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Representation of Linear Time Delay Systems: Multimodel Approach

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Abstract. The multimodel approach is recently developed in order to resolve the complexity of many industrial process. Nevertheless, this approach is often confronted to several difficulties, such as, the determination of the useful models'base. A new approach for a determination of a models'base for the representation of linear time delay systems is presented. The effectiveness of this method has been illustrated through simulation.

1 Introduction

Several researches involve analyse, modeling and control of time delay systems. To name a few, the monographic gives examples in biology, chemistry, economics, mechanics, physics, physiology, population dynamics, as well as in engineering sciences.

Unlike ordinary differential equations, delay systems are infinite dimensional in nature [8] and time delay is, in many cases, a source of complex behavior [8](oscillations, instability, bad performance). It is known that necessary and sufficient conditions can be derived in the case of a known constant time delay, but, if the value of the time delay is not available, then the time delay estimation constitutes the greatest challenge. The stability issue and the performance of control systems with time delay are, therefore, both of theoretical and practical importance [5] [13].

Moreover, the problem of identifying the time delay in real time presents a one of the most crucial open problems in the field of time delay systems, indeed, several control techniques can be applied only if the delay is well known. Literatures have abundant identification methods when the time delay is unknown. Some typical approaches are proposed, we cite the principal one to identify time delay system. One approach based on the approximation of the time delay by a rational transfer function or Pade approximation is proposed in [7] [11]. Such approach requires estimation of more parameters because the order of the approximated system model is increased and an unacceptable approximation error may occur when the system has a large delay.

Another method that identifies the delay and the system parameters is presented

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in [1]. This method is based on the correlation functions analysis. The method developed in [19] evolves the identification of delay and the parameters of a system operating in the presence of colored noise. This method based also on correlation analysis [4]. In a somewhat dual way, another [6] which suggest an algorithm to recursively update the value of a small delay by inspection of the phase contribution of the real negative zero arising in the corresponding sampled system. The main drawback of these methods is that iteration on time delay is needed to estimate the parameters and this makes on line implementation difficult.

The two-step procedure [3], first assumes that a known time delay and estimates the other transfer parameters, then minimizes the least squares error performance index with respect to the delay value.

Nevertheless, there are few algorithms that address the recursive identification of unknown time varying delay (time varying parameters and delay) [16] [18]. This work exploits the multimodel approach for the representation of the time varying time delay systems.

The multimodel approach was suggested to resolve some difficulties of both modelling and control. The multimodel approach consists in representing the system by different simple models having each a given validity domain. These models form the models'base.

In this paper, the on line estimation problem of time delay was mentioned through a minimum variance generalized adaptive control scheme. To overcome this problem, the multimodel approach was exploited. In fact, a models' base generation method was proposed for the multimodel representation of this kind of systems.

This paper is organized as follows. In section 2, we present the recursive estimation of time delay system. The multimodel representation for class of time delay systems is investigated in section 3. Section 4 contains several illustrative examples.

2 Recursive estimation of time delay system

In this part, a recursive identification method (simultaneously the parameters and the delay) will be presented. This method is a modified version of any recursive parameter estimator (Least squares, instrumental variable, maximum likelihood, etc.) [3].

Consider the ARMAX model given by:

$$A(q^{-1})y(k) = q^{-d(k)}B(q^{-1})u(k) + v(k) \tag{1}$$

where,

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{na} q^{-na}$$

$$B(q^{-1}) = b_1 q^{-1} + ... + b_{nb} q^{-nb}$$

with $y(k)$ is the system output signal, $u(k)$ is its input signal $v(k)$ is the disturbance signal (random data sequence with zero mean and finite variance) and $d(k)$ is the time varying delay.
The equation (1) is rewritten:

$$y(k) = \varphi(k, d(k))\theta + v(k) \tag{2}$$

where:

$$\begin{aligned}\varphi(k, d(k)) = [&-y(k-1), -y(k-2), ..., -y(k-n_a),\\ &u(k-d(k)-1), ..., u(k-d(k)-n_b)]\\ \theta = [a_1, a_2&, ..., a_{n_a}, b_1, b_2, ..., b_{n_b}]^T\end{aligned} \tag{3}$$

