# Estimation of Vehicle Longitudinal Tire Force with FOSMO & SOSMO

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**Abstract.** In this paper, we propose two SM observer to estimate the longitudinal tire forces  $(F_x)$  using only wheel angular positions measurement. The robust estimations are based on First and Second Order Sliding Mode Observer (FOSMO, SOSMO). The estimation performance is tested and validated using a car driving simulator SCANeR<sup>TM</sup>-Studio (www.oktal.fr). The simulation results show effectiveness and robustness of the proposed method.

**Keywords.** Vehicle dynamic, Sliding Mode Observer, Inputs Estimation, Tire Forces Estimation.

#### 1 Introduction

The estimation of tire road force has become an intensive research area as the interest in information technology in vehicles grows. More and more new safety technologies and approaches are introduced in the automotive environment. Therefore, the problem of traction control for ground vehicles is of enormous importance to automotive industry.

In the literature, many studies deal with observers for vehicle dynamics. The objective may be comfort analysis, design or increase in safety and by means of this controllability of the car on the road [1][2][3]. Various tire models for automotive vehicle are proposed for estimation of involved variables [4][5][6][7]. In [8], Bakker describe two analytical models that are intensively used by researchers in the field. In [9], Eiichi Ono estimate friction force characteristics using XBS (extended braking stiffness) method by applying online least-squares method. This method is applied also in [10][11]. Kiencke presents in [12] a procedure for real-time estimation of adherence  $\mu$  ( $\mu = \frac{Friction force}{Normal force}$ ). He develops a relation between  $\mu$  and the wheel slip s. In [13] and [14], Ray estimates the forces on the tires with an extended Kalman filter. Villagra presents in [15] an estimation

of the road maximum adherence  $\mu$  based on algebraic approach using ALIEN algorithms. In [16], Gustafsson derives a scheme to identify different classes of roads. He assumes that combining the slip and the initial slope of the road adhesion versus curve it is possible to distinguish between different road surfaces. Recently, many studies have been performed on estimation of the frictions and contact forces between tires and road [17][18][19][20][21]. However, these inputs are difficult to get and not easy to measure. There are some difficulties to have them in real time. Then, it is necessary to estimate contact forces by robust observers with fast convergence.

In this paper, we use  $SCANeR^{TM}$ -Studio, a professional driving simulator proposed by Oktal (see www.oktal.fr), to validate the developed control systems using in the field of autonomous vehicles. This driving simulator can be used for prototyping and for design of vehicle functionalities. It will help us to evaluate performance of our observers.

This paper is organized as follows. Section 2 presents, after this introduction, the problem formulation and the models we have used to design our observers. The design of the First Order Sliding Mode Observer is then considered in section 3. This provide robust and fast estimations which are good indications for vehicle controllability despite the chattering effect which may appear. The Second Order sliding Mode Observer and its convergence analysis are presented in Section 4 to tackle the chattering problems. The description of the car driving simulator environment SCANeR $^{TM}$ -Studio and some results about the states observation and longitudinal tire force estimation by means of proposed observer are presented in Section 5. Finally, some remarks and perspectives are given in a concluding section.

# 2 Vehicle Road Interaction and Problem Formulation

# 2.1 bakground

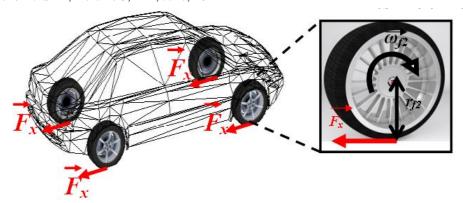
In this part, we use of a simple nominal partial model for description of wheels dynamics (see Figure (1)) [11][20][21]. This model is then parametrized for state observation and force estimation. The dynamic equations of the vehicle dynamics have been studied in [20][21][23] assuming the car body rigid and pneumatic contact permanent (see Figure(1)).

