}) = \frac{1}{2}\sum_{i=1}^{p}(s_i^2 + \dot{s}_i^2) \tag{15}$$

the derivative of V is:

$$\dot{V} = \sum_{i=1}^{p}(\dot{s}_i s_i + \dot{s}_i \ddot{s}_i) = (s + \ddot{s})^T \dot{s} \tag{16}$$

Remplacing $\dot{s}$ by its value in (13) and assuming the control given by(14)

$$\dot{s} = h + bu = -\begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ \vdots \\ k_p sign(s_p + \ddot{s}_p) \end{bmatrix} \tag{17}$$

The expression of $\dot{V}$ becomes:

$$\dot{V} = -(s + \ddot{s})^T \begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ \vdots \\ k_p sign(s_p + \ddot{s}_p) \end{bmatrix} \tag{18}$$

then

$$\dot{V} = -\sum_{i=1}^{p} k_i(s_i + \ddot{s}_i) sign(s_i + \ddot{s}_i) \tag{19}$$

$$\dot{V} = -\sum_{i=1}^{p} k_i |s_i + \ddot{s}_i| \leq 0 \tag{20}$$

Then, the Lyapunov function V tends to 0 and therefore all surfaces s_i tend to zero and their derivative, hence the existence of second order sliding mode.

Remark. The problem in the realisation of this control is that $\ddot{s}_i$ is not available and must be estimated. To do that we use p second order differentiators [13].

$$\begin{aligned}
&\dot{z}_{i0} = \nu_{i0}, \quad \nu_{i0} = -\lambda_{i0}\,|z_{i0} - s_i(t)|^{\frac{2}{3}}\, sign(z_{i0} - s_i(t)) + z_{i1} \\
&\dot{z}_{i1} = \nu_{i1}, \quad \nu_{i1} = -\lambda_i 1 |z_{i1} - \nu_{i0}|^{\frac{1}{2}} sign(z_{i1} - \nu_{i0}) + z_{i2} \\
&\dot{z}_{i2} = -\lambda_{i2} sign(z_{i2} - \nu_{i1})
\end{aligned}$$

which supplies us with z_i0, z_i1 and z_i2 the estimates of s_i, $\dot{s}_i$, $\ddot{s}_i$, for $i = 1$ to p. (p is the number of sliding surfaces)

3 Robustness to parametric uncertainties and external disturbances

Consider au uncertain MIMO nonlinear system :

$$\dot{x} = \hat{f}(t,x) + \Delta f(t,x) + (\hat{g}(t,x) + \Delta g(t,x))u + d \tag{21}$$

Where $x \in \Re^n$ is the state vector, $u \in \Re^p$ is the input control bounded as $|u_i| \leq u_{imax}$ for i=1 to p, the vector field $f = \hat{f}(t,x) + \Delta f(t,x)$, is continuous and smooth, Where $\hat{f}(t,x)$ is the nominal part and $\Delta f(t,x)$ is the uncertain part bounded by a known function,$d \in D \subset \Re^p$ represent the disturbances.
The dynamic $g(t,x)$, are not exactly known and it is written as the sum of the nominal part $\hat{g}$ and the uncertain part Δg.
with:

$$\hat{f}(t,x) = \begin{bmatrix} \hat{f}_1(t,x) \\ \vdots \\ \hat{f}_n(t,x) \end{bmatrix}, \quad \Delta f(t,x) = \begin{bmatrix} \Delta f_1(t,x) \\ \vdots \\ \Delta f_n(t,x) \end{bmatrix},$$

$$d = \begin{bmatrix} d_1 \\ \vdots \\ d_p \end{bmatrix}, \quad \hat{g} = \begin{bmatrix} \hat{g}_{11}(t,x) & \cdots & \hat{g}_{1p}(t,x) \\ \vdots & \ddots & \vdots \\ \hat{g}_{p1}(t,x) & \cdots & \hat{g}_{pp}(t,x) \end{bmatrix}$$

$$and \quad \Delta g = \begin{bmatrix} \Delta g_{11}(t,x) & \cdots & \Delta g_{1p}(t,x) \\ \vdots & \ddots & \vdots \\ \Delta g_{p1}(t,x) & \cdots & \Delta g_{pp}(t,x) \end{bmatrix}.$$