θ is the vector parameters to be estimated and $\varphi(k, d(k))$ is the observation vector.
The estimated output is given by:

$$\hat{y}(k) = \hat{\varphi}(k, \hat{d}(k))\hat{\theta} \tag{4}$$

where:

$$\begin{aligned}\hat{\varphi}(k, \hat{d}(k)) = [&-y(k-1), -y(k-2), ..., -y(k-n_a),\\ &u(k-\hat{d}(k)-1), ..., u(k-\hat{d}(k)-n_b)\Big]\\ \hat{\theta} = \Big[\hat{a}_1, \hat{a}_2&, ..., \hat{a}_{n_a}, \hat{b}_1, \hat{b}_2, ..., \hat{b}_{n_b}\Big]^T\end{aligned} \tag{5}$$

$\hat{\theta}$ and $\hat{d}$ represent the estimated parameter vector and the estimated delay. The prediction error is defined by:

$$e(k) = y(k) - \hat{y}(k) = y(k) - \hat{\varphi}(k, \hat{d}(k))\hat{\theta} \tag{6}$$

The criterion to be minimized is given by:

$$J = \sum_{i=0}^{k} [e(i)]^2 \tag{7}$$

Minimizing J with respect to the parameters and the delay can identify $\hat{\theta}$ and $\hat{d}$. i.e,

$$\frac{\partial J}{\partial \hat{\theta}} = 0 \tag{8}$$

and

$$J(\hat{d}) = \min_{\hat{d},\hat{\theta}} [J(d_i)] \, \forall d_i \in [d_{\min}, d_{\max}] \tag{9}$$

where $[d_{\min}, d_{\max}]$ is the delay margin evolution. If the parameters and delays are correctly estimated, the equation (9) has a minimum equal to zero. In the presence of measurement errors, equation (9) is sufficient to identify correctly the delay.
The estimation algorithm is based on two steps. The first one is to use the

recursive least squares estimator to estimate the parameters assuming that the latest delay is correctly estimated. The second one is to estimate the delay by considering the last parameters identification.
Thus, by the minimization of the criterion, we lead finally to the modified version of recursive least squares algorithm given by the following equations:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)(y(k) - \hat{\varphi}(k)\hat{\theta}(k-1)) \tag{10}$$

$$K(k) = P(k-1)\hat{\varphi}^T(k)\left[\lambda_i + \hat{\varphi}(k)P(k-1)\hat{\varphi}^T(k)\right]^{-1} \tag{11}$$

$$P(k) = \left[I - K(k)\hat{\varphi}(k)\right]P(k-1)/\lambda_i \tag{12}$$

$$\begin{gathered} J(k,d) = \lambda_i J(k-1,d) + \left[y(k) - \varphi(k,d)\hat{\theta}(k-1)\right]^2 \\ d \in [d_{\min}, d_{\max}] \end{gathered} \tag{13}$$

$$J(k,\hat{d}) = \min_{\hat{d},\hat{\theta}}\left[J(k,d)\right] \forall d \in [d_{\min}, d_{\max}] \tag{14}$$

λ_i is a weight between 0 and 1. To ensure the convergence of the parameters, we choose a gain $P(k)$ initially high $P(0) = CI_d$ where the constant $C \simeq 10^3$ and I_d is the identity matrix [12].
Equations (10), (11), (12) are, obviously, the standard recursive least squares algorithm for a constant delay. The equations (13) and (14) deal with the delay estimation. The implementation of this additional equations requires minimum storage and computation time as they contain only simple multiplications and additions and a simple search routine.
By examining this algorithm we can see that the estimated value of the delay will not change at every iteration as do the estimated values of parameters. In fact, with increasing the data picked out on the system, the parameters converge to the real parameters and their variation is relatively slow and at the same time the period in which the estimated delay is unchanged becoming larger. When the parameters converge to their final values, the estimated delay reaches the correct value and leave the parameters converge to the correct values quickly. This behavior requires the adjustment of the covariance matrix $P(k)$.

3 Multimodel representation for class of time delay systems

The implementation of the multi-model approach is necessary for the generation phase of base model's that can reproduce the behavior of the system in its entire operating range.