The generalized coordinate vector  $q \in \mathbb{R}^{16}$  is defined as follows for he Nominal Global Dynamical Model (see [21] for details):

$$\tau = M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + V(q, \dot{q}) + \eta_o(t, q, \dot{q})$$
 (1)

$$\tau = T_e + \Gamma = T_e + J^T F \tag{2}$$

The vectors  $\dot{q}, \ddot{q} \in R^{16}$  are velocities and corresponding accelerations respectively. The M(q) and  $C(q, \dot{q})$  are the inertia and the Coriolis and Centrifugal matrices respectively. The vector  $V(q, \dot{q})$  summarize the suspensions and gravitation forces with  $K_v$  and  $K_p$  are respectively the damping and the stiffness matrices, G(q) is the gravity term.  $\eta_o(t,q,\dot{q})$  represents the uncertainties and the neglected dynamics.



**Fig. 1.** Longitudinal Tire Forces  $F_x$  of Vehicle

#### 2.2Wheels Contact models

From equation (1) the wheels dynamic can be extracted in a partial model. The corresponding coordinate vector is  $x_1$ , velocities  $x_2$  and longitudinal forces  $x_3$ 

$$x_1 = (x_{11} \ x_{12} \ x_{13} \ x_{14})^T = (\varphi_{f1} \ \varphi_{f2} \ \varphi_{r1} \ \varphi_{r2})^T$$
(3)

$$x_2 = (x_{21} \ x_{22} \ x_{23} \ x_{24})^T = (\omega_{f1} \ \omega_{f2} \ \omega_{r1} \ \omega_{r2})^T \tag{4}$$

$$x_3 = (x_{31} \ x_{32} \ x_{33} \ x_{34})^T = (F_{xf1} \ F_{xf2} \ F_{xr1} \ F_{xr2})^T$$
 (5)

- $x_1$  are the angular positions of the four wheel.
- $x_2$  are the angular velocities of the four wheel.
- $x_3$  are the longitudinal forces applied to wheels  $y = (\varphi_{f1} \ \varphi_{f2} \ \varphi_{r1} \ \varphi_{r2})^T \ (y \in \mathbb{R}^4)$  is the vector of measured system outputs.

The wheel angular motion is given by:

$$\begin{cases}
\dot{\omega}_{f1} = \frac{1}{J_{rf1}} \left( C_m - T_{f1} - r_{f1} F_{xf1} \right) - \mu_{f1} \\
\dot{\omega}_{f2} = \frac{1}{J_{rf2}} \left( C_m - T_{f2} - r_{f2} F_{xf2} \right) - \mu_{f2} \\
\dot{\omega}_{r1} = \frac{1}{J_{rr1}} \left( -T_{r1} - r_{r1} F_{xr1} \right) - \mu_{r1} \\
\dot{\omega}_{r2} = \frac{1}{J_{rr2}} \left( -T_{r2} - r_{r2} F_{xr2} \right) - \mu_{r2}
\end{cases}$$
(6)

Where  $\omega_{f1}$ ,  $\omega_{f2}$ ,  $\omega_{r1}$  and  $\omega_{r2}$  are wheel rotational velocities for, respectively, the front left, front right, rear left and rear right wheels.  $J_{rf1}$ ,  $J_{rf2}$ ,  $J_{rr1}$  and  $J_{rr2}$ are inertia of, respectively, the front left, front right, rear left and rear right tyre in vehicle.  $C_m$  is the motor traction torque.  $T_{ij}$ ,  $r_{ij}$ ,  $F_{ij}$  (i = f, r and j = 1, 2)are respectively braking, wheel nominal radius and longitudinal tire force.  $\mu_w$  is the coupling term due to dynamics of the other subsystems and the neglected dynamics  $\eta_0(t,q,\dot{q})$  (see [21]).

#### 2.3 State Space model with unknown inputs

Approximations are made when neglecting the coupling terms  $\mu$  du to connections with the other sub systems. We can verify that these  $\mu$  are bounded such and as  $|\mu_i| < k_i \ \forall \ t, i = (1, ..., 4)$  [21][22][23].