Then the derivative of the sliding surface takes the following form:

$$\frac{ds_i}{dt} = \hat{h}_i + \Delta h_i + \sum_{k=1}^{p} (\hat{b}_{ik} + \Delta b_{ik}) u_k \tag{22}$$

with:

$$\hat{h}_i = \frac{\partial\, s_i}{\partial\, t} + \sum_{j=1}^{n} \frac{\partial\, s_i}{\partial\, x_j} \hat{f}_j, \quad \Delta h_i = \sum_{j=1}^{n} \frac{\partial\, s_i}{\partial\, x_j} \Delta f_j,$$

$$\hat{b}_{ik} = \sum_{j=1}^{n} \frac{\partial\, s_i}{\partial\, x_j} \hat{g}_{jk}, \quad \Delta b_{ik} = \sum_{j=1}^{n} \frac{\partial\, s_i}{\partial\, x_j} \Delta g_{jk}, \quad and \quad \delta_i = \sum_{j=1}^{n} \frac{\partial\, s_i}{\partial\, x_j} d_i.$$

We can rewrite the derivative of the surface as follows:

$$\dot{s} = \hat{h} + \Delta h + (\hat{b} + \Delta b)u + \delta \tag{23}$$

with :

$$\hat{h}(t,x) = \begin{bmatrix} \hat{h}_1(t,x) \\ \vdots \\ \hat{h}_p(t,x) \end{bmatrix}, \quad \Delta h(t,x) = \begin{bmatrix} \Delta h_1(t,x) \\ \vdots \\ \Delta h_p(t,x) \end{bmatrix},$$

$$\delta = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_p \end{bmatrix}, \quad \hat{b} = \begin{bmatrix} \hat{b}_{11}(t,x) & \cdots & \hat{b}_{1p}(t,x) \\ \vdots & \ddots & \vdots \\ \hat{b}_{p1}(t,x) & \cdots & \hat{b}_{pp}(t,x) \end{bmatrix}$$

$$and \quad \Delta b = \begin{bmatrix} \Delta b_{11}(t,x) & \cdots & \Delta b_{1p}(t,x) \\ \vdots & \ddots & \vdots \\ \Delta b_{p1}(t,x) & \cdots & \Delta b_{pp}(t,x) \end{bmatrix}$$

Theorem 2. *Consider the uncertain system defined by equation (21). The control law :*

$$u = -\hat{b}^{-1} \left(\hat{h} + \begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ \vdots \\ k_p sign(s_p + \ddot{s}_p) \end{bmatrix} \right) \tag{24}$$

with k_i satisfying:

$$k_i > \alpha_i + \delta^* + \beta_{i1} u_{1max} + \cdots + \beta_{ip} u_{pmax} \tag{25}$$

where:

$$|\Delta h_i| < \alpha_i, \quad |\Delta b_{ij}| < \beta_{ij}, \quad |\delta_i| < \delta_i^* \quad and \quad |u_i| < u_{imax} \tag{26}$$

ensures the convergence of the sliding surface and its derivative to zero.

Proof. Using the control law (24) the expression of the derivative of the surface becomes:

$$u = -\hat{b}^{-1} \left(\hat{h} + \begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ \vdots \\ k_p sign(s_p + \ddot{s}_p) \end{bmatrix} \right)$$

The expression of the derivative of the surface becomes:

$$\dot{s} = \hat{h} + \hat{b}u + \Delta h + \Delta b u + \delta \tag{27}$$

The derivative of the surface s_i is then written:

$$\dot{s_i} = -k_i sign(s_i + \ddot{s}_i) + \sum_{k=1}^{p} \Delta b_{ik} u_k + \delta_i \tag{28}$$

if $s_i + \ddot{s_i} > 0$, we must have
$\Delta h_i - k_i + \Delta b_{i1} u_1 + \sum_{k=1}^{p} \Delta b_{ik} u_k + \delta_i < 0$
then:

$$k_i > \Delta h_i + \sum_{k=1}^{p} \Delta b_{ik} u_k + \delta_i \quad (29)$$

if $s_i + \ddot{s_i} < 0$, we must have
$\Delta h_i + k_i + \sum_{k=1}^{p} \Delta b_{ik} u_k + \delta_i > 0$
then:

$$k_i > -\Delta h_i - \sum_{k=1}^{p} \Delta b_{ik} u_k - \delta_i \quad (30)$$

The condition (29) and (30) are satisfied if:

$$k_i > \alpha_i + \sum_{k=1}^{p} \beta_{ik} u_{kmax} + \delta_i^* \quad (31)$$

then $\dot{V} < 0$, which end the proof.