3.1 The model's base generation

This method exploits the data classification method proposed by Chiu. It is divided into two steps, the first is to classify the data set obtained from identification measurements. The second treats the structural and parametric identification exploiting the data relating of each cluster obtained from the classification phase [9] [10].

Construction of the classification data The classification procedure consists to select among a set of data points representing classes that will be centers of classes through a computation of potentials.
The classification data $s(k)$ is obtained by a combination of two terms. The first term focus the classification according to the time delay, whereas, the second term focus the classification according to the parameters of the system. The first term is obtained from maximizing the crosscorrelation function between the input and the outputs increments.

$$R(k,h) = \frac{1}{k}\sum_{j}^{k} u(j-h)(y(j+1)-y(j)) \tag{15}$$

$$s_d(k) = max(R(k)) \tag{16}$$

The second term is a normalization of the output:

$$s_p(k) = \frac{y(k)}{y_{max}} \tag{17}$$

The data $s(k)$ are calculated by the formulation:

$$s(k) = \alpha s_d(k) + (1-\alpha)s_p(k) \tag{18}$$

where α is a ponderation between 0 and 1.
If α is equal to 1, then we focus the classification according to the time delay, else, we allows the classification according to the system parameters.

Classification of the identification data Having a set identification data $(s_i,\, i = 1, ..., N)$, the classification procedure consists to associate to each datum s_i a potential P_i given by the following expression:

$$P_i = \sum_{j=1}^{M} e^{(\frac{-4\|s_i - s_j\|^2}{r_a^2})} \tag{19}$$

where r_a is a positive parameter controlling the decrease ratio of the potential. The potential decreases exponentially as s_j away from s_i. The first cluster center that we call s_{c1} is the datum whose potential P_1^* is the maximum.
To avoid selecting the first center s_{c1} and its neighborhood as other cluster centers, the procedure assigns to each potential P_i^* the following new value:

$$P_i \leftarrow P_i - P_1^* e^{(\frac{-4\|s_i - s_{c1}\|^2}{r_b^2})} \tag{20}$$

The parameter r_b ($r_b > 0$ must be selected larger than r_a to favor the operation related to the selection of the other cluster center completely different from the last one.
Next, we select, as second cluster center be s_{c2} whose modified potential given

by the relation (20). As similar, we choose the c^{th} cluster center s_{cc} with the maximum potential P_c^* and modify the potentials as follows:

$$P_i \leftarrow P_i - P_c^* e^{\left(\frac{-4\|s_i - s_{cc}\|^2}{r_b^2}\right)} \tag{21}$$

However, the selection of centers obeyed at each iteration, the following conditions:

- if $P_c^* > \varepsilon_1 P_1^*$, the selection is permitted.
- if $P_c^* < \varepsilon_2 P_1^*$, the selection is completed.
- if $\varepsilon_2 P_1^* \leq P_c^* \leq \varepsilon_1 P_1^*$ et if:

$$\frac{Min\,|s_{cc} - s_{ci}|}{r_a} < 1 - \frac{P_c^*}{P_1^*}, \qquad i = 1, ..., c-1. \tag{22}$$

where ε_1 and ε_2 are two positive parameters ($\varepsilon_1 > \varepsilon_2$) introduced by Chiu, s_{cc} is the current center and s_{c1},s_{c2},...,$s_{c(c-1)}$ are the last selected ones, the center to be retained corresponds, in this case, to the maximum value of the potentials after rejecting the current value P_c^*.

After the selection of the cluster center, we searched the elements belonging to each class by the calculate of distance between s_i and s_{cc} and classify y_i into the class whose distance is minimum.

Clusters' modeling After the collection of data for each class c (c,1,...,N), a structural and parametric identification must be carried out to elaborate the local model.
We consider the ARX structure giving by the following structure:

$$y_{M_c}(k) = -\sum_{l=1}^{n_a} a_{cl} y(k-l) + \sum_{j=1}^{n_b} b_{cj} u(k-j-d) \tag{23}$$

In this paper, we suppose the knowledge of the structure (the orders n_a and n_b are known).
The parametric identification use the modified recursive least square method explained in the section II and exploits the observation vector relating to the same cluster c.