Contact forces are the most difficult part to model in vehicle applications. Some assumptions are often used for existing models, like, for example, in the case of magic formula [24]:

- Assume wheel velocities constant, then slip angle and steering are constant,
- Assume that the pneumatic stiffness  $C_x$ ,  $C_y$ , are invariant and constant
- $\bullet$  Assume that the normal forces  $F_z$  exerted on the wheels are constant
- Assume that the behavior is uniform.

Actual situation are different from steady state conditions considered when modelling tire contact for steady states experiments. This means that at least the road characteristics have some perturbations and variables have small random variation, perturbations or additional noise e(t). In this work, without any knowledge on force characteristics, we consider the case of driving in an homogenous straight road like in a highway. It can be justified if we get acceptable results and performance. We assume to have slow variations in average.

Let us consider that we have very small variations in the mean:  $\dot{F} \simeq 0$ . So we can use  $\dot{x}_3 = e(t)$  where e(t) is a centred random noise with a small mean average.

The system (6) can be written in state space form as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(t) + g(t, x_3) - \mu \\ \dot{x}_3 = e(t)y = x_1 \end{cases}$$
 (7)

with

$$f(t) = \left[ \frac{C_m - T_{f1}}{J_{rf1}} \quad \frac{C_m - T_{f2}}{J_{rf2}} \quad -\frac{T_{r1}}{J_{rr1}} \quad -\frac{T_{r2}}{J_{rr2}} \right]^T \tag{8}$$

$$g(t, x_3) = \begin{bmatrix} -r_{f1} & 0 & 0 & 0\\ 0 & -r_{f2} & 0 & 0\\ 0 & 0 & -r_{r1} & 0\\ 0 & 0 & 0 & -r_{r2} \end{bmatrix} \begin{bmatrix} x_{31}\\ x_{32}\\ x_{33}\\ x_{34} \end{bmatrix} = U.x_3$$

$$\mu = \begin{bmatrix} \mu_{f1} & \mu_{f2} & \mu_{r1} & \mu_{r2} \end{bmatrix}^T \tag{9}$$

# 3 First Order Sliding Mode Observer (FOSMO)

#### 3.1 SM Observer

In this section, we develop a robust First Order Sliding Mode Observer. This technique is an attractive approach for robustness [23][25][26][27]. The proposed FOSMO has the form

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - \Lambda_1 sign(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_2 = \hat{f}(t) + \hat{g}(t, \hat{x}_3) - \Lambda_2 sign(\hat{x}_1 - x_1) \\ \dot{\hat{x}}_3 = -\Lambda_3 sign(\hat{x}_1 - x_1) \end{cases}$$
(10)

where  $\hat{x}_1$ ,  $\hat{x}_2$  and  $\hat{x}_3$  are respectively the state estimations of positions, velocities and input forces,  $\Lambda_1$ ,  $\Lambda_2$  and  $\Lambda_3$  are the observer gains matrices ( $\Lambda_i \in \mathbb{R}^4$ ; i = 1, 2, 3),  $\hat{f}(t)$  and  $\hat{g}(t, \hat{x}_3) = \hat{U}\hat{x}_3$  are respectively the estimation of f(t) and  $g(t, x_3)$ .

$$\begin{split} &\Lambda_1 = diag\{\lambda_{11},\lambda_{12},\lambda_{13},\lambda_{14}\}\\ &\Lambda_2 = diag\{\lambda_{21},\lambda_{22},\lambda_{23},\lambda_{24}\}\\ &\Lambda_3 = diag\{\lambda_{31},\lambda_{32},\lambda_{33},\lambda_{34}\} \end{split}$$