4 Coupled-Tanks System

4.1 Sytem description and modeling

The process considered is a three tank system, which have two inputs and three outputs. It consists of three cylindrical tanks with identical section a supplied with distilled water, which are serially interconnected by two cylindrical pipes of identical sections S_n. The pipes of communication between the tanks T_1 and T_2 are equipped with manually adjustable values; the flow rates of the connection pipes can be controlled using ball valves az_1 and az_2. The plant has one outlet pipe located at the bottom of tank T_3. There are three other pipes installed at the bottom of each Tank, they are provided with a direct connection (outflow rate) to the reservoir with ball valves bz_1, bz_2 and bz_3, respectively, it can only be manipulated manually. The pumps 1 and 2 are supplied by water from the water tank below the three tanks with flow rates $Q_1(t)$ and $Q_2(t)$, respectively. The necessary level measurements $h_1(t)$, $h_2(t)$ and $h_3(t)$ are carried out by the piezo-resistive differential pressure sensors. The state equations are obtained by writing that the variation of the volume of water in a tank is equal to the sum of the incoming flow minus the sum of outgoing flows, that means, the water of the tanks 1and 2 can flow toward the tank 3. Then, the system can be represented by the following equations:

$$\dot{h}_i(t) = \frac{1}{A}(Q_i^{in}(t) - Q_{ij}^{out1}(t) - Q_{ij}^{out2}(t))\; i,j = 1,2,3 \quad (32)$$

where $Q_{ij}^{out1}(t)$ represents the flow rates of water between the tanks i and j $(i,j = 1,2,3\forall \quad i \neq j)$, and can be expressed while using the law of Torricelli[12].

$$Q_{ij}^{out1}(t) = a_{zi} S_n sign(h_i - h_j)\sqrt{2g|h_i - h_j|}, i = 1,3 \quad (33)$$

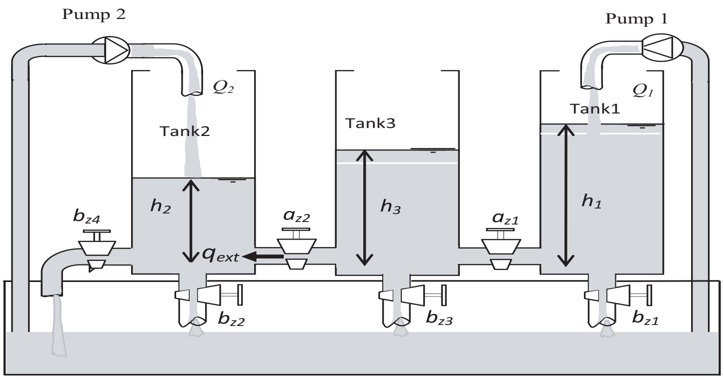

Fig. 1. Laboratory experiment : Thtree tank system

and $Q_{ij}^{out2}(t)$ represent the outflow rate, given by:

$$Q_{ij}^{out2}(t) = b_{zj} S_L \sqrt{2gh_i} \qquad j = 1, 2, 3, 4 \tag{34}$$

Where $h_i(t)$, $Q_i^{in}(t)$ and $Q_{ij}^{out}(t)$ are respectively the levels of water, the input flow and the output flow rates.
The controlled signal is the water levels (h_2, h_3) of tank 2 and tank 3. This level is controlled by two pumps P_1 and P_2. The system can be considered as a multi inputs multi outputs system (MIMO) where the inputs is inflow rate Q_1, Q_2 and outputs is liquid levels h_2, h_3. Then the three tanks system can be modeled by the following three differential equations:

$$\frac{dh_1}{dt} = -c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} + \frac{Q_1}{a} - B_1\sqrt{h_1} \tag{35}$$

$$\frac{dh_2}{dt} = c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2|} + \frac{Q_2}{a} - (B_4 + B_2)\sqrt{h_2} \tag{36}$$

$$\frac{dh_3}{dt} = c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} - B_3\sqrt{h_3} - c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2|} \tag{37}$$