3.2 Generation of the multimodel output

The multimodel output $y_m(k)$ is obtained by the fusion of the elementary outputs of the generated models and it is written:

$$y_m(k) = \sum_{c=1}^{N} v_c(k) y_c(k) \tag{24}$$

Where $y_c(k)$ is the elementary output of the model M_i and $v_c(k)$ is the validity of this model which is carried out using the residue approach.

3.3 Validities estimation based on classical residue approach

The validities of models $M_i (i = 1, ..., c)$ are calculated using the residues approach formulated by the relations (25)-(29):

$$r_c(k) = |y(k) - y_{M_c}(k)| \,; \qquad c = 1, ...N \tag{25}$$

where N represents the number of models based [9].

This expression must be normalized to have a residue between 0 and 1:

$$r'_c(k) = \frac{r_c(k)}{\sum\limits_{j=1}^{N} r_j(k)} \tag{26}$$

The validity which varies in the contrary sense of the residue, can then be expressed by:

$$v'_c(k) = \frac{v_c(k)}{N-1} \tag{27}$$

In order to reduce the perturbation phenomenon due to the inadequate models, we reinforce the validities as follow:

$$v^r_c(k) = v'_c(k) \prod_{\substack{j=1 \\ j \neq c}}^{N} (1 - e^{-(\frac{r'_j(k)}{\sigma})^2}) \tag{28}$$

with σ is a positive number to control the transition between the different models of the database:

$$0 < \sigma \leq 1$$

The normalized reinforced validities are given by:

$$v^r_{cn}(k) = \frac{v^r_c(k)}{\sum\limits_{j=1}^{N} v^r_j(k)} \tag{29}$$

4 Simulation result

To demonstrate the interest and the contribution in performance of the proposed approach, the deterministic and stochastic cases are investigated.
We consider a discrete non stationary second order system with time varying delay described by the following equation:

$$\begin{aligned} y(k) &= -a_1(k)y(k-1) - a_2(k)y(k-2) \\ &+ b_1(k)u(k-1-d(k)) + b_2(k)u(k-2-d(k)) \end{aligned}$$

$a_i(k)$, $b_i(k)$ and d are the time varying parameters and the time delay. Their variation law is given by this table:

Les itérations	$a_1(k)$	$a_2(k)$	$\hat{b}_1(k)$	$\hat{b}_2(k)$	$d(k)$
$0 < k < 500$	-1.2	0.25	0.35	0.5	1
$500 < k < 1000$	-1.3	0.35	0.2	0.25	2
$1000 < k < 1500$	-1.40	0.45	0.1	0.35	3

Table 1. The variation law of the parameters and the time delay.

4.1 Deterministic case

The system is excited in it's full operating range by a pseudo random binary sequence so as to generate the necessary classification data. The Fig 1 illustrate the evolution of the classification data:

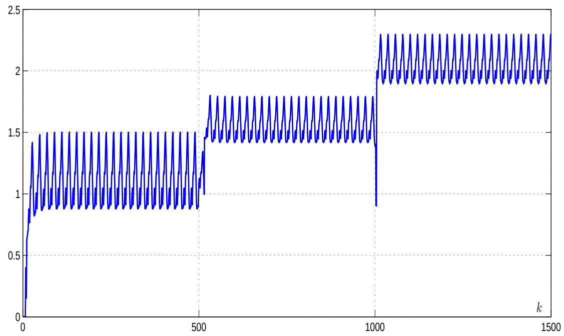

Fig. 1. The evolution of the classification data $s(k)$.

By examining the Fig 1, we can release the presence of three classes.
Besides, the application of the classification method leads to three clusters focused on the cluster centers s_{c_1}, s_{c_2} and s_{c_3}. The Fig 2 shows the evolution of the potentials of the different data during the classification procedure with the emplacements of the cluster centers.