For the convergence analysis, we take  $H(t, \hat{x}_3, x_3) = \hat{f}(t) - f(t) + g(t, \hat{x}_3) - g(t, x_3) + \mu$ ,  $\tilde{x}_1 = \hat{x}_1 - x_1$  and  $\tilde{x}_2 = \hat{x}_2 - x_2$ . The observation error dynamic is then:

$$\begin{cases}
\dot{\tilde{x}}_1 = \tilde{x}_2 - \Lambda_1 sign(\tilde{x}_1) \\
\dot{\tilde{x}}_2 = H(t, \hat{x}_3, x_3) - \Lambda_2 sign(\tilde{x}_1) \\
\dot{\tilde{x}}_3 = -\Lambda_3 sign(\tilde{x}_1)
\end{cases}$$
(11)

#### 3.2 Convergence analysis

The Lyapunov candidate function  $V_1 = \frac{1}{2}\tilde{x}_1^T\tilde{x}_1$ , in a first step, shows that  $\tilde{x}_1 = 0$  is an attractive surface if we ensure that  $\dot{V}_1 < 0$  by means of choices of  $\lambda_{1i}$  (i = 1, ..., 4)

$$\dot{V}_1 = \tilde{x}_1^T (\tilde{x}_2 - \Lambda_1 sign(\tilde{x}_1)) \tag{12}$$

$$\lambda_{1i} > |\tilde{x}_{2i}| \quad for \ i = 1, ..., 4$$
 (13)

The convergence, in finite time  $(t_0)$  for the first system state is obtained:  $\hat{x}_1$  goes to  $x_1$ , so  $\dot{\tilde{x}}_1 = 0 \ \forall t > t_0$ . We can then deduce, in average, from the first equation of system (11) that  $\tilde{x}_2 = \Lambda_1 sign_{eq}(\tilde{x}_1)$ , with  $sign_{eq}$  the average value of function "sign" in the sliding surface. Then we obtain the following average for the over equations:

$$\dot{\tilde{x}}_2 = H(t, \hat{x}_3, x_3) - \Lambda_2 \Lambda_1^{-1} \tilde{x}_2 
\dot{\tilde{x}}_3 = -\Lambda_3 \Lambda_1^{-1} \tilde{x}_2$$
(14)

with

$$H(t, \hat{x}_3, x_3) = \tilde{f}(t) + \hat{U}\hat{x}_3 - Ux_3 \pm U\hat{x}_3 + \mu \tag{15}$$

$$= \tilde{f}(t) + \tilde{U}\hat{x}_3 - U\tilde{x}_3 + \mu \tag{16}$$

In the second step, we take as Lyapunov candidate function  $V_2 = \frac{1}{2}\tilde{x}_2^T\tilde{x}_2 + \frac{1}{2}\tilde{x}_3^T\tilde{x}_3$ , the derivative of this function is:

$$\dot{V}_2 = \tilde{x}_2^T (H - \Lambda_2 \Lambda_1^{-1} \tilde{x}_2) - \tilde{x}_3^T \Lambda_3 \Lambda_1^{-1} \tilde{x}_2$$
(17)

if  $\Lambda_3 = U^T \Lambda_1$ , the  $\dot{V}_2$  becomes:

$$\dot{V}_2 = \tilde{x}_2^T (\tilde{f}(t) + \tilde{U}\hat{x}_3 + \mu) - \tilde{x}_2^T \Lambda_2 \Lambda_1^{-1} \tilde{x}_2$$
(18)

In the following,  $N(t,\hat{x}_3) = \tilde{f}(t) + \tilde{U}\tilde{x}_3 + \mu$  and  $\tilde{x}_2$  will be supposed to be bounded for all t as follows:

$$|N(t, \hat{x}_3)| \le N_{max} \tag{19}$$

$$|\hat{x}_2^T| \le X_{max} \tag{20}$$

if  $\Lambda_2 > N_{max} X_{max}^T \Lambda_1$ , then  $\dot{V}_2 < 0$  and consequently  $\hat{x}_2$  and  $\hat{x}_3$  goes respectively to  $x_2$  and  $x_3$ .