While taking $B_1 = B_2 = B_3 = 0$, the three equations of the system become:

$$\frac{dh_1}{dt} = -c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} + \frac{Q_1}{a} \tag{38}$$

$$\frac{dh_2}{dt} = c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2|} + \frac{Q_2}{a} - B_4\sqrt{h_2} \tag{39}$$

$$\frac{dh_3}{dt} = c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} - c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2|} \tag{40}$$

Where the parameters are defined by:

$$c_i = \frac{1}{a} a_{zi} S_n \sqrt{2g} \quad i = 1, 3$$

$$B_j = \frac{1}{a} b_{zj} S_L \sqrt{2g} \quad j = 1, 2, 3, 4$$

At equilibrium, for constant water level set point, the level derivatives must be zero.

$$\dot{h}_1 = \dot{h}_2 = \dot{h}_3 = 0 \tag{41}$$

Therefore, using (45) in the steady state, the following algebraic relationship holds.

$$\begin{cases} -c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} + \frac{Q_1}{a} = 0 \\ c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} - c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2} = 0 \end{cases} \tag{42}$$

then:

$$\begin{cases} -c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} = \frac{Q_1}{a} \\ c_1 sign(h_1 - h_3)\sqrt{|h_1 - h_3|} = c_3 sign(h_3 - h_2)\sqrt{|h_3 - h_2} \end{cases} \tag{43}$$

For the coupled tanks system, the fluid flow, Q_1, into Tank 1, cannot be negative because the pump can only drive water into the tank. Therefore, the constraint on the inflow rate is given by:

$$Q_1 \geq 0 \tag{44}$$

From (47) and to satisfy the constraint (48) on the input flow rate, we should have $c_1 sign(h_1 - h_3) \geq 0$ and $c_3 sign(h_3 - h_2) \geq 0$, which implies:

$$sign(h_1 - h_3) = sign(h_3 - h_2) = 1 \tag{45}$$

One puts:

$$x_1 = h_1, \quad x_2 = h_2, \quad u_1 = Q_1 \ and \ u_2 = Q_2 \tag{46}$$

Then we obtain:

$$\dot{x}_1 \ = -c_1 sign(x_1 - x_3)\sqrt{|x_1 - x_3|} + \frac{u_1}{a} \tag{47}$$

$$\dot{x}_2 = c_3 sign(x_3 - x_2)\sqrt{|x_3 - x_2|} - B_4\sqrt{x_2} + \frac{u_2}{a} \tag{48}$$

$$\dot{x}_3 = c_1 sign(x_1 - x_3)\sqrt{|x_1 - x_3|} - c_3 sign(x_3 - x_2)\sqrt{|x_3 - x_2|} \tag{49}$$

$$\begin{cases} \dot{x} = f(t, x) + gu \\ y = cx \end{cases} \tag{50}$$

where: $x = [x_1 \ x_2 \ x_3]^T$, $u = [u_1 \ u_2]^T$, $y = [x_2 \ x_3]^T$,

$$\mathrm{f} = \begin{pmatrix} -c_1\sqrt{x_1 - x_3} \\ c_3\sqrt{x_3 - x_2} - B_4\sqrt{x_2} \\ c_1\sqrt{x_1 - x_3} - c_3\sqrt{x_3 - x_2} \end{pmatrix}$$

and

$$g = \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{a} \\ 0 & 0 \end{pmatrix}$$

4.2 Sliding Mode Control of the three tanks system:

The objective is to regulate the two water levels of tank 2 and tank 3 by using sliding mode control. The vector of the sliding surface is given by:

$s = [s_1 \;\; s_2]$
where:

$s_1 = (x_2 - x_{2d})$ and $s_2 = \lambda(x_3 - x_{3d}) + (\dot{x}_3 - \dot{x}_{3d})$

x_{2d} and x_{3d}: are the desired water levels of tank 2 and 3.

The derivatives of the sliding surfaces s_1 can written as follows:

$$\dot{s}_1 = h_1 + b_{12}u_2 \tag{51}$$

with:

$h_1 = (c_3\sqrt{(x_3 - x_2)} - B_4\sqrt{x_2} - \dot{x}_{2d})$ and $b_{12} = \frac{1}{a}$.