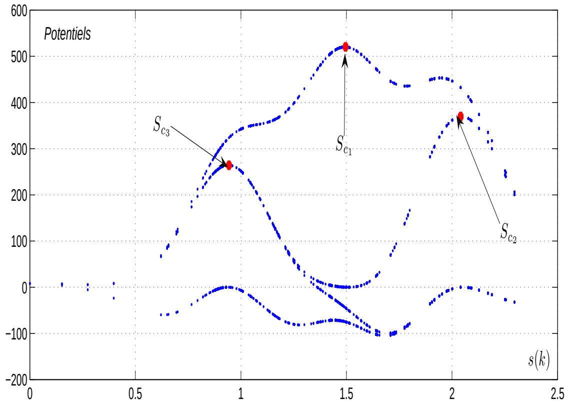

Fig. 2. The evolution of the potentials with the emplacements of the cluster centers.

After obtaining the classes and the corresponding elements, we carry out an identification of the obtained models (M_c, c=1...,3) realized by the least squares modified algorithm.
The table 2 present the three models:

Models	$\hat{a}_1(k)$	$\hat{a}_2(k)$	$\hat{b}_1(k)$	$\hat{b}_2(k)$	$\hat{d}(k)$
M_1	-1.2	0.25	0.35	0.5	1
M_2	-1.3	0.35	0.2	0.25	2
M_3	-1.40	0.45	0.1	0.35	3

Table 2. The models.

In addition, by exploiting all the data, we identify a global model using classical recursive least square algorithm [12]. A validation of the model base, using a sinusoidal input described by the relation (30), is realized.

$$u(k) = \begin{cases} e^{-0.005k}(1+\sin(k/10)) & k \leqslant NI/2 \\ u(k-NI/2) & \text{k ¿NI/2} \end{cases} \tag{30}$$

The fig 3 gives the evolution of real output $y(k)$, multimodel output $y_m(k)$ and global output $y_g(k)$.

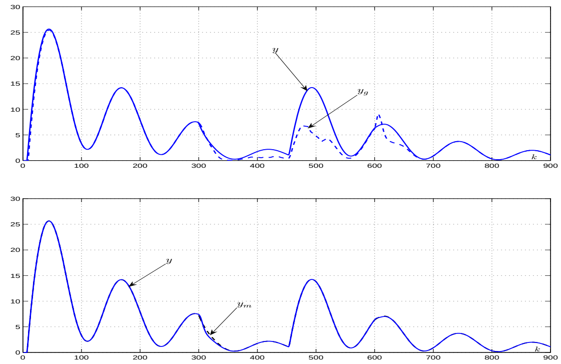

Fig. 3. The multimodel, the global and the real outputs evolutions.

This figure show that the multimodel output $y_m(k)$, which is generated by the fusion of the elementary outputs of the local models is practically equal to the real output. Compared to the classical method, we find out the contribution of the multimodel approach.
The reinforced validities is given in Fig 4:

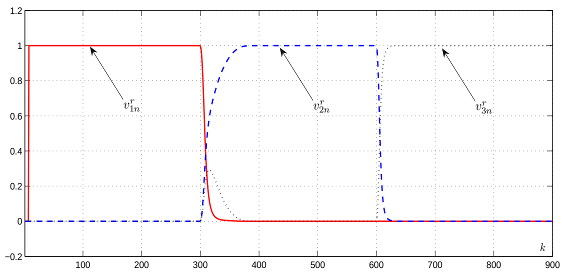

Fig. 4. The evolution of the potentials with the emplacements of the cluster centers.

This figure illustrate the contribution of each model in the representation of the system behavior in their function area and their complementarity. This contribution is quantified by the delay value of each model.

4.2 Stochastic case

We consider the same system with the same parameters and delay variation and a signal of noise is added to the system's output as given by the relation (31) to evaluate the robustness of the suggested method:

$$\begin{aligned} y(k) &= -a_1(k)y(k-1) - a_2(k)y(k-2) \\ &+ b_1(k)u(k-1-d(k)) + b_2(k)u(k-2-d(k)) + v(k) \end{aligned} \tag{31}$$

where $v(k)$ is a white noise $(0, \sigma)$ with $\sigma^2 = 0.1$
The Fig 5 records the evolution of the classification data:

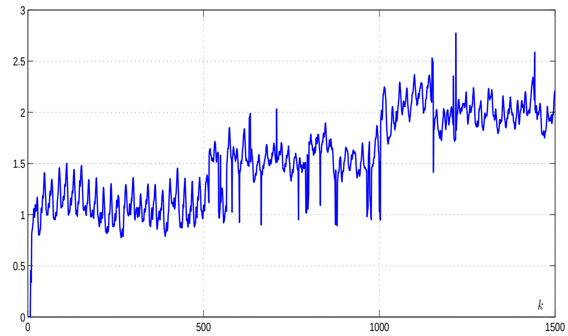

Fig. 5. The evolution of the classification data $s(k)$.