Under some conditions, the system state estimation error converges, in a finite time, to vicinity of zero, the velocity errors go asymptotically to zero and the force estimation errors are bounded and decreasing. In Conclusion, the conditions to get the convergence of FOSMO can be summarized as:

- $\begin{array}{l} * \ \varLambda_1 > \mid \tilde{x}_2 \mid \\ * \ \varLambda_2 > N_{max} X_{max}^T \varLambda_1 \\ * \ \varLambda_3 = U^T \varLambda_1 \end{array}$

# Second Order Sliding Mode Observer (SOSMO)

# **Robust Differential Estimator**

In order to estimate the state vector and to deduce the unknown input forces vector  $(x_3 = F_x)$ , the following Second Order Sliding Mode Observer SOSMO (using the super-twisting algorithm) is proposed: [28][26][29][30][31][22]

$$\dot{\hat{x}}_1 = \hat{x}_2 + z_1 \tag{21}$$

$$\dot{\hat{x}}_2 = \hat{f}(t) + z_2 \tag{22}$$

where  $\hat{x}_1$ ,  $\hat{x}_2$  and  $\hat{x}_3$  are the state estimations,  $\alpha$  and  $\beta$  ( $\in \mathbb{R}^4$ ) are the observer gains and the correction variables  $z_1$  and  $z_2$  are calculated by:

$$z_1 = -\alpha \mid \hat{x}_1 - x_1 \mid^{\frac{1}{2}} sign(\hat{x}_1 - x_1) z_2 = -\beta sign(\hat{x}_1 - x_1)$$

# Convergence analysis

At the initial moment we take  $\hat{x}_{1(t=0)} = x_{1(t=0)}$  and  $\hat{x}_{2(t=0)} = 0$ . It is important to note that in a first step, input effects on the dynamic are rejected by the proposed observer like a perturbation. The dynamics estimation errors for SOSMO, with  $\tilde{x}_1 = \hat{x}_1 - x_1$  and  $\tilde{x}_2 = \hat{x}_2 - x_2$ , are calculated as follows:

$$\begin{cases} \dot{\tilde{x}}_1 = \tilde{x}_2 - \alpha \sqrt{|\tilde{x}_1|} sign(\tilde{x}_1) \\ \dot{\tilde{x}}_2 = \Phi(t, \hat{x}_3, x_3) - \beta sign(\tilde{x}_1) \end{cases}$$
(23)

Where  $\Phi(t, \hat{x}_3, x_3) = \hat{f}(t) - f(t) - g(t, x_3) + \mu$ . In our case, we suppose that the system states can be assumed bounded, then the existence of a constant bound  $g^+$  is ensured, such that the inequality

$$|\Phi(t,\hat{x}_3,x_3)| < \Phi^+ \tag{24}$$

In order to define the bound  $\Phi^+$  let us consider the system physical properties. Consider  $x_{3max}$  and  $f_{max}$  are defined such that  $\forall t \in \mathbb{R}^+, \forall \hat{x}_3, \mid \hat{x}_3 \mid \leq 2x_{3max}$  and  $\forall \tilde{f}(t), \mid \tilde{f}(t) \mid \leq f_{max}$ . The state boundedness is true, because mechanical systems are passive and BIBS stable, and the control action input  $x_3$  is bounded. The maximal possible acceleration in the system is a priori known and it coincides with the bound  $\Phi^+$ . We have:

$$UI \le U \le \overline{U}I \tag{25}$$

where  $\underline{U}$  is the minimal eigenvalue and  $\overline{U}$  the maximal one. The  $\Phi^+$  can be written as

$$\Phi^{+} = f_{max} + \overline{U}x_{3max} \tag{26}$$

Let  $\alpha$  and  $\beta$  satisfy the following inequalities, where p is some chosen constant, 0 (see [32])

$$\beta > \Phi^+ \tag{27}$$

$$\alpha > \sqrt{\frac{2}{\beta - f_{max}}} \frac{(\beta + f_{max})(1+p)}{(1-p)} \tag{28}$$

