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On Robust Stability of Uncertain Neutral Systems: A Novel Lyapunov-Krasovskii Functional

Dhaou SOUDANI [1], Issam AMRI [1], Sahbi MEZLINI [1] and Mohamed BENREJEB [1]

[1]L.A.R.A. Laboratory, Ecole Nationale d'Ingénieurs de Tunis, ENIT, BP 37, 1002 Tunis, Tunisie.

amri_issam@yahoo.fr, mezlini.sahbi@yahoo.fr, dhaou.soudani@enit.rnu.tn, mohamed.benrejeb@enit.rnu.tn

Abstract. *This paper studies the problem of robust exponential stability of uncertain perturbed neutral system with time varying delay. By introducing a novel Lyapunov-Krasovskii functional, new sufficient delay dependent stability conditions have been derived in terms of Linear Matrix Inequalities LMIs. Numerical examples are given to illustrate the theoretical developments.*

Keywords. *Delay dependent stability, Exponential stability, Lyapunov-Krasovskii Functional, Time varying delays, Uncertain Neutral systems.*

1. Introduction

Time delays and uncertainties are common phenomena frequently encountered in engineering dynamic systems, for instance in chemical process, in network control systems, in long transmission and so on. They are major sources of instability, oscillations and poor performance. In view of this, considerable attention has been devoted to the problem of stability and robustness of time delays systems for several decades, see for example [1-22], and the references therein. The developed stability results can be classified into two types: delay dependent stability results, which are concerned with the size of the delay and usually give the maximum delay bounds for making the system stable, and the delay independent stability results, which can be applied with arbitrary delay's size. Generally, delay dependent stability conditions are less conservative than delay independent ones, especially when the time delays are small.

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In practice, system models can be described by functional differential equation of neutral type, which have delays in both its state and the derivatives of its states. Furthermore, practical systems almost present some uncertainties and nonlinear perturbations, thus the problem of robust stability analysis has been widely investigated in many reports [1-3, 8, 9], and many approaches have been developed to solve the stability problem for neutral time-delay systems.

Many results have been reported in the literature in order to reduce the conservatism of stability results and by using various methods. Based on descriptor model transformation combined with a matrix decomposition approach, [9] presents robust delay dependent stability criteria by using an integral inequality. However, these model transformations and bounding techniques often introduce additional dynamics which leads to relatively conservative results. Furthermore, free weighting matrices introduced by [18] have been employed in a lot of papers, such as [3, 10, 13, 16], to investigate the delay dependent stability, in which many lack variables are introduced by using the Newton–Leibniz formula. This method plays a key role to increase freedom to search the Lyapunov matrices and may improve the feasibility region of stability criterion. But this technique may involve much more decision variables must be decided which may increase the complexity of the computation.

The purpose of this paper is to investigate the problems of robust exponential delay dependent stability for uncertain neutral system with time varying delays and nonlinear perturbations. A new class of Lyapunov-Krasovskii functional containing both triple and quadruple integrals is proposed based on the use of novel triple integral inequality extended from Jensen integral inequality. Some numerical examples are given to illustrate the effectiveness of our results.

Notation. Throughout this paper, a real symmetric matrix $S > 0$ denotes S being a positive definite matrix. The superscript ‘‘T’’ is used for the transpose of a matrix. The maximum and minimum eigenvalues of a matrix S are represented as $\lambda_{Max}(S)$ and $\lambda_{Min}(S)$ respectively. $\|.\|$ denotes the Euclidian norm vector. The symmetric terms in a symmetric matrix are denoted by $(*)$.

2. System description and preliminaries

Consider the following neutral system with time varying delays and nonlinear perturbations:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h(t)) + C\dot{x}(t-h(t)) + F_1 f_1(t,x(t)) + F_2 f_2(t,x(t-h(t))) + F_3 f_3(t,\dot{x}(t-h(t))) \\ x(t) = \phi(t), \quad \dot{x}(t) = \varphi(t), \quad \forall t \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, and A, A_h, C, F_1, F_2, F_3 are known constant matrices with appropriate dimensions. $h(t)$ is a continuous time-varying delay satisfying

$$0 < h(t) \le h, \quad \dot{h}(t) \le d \tag{2}$$

The initial conditions $\phi(.)$ and $\varphi(.)$ are continuously differentiable vector-valued function of t, $t \in [-h, 0]$. $f_1(t, x(t))$, $f_2(t, x(t-h(t)))$ and $f_3(t, \dot{x}(t-h(t)))$ are unknown nonlinear perturbations. They satisfy $f_i(t, 0) = 0$, $i = 1, 2, 3$, and