The application of the suggested method on the noisy data set $s(k)$ leads to three different classes.
This is illustrated by the evolution of the potentials and the cluster centers presented by Fig 6:

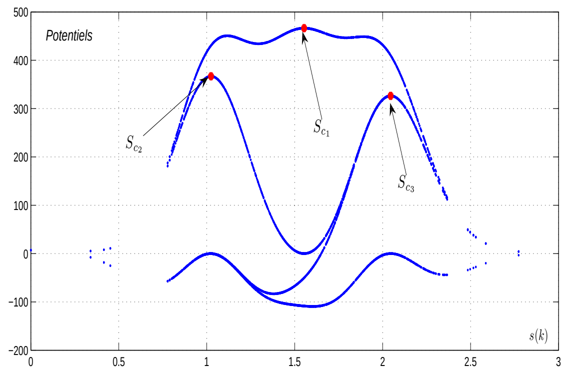

Fig. 6. The evolution of the potentials with the emplacements of the cluster centers.

Indeed, the classification procedure yields to three clusters which are modeled by three models given in the table 2:

Models	$\hat{a}_1(k)$	$\hat{a}_2(k)$	$\hat{b}_1(k)$	$\hat{b}_2(k)$	$\hat{d}(k)$
M_1	-1.1802	0.2315	0.3547	0.4926	1
M_2	-1.3328	0.3918	0.1838	0.2614	2
M_3	-1.4012	0.4398	0.1118	0.3267	3

Table 3. The models.

By exploiting all the noise data collected idendification on the system, we identify a global model. The same signal of validation $u(k)$ is considered to validate the models base.

The Fig 7 shows the evolution of the multimodel output generated by the fusion of the elementary of each model and the output of the global model y_g.

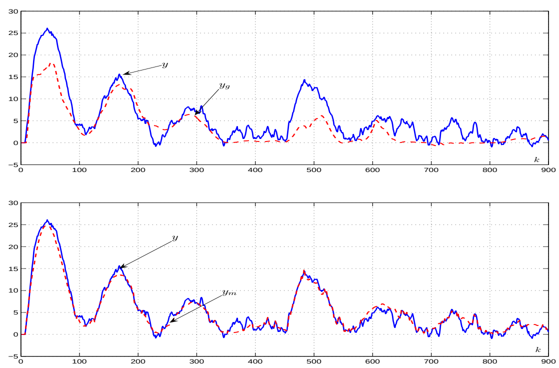

Fig. 7. The multimodel, the global and the real outputs evolutions.

This figure shows a relatively accurate modeling by exploitation of the multimodel approach. In fact, we can see clearly that the multimodal output still describe the real output of the system. The reinforced validities is given in Fig 8:

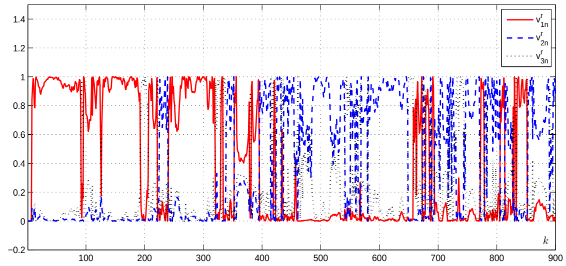

Fig. 8. The evolution of the potentials with the emplacements of the cluster centers.

5 Conclusion

The multimodel approach is an effective tool, particularly well suited to modeling systems with variable delays. A method for generating a basic model for the representation of delay systems is developed in this paper. The numerical simulation on time varying delay system in the deterministic and stochastic case are very satisfactory compared to the classical modeling.

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