The presented observer ensures that in finite time, we have  $\tilde{x}_2 \simeq 0$ , then  $\dot{\tilde{x}}_2 \simeq 0$  holds after some finite time

$$\dot{\tilde{x}}_2 = \tilde{f}(t) - g(t, x_3) + \mu + z_2 = 0 \tag{29}$$

It was assumed that the term  $z_2$  changes at a high (infinite) frequency. However, in reality, various imperfections make the state oscillate in some vicinity of the intersection and components of  $z_2$  are switched at finite frequency, this oscillations have high and slow frequency components. The high frequency term  $z_2$  is filtered out and the motion in the sliding mode is determined by the slow components. The equivalent control is close to the slow component and may be derived by low pass filtering. The filter time constant should be sufficiently small to preserve the slow components undistorted but large enough to eliminate the high frequency component [25][27]. Let us take a low-pass filtering of  $z_2$ , then we obtain in the mean average:

$$\overline{z}_2 = g(t, x_3) - \xi(t) \tag{30}$$

where  $\xi(t) = \tilde{f}(t) + \mu$  and  $z_2$  is filtered signal of  $z_2$ . Then we obtain

$$x_3 = U^{-1}(\overline{z}_2 + \xi(t)) \tag{31}$$

As  $\xi(t)$  is unknown function and is assumed small, then we can written an estimation of longitudinal tire force  $\hat{x}_3$  as following

$$\hat{x}_3 = U^{-1}\overline{z}_2 \tag{32}$$

In Conclusion, the conditions to get the convergence of SOSMO can be summarized as:

$$\begin{array}{l} * \; \beta > \varPhi^+ \\ * \; \alpha > \sqrt{\frac{2}{\beta - f_{max}}} \frac{(\beta + f_{max})(1 + p)}{(1 - p)} \\ * \; \hat{x}_3 = U^{-1} \overline{z}_2 \; \text{is an approximate estimation of contact longitudinal forces.} \end{array}$$

# **Experimental Results**

# Description of SCANeR $^{TM}$ studio

 $SCANeR^{TM}$  software is compliant with any kind of driving system simulator from desktop system to full scale motion based system. OKTAL<sup>1</sup> is able to provide you with different types of products:

- To end users requiring a turn key solution, OKTAL provides relevant driving or testing simulator.
- To systems integrators or end users requiring only a software solution, OK-TAL provides them with SCANeR $^{TM}$  software and associated services

OKTAL has packaged two different softwares:

- \* SCANe $\mathbf{R}^{TM}$ studio dedicated to engineering & research.
- \* SCANeR<sup>TM</sup>DT dedicated to training and safety awareness.

 $SCANeR^{TM}$  studio software is a modular configuration, which means that you can use and add a module or bundle depending on your needs. The main characteristics of SCANe $\mathbf{R}^{TM}$ studio software are:

- Distributed architecture.
- Ethernet based communication back-bone,
- Multi-platform (Windows $^{TM}$  XP, Vista, Windows 7),
- Centralised user-friendly supervising application.

The SCANeR $^{TM}$  studio features are regrouped according to different components to run experimentations: ▶ Core, ▶ Driver, ▶ Environment, ▶ Vehicle, ▶ Simulation. In this section, we give some results in order to test and validate our approach an the proposed observers. We use the simulator of vehicle  $SCANeR^{TM}$ -Studio which is in laboratory LSIS. The Figures (3-a) and (3-b) present respectively the various constituents of simulator and the view module of simulator.