Similarly, the derivative of s_2 is as follows:

$$\dot{s}_2 = h_2 + b_{21}u_1 + b_{22}u_2 \tag{52}$$

with:

$h_2 = \lambda(c_1\sqrt{(x_1 - x_3)} - c_3\sqrt{(x_3 - x_2)} - \dot{x}_{3d}) + c_1\frac{2c_1\sqrt{(x_1-x_3)}-c_3\sqrt{(x_3-x_2)}}{2\sqrt{(x_1-x_3)}}$
$- c_3\frac{-c_1\sqrt{(x_1-x_3)}+2c_3\sqrt{(x_3-x_2)}}{2\sqrt{(x_1-x_3)}+B_4\sqrt{x_2}} - \dot{x}_{3d}$, $b_{21} = c_1\frac{1}{2a\sqrt{(x_1-x_3)}}$ and $b_{22} = c_3\frac{1}{2a\sqrt{(x_3-x_2)}}$

then:

$$\dot{s} = h + bu \tag{53}$$

with:

$b = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ and $b = \begin{pmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

The control vector of SOSMC is:

$$u = -b^{-1}(h + \begin{bmatrix} k_1 sign(s_1 + \ddot{s}_1) \\ k_2 sign(s_2 + \ddot{s}_2) \end{bmatrix}) \tag{54}$$

with:

$b^{-1} = \begin{pmatrix} \frac{ac_3\sqrt{x_1-x_3}}{c_1\sqrt{x_3-x_2}} & \frac{2a\sqrt{x_1-x_3}}{c_1} \\ a & 0 \end{pmatrix}$

5 Exprimental results

The proposed control algorithms were tested on the physical laboratory plant (Fig. 2) consisting of interconnected three tanks system. The objective is to control the liquid level of tanks two and three. The experimental schemes have been done under Matlab/Simulink, using Real-Time Interface, and run on the DS1102 DSPACE system, which is equipped by a power PC processor. The control algorithm is implemented on DSP (TMS 320C31). For given references we

Fig. 2. Laboratory experiment : Thtree tank system

remark that water levels h_{2d} and h_{3d}reach their references without overshooting. When we change the references we obtain the same response. In order to test the robustness of our strategy with respect to parameter uncertainties and disturbances, we varied the parameters c_1 and c_3 by closing and opening a little bit the valves az_1 and az_2 and we introduce a permanent leakage in the outflow pipes of tank 2 and tank 3 at $t = 1500s$. We can see fig 2 and fig 3 that, in spite of all these changes, the controller ensure the convergence of water levels to their references.
Moreover, we can observe that the control inputs Q_1 and Q_2 are smooth, the advantage and the chattering phenomenon is almost eliminated (Fig.5 and Fig. 6).