<sup>1</sup> www.oktal.fr

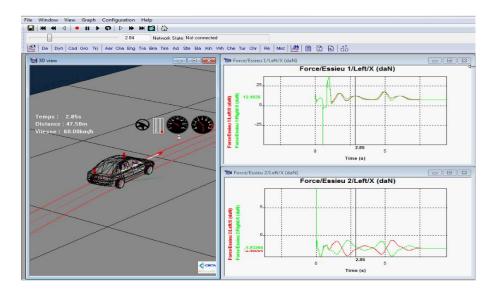


Fig. 2. Analysing Tool example

# 5.2 Results of *FOSMO*

In this simulation, the vehicle crosses the chicane trajectory (Figure (4)) with two constant speed at 60km/h = 16.66m/s and at 80km/h = 22.22m/s (Figure (5-c)). The tire characteristic used here is 185/70/R14. The road is considered plane, no slope and no banking road. The forces are generated by use of the Magic Formula tire model. The inputs of vehicle model are the steering angle for the two front wheels, presented by the Figure (5-a), and the angular position of the four wheels. The Figure (5-b) present the longitudinal acceleration used in simulation.

	FOSMO	
	60  km/h	80  km/h
$\lambda_{11}$	1.2	1.2
$\lambda_{12}$	1.2	1.2
$\lambda_{13}$	1.2	1.2
$\lambda_{14}$	1.2	1.2
$\lambda_{21}$	75	150
$\lambda_{22}$	75	150
$\lambda_{23}$	75	150
$\lambda_{24}$	75	150

Tableau 1. FOSMO parameters

	SOSMO	
	60  km/h	80  km/h
$\alpha_{11}$	1.5	10
$\alpha_{12}$	1.5	10
$\alpha_{13}$	1.5	10
$\alpha_{14}$	1.5	10
$\beta_{21}$	150	150
$\beta_{22}$	150	150
$\beta_{23}$	150	150
$\beta_{24}$	150	150

Tableau 2. SOSMO parameters



Fig. 3. Car simulator  $SCANeR^{TM}$  studio

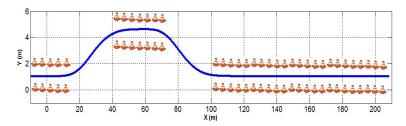


Fig. 4. Trajectory

The FOSMO Parameters are presented by the table (1). The Figures (6-a), (6-b), (6-c) and (6-d) presents the real and the estimation angular velocity of the four wheel: Front Left, Front Right, Rear Left and Rear Right wheel with a longitudinal velocity at 60km/h. We show the asymptotic convergence of the angular velocity. With the same longitudinal velocity  $V_x = 60km/h$ , the figures (8-a), (8-b), (8-c) and (8-d) show the convergence of the estimated longitudinal force to their actual value in finite time for respectively the front left, front right, rear left and rear right tire of vehicle using the observer FOSMO.

The performance of this estimation approach is satisfactory since the estimation error is minimal for angular velocities and longitudinal tire force at 60km/h. Nevertheless, we remark the appearance of a chattering for  $t \in [0, 2s]$  that comes from to the sign functions. For  $V_x = 80km/h$ , we present respectively in Figure (7) and Figure (9), the real and observed angular velocities for the four wheels in vehicle and the longitudinal tire force  $F_x$ . We note also a good convergence despite the chattering phenomena.

### 5.3 Results of SOSMO

in this part, we present the experimental results obtained for the observer *SOSMO*. The table (2) present the parameters used in our simulation. We show, respectively, in Figure (10) and Figure (11) a good convergence of the angular velocities