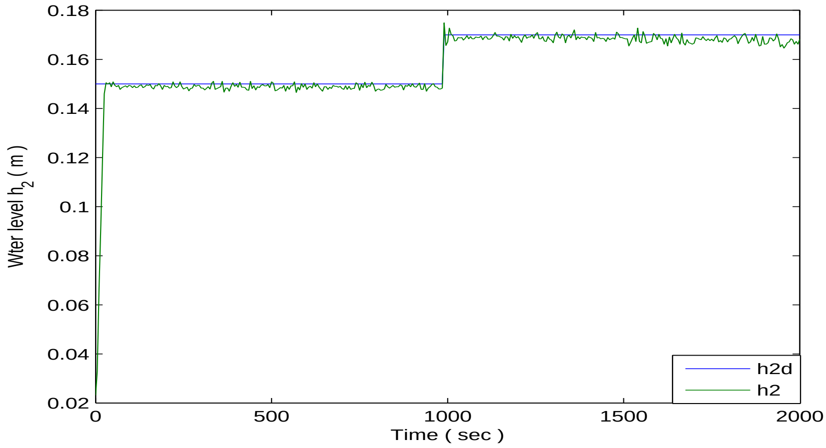

Fig. 3. Liquid level in tank2

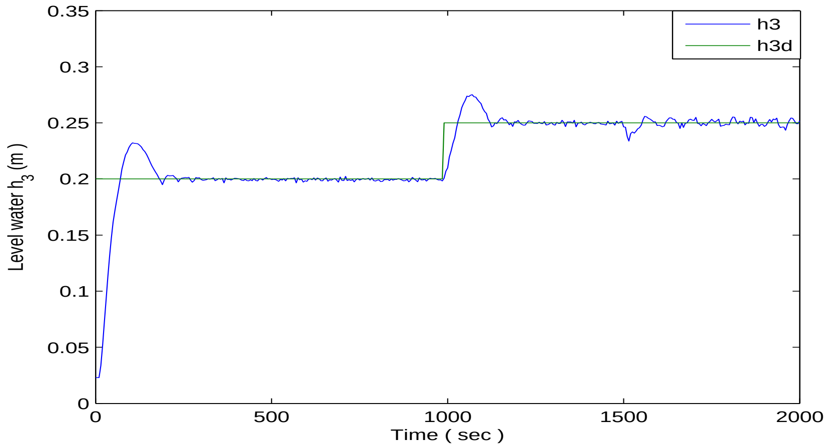

Fig. 4. Liquid level in tank3

For given references we remark that water levels h_{2d} and h_{3d}reach their references without overshooting. When we change the references we obtain the same response. In order to test the robustness of our strategy with respect to parameter uncertainties and disturbances, we varied the parameters c_1 and c_3 by closing and opening a little bit the valves az_1 and az_2 and we introduce a permanent leakage in the outflow pipes of tank 2 and tank 3 at $t = 1500s$. We can see fig 2 and fig 3 that, in spite of all these changes, the controller ensure the convergence of water levels to their references.

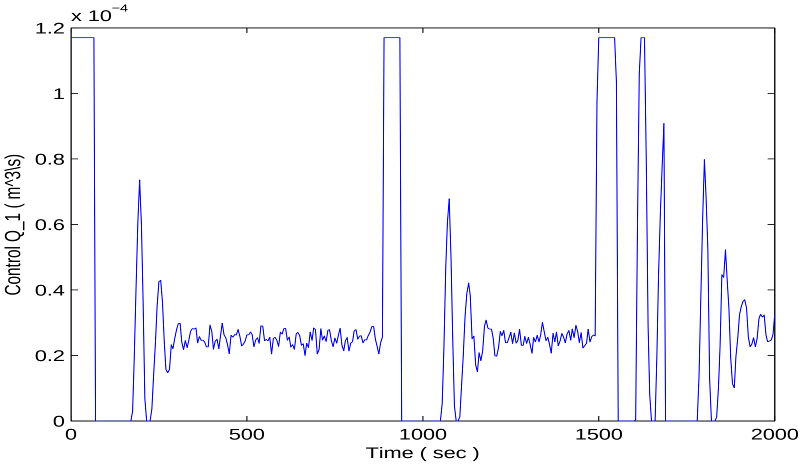

Fig. 5. The control input Q_1

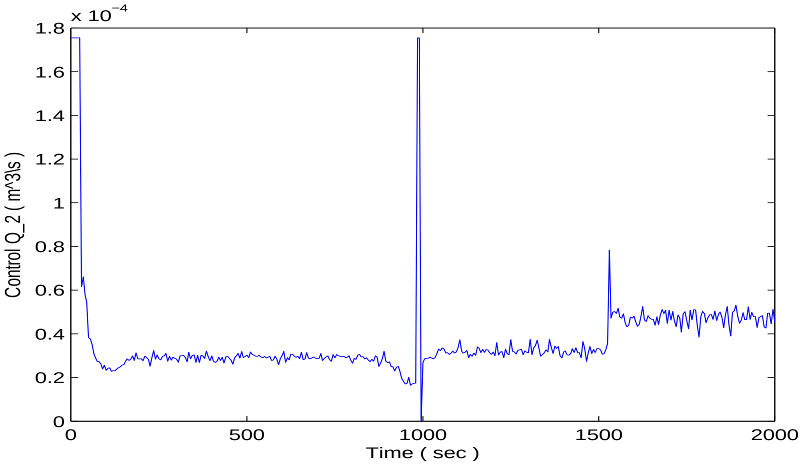

Fig. 6. The control input Q_2

Moreover, we can observe that the control inputs Q_1 and Q_2 are smooth, the advantage and the chattering phenomenon is almost eliminated (Fig.5 and Fig. 6).

6 Conclusion

In this paper, a new second order sliding mode control for MIMO nonlinear uncertain systems was proposed. This control has allowed the taking into account parameters uncertainties and disturbances. In order to prove the effectiveness and the efficiency of our control on an actual process, we implemented it on the bench of an interconnected three tanks system. Experimental results had shown its robustness with respect to parameters uncertainties and disturbances.

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