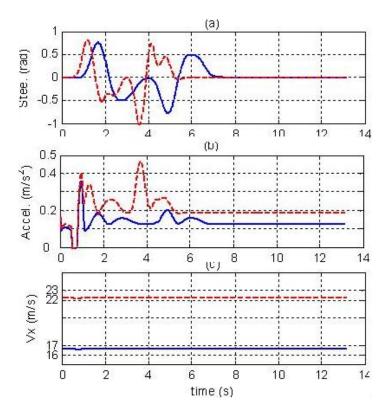


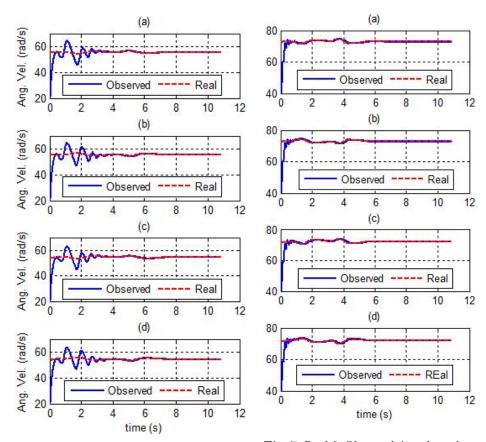
Fig. 5. Vehicle dynamic states (--:80km/h, -:60km/h)

for the four wheels of vehicle at longitudinal velocity 60km/h and 80km/h. The Figures (12) and (13) presents respectively the estimation of the longitudinal tire force Fx at 60km/h and 80km/h. We remark a good convergence in finite time and we see that the chattering phenomena is reduced.

# 6 Conclusions

In this paper, we present a First and Second Order Sliding Mode Observer to estimate the longitudinal tire force in vehicle. This approach allows us to avoid the observability problems dealing with inappropriate use of modelling equations. For vehicle systems, it is very hard to build up a complete and appropriate model for observation of all the system states. The finite time convergence of the two observer, *FOSMO* and *SOSMO*, is proved.

The gains of the proposed observer are chosen very easily ignoring the system parameters. Validation of performance has been proven using a driving simulator SCANeR<sup>TM</sup> Studio with two longitudinal velocities of vehicle  $V_x$  at 60km/h and at 80km/h.

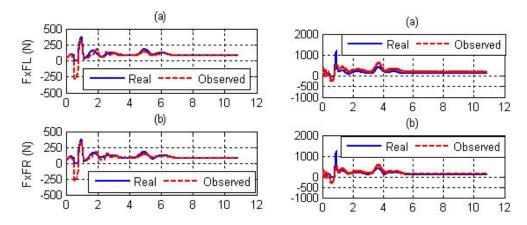


**Fig. 6.** Real & Observed Angular velocities of wheels with FOSMO at 60km/h

 ${\bf Fig.\,7.}$  Real & Observed Angular velocities of wheels with FOSMO at 80km/h

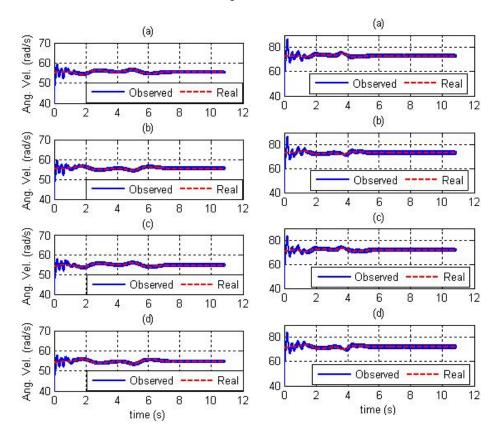
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**Fig. 8.** Real & Observed  $F_x$  with FOSMO at **Fig. 9.** Real & Observed  $F_x$  with 60km/h FOSMO at 80km/h

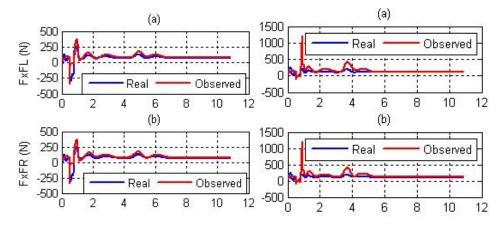
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**Fig. 10.** Real & Observed Angular velocities of wheels with SOSMO at 60km/h

Fig. 11. Real & Observed Angular velocities of wheels with SOSMO at 80km/h

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**Fig. 12.** Real & Observed  $F_x$  with SOSMO **Fig. 13.** Real & Observed  $F_x$  with at 60km/h SOSMO at 80km/h

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