$$\begin{aligned} &f_1^T(t, x(t))\, f_1(t, x(t)) \le \beta_1^2 x^T(t) x(t) \\ &f_2^T(t, x(t-h(t)))\, f_2(t, x(t-h(t))) \le \beta_2^2 x^T(t-h(t)) x(t-h(t)) \\ &f_3^T(t, \dot{x}(t-h(t)))\, f_3(t, \dot{x}(t-h(t))) \le \beta_3^2 \dot{x}^T(t-h(t)) \dot{x}(t-h(t)) \end{aligned} \tag{3}$$

where β_i, $i = 1, 2, 3$, are positive given constant, for simplicity, we note:
$f_1 := f_1(t, x(t))$, $f_2 := f_2(t, x(t-h(t)))$, $f_3 := f_3(t, \dot{x}(t-h(t)))$.

The following definition and lemmas will be needed for deriving our main results.

Definition 1. [6]

The system (1) is exponentially stable, if there exist positive constants α and δ, $\delta \ge 1$ such that for all $x(t)$, the following inequality holds

$$\|x(t)\| \le \delta\, e^{-\alpha t} \|\psi\|_h, \ \forall t \ge 0$$

where α is the decay rate and $\|\psi\|_h = \sup_{-h \le \theta \le 0} \sqrt{\|\varphi(\theta)\|^2 + \|\phi(\theta)\|^2}$.

Lemma 1.

For any constant matrix $\Theta = \Theta^T > 0$, scalar $h := h(t) > 0$, and vector function $x(.) : [-h, 0] \to \Re^n$ such that the following integral is defined, then

(a) $h \int_{t-h}^{t} \eta^T(s)\, \Theta\, \eta(s) ds \ge \int_{t-h}^{t} \eta^T(s) ds\, \Theta \int_{t-h}^{t} \eta(s) ds,$

(b) $\frac{h^2}{2} \int_{-h}^{0} \int_{t+\theta}^{t} \eta^T(s)\, \Theta\, \eta(s) ds\, d\theta \ge \int_{-h}^{0} \int_{t+\theta}^{t} \eta^T(s) ds\, d\theta\, \Theta \int_{-h}^{0} \int_{t+\theta}^{t} \eta(s) ds\, d\theta,$

(c) $\frac{h^3}{6} \int_{-h}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} \eta^T(s)\, \Theta\, \eta(s) ds\, d\theta\, d\lambda \ge \int_{-h}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} \eta^T(s) ds\, d\theta\, d\lambda\, \Theta \int_{-h}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} \eta(s) ds\, d\theta\, d\lambda.$

Proof. Inequalities (a) and (b) were proposed in [7] and [17] respectively. For inequality (c), it is easy to see, using Shur's complement, that

$$\begin{bmatrix} \eta^T(s)\,\Theta\,\eta(s) & \eta^T(s) \\ \eta(s) & \Theta^{-1} \end{bmatrix} \geq 0 \tag{4}$$

The integration of inequality (4) from $t+\theta$ to θ, yields

$$\begin{bmatrix} \int_{t+\theta}^{t} \eta^T(s)\,\Theta\,\eta(s)\,ds & \int_{t+\theta}^{t} \eta^T(s)\,ds \\ \int_{t+\theta}^{t} \eta(s)\,ds & -\theta\,\Theta^{-1} \end{bmatrix} \geq 0 \tag{5}$$

A second integration of inequality (5) from λ to 0, with $\lambda \in [-h, 0]$, leads to

$$\begin{bmatrix} \int_{\lambda}^{0}\int_{t+\theta}^{t} \eta^T(s)\,\Theta\,\eta(s)\,ds\,d\theta & \int_{\lambda}^{0}\int_{t+\theta}^{t} \eta^T(s)\,ds\,d\theta \\ \int_{\lambda}^{0}\int_{t+\theta}^{t} \eta(s)\,ds\,d\theta & \frac{\lambda^2}{2}\Theta^{-1} \end{bmatrix} \geq 0 \tag{6}$$

Finally, the integration of inequality (6) from λ to 0 yields

$$\begin{bmatrix} \int_{-h}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t} \eta^T(s)\,\Theta\,\eta(s)\,ds\,d\theta\,d\lambda & \int_{-h}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t} \eta^T(s)\,ds\,d\theta\,d\lambda \\ \int_{-h}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t} \eta(s)\,ds\,d\theta\,d\lambda & \frac{h^3}{6}\Theta^{-1} \end{bmatrix} \geq 0 \tag{7}$$

By applying the Schur's complement again, one can obtain the inequality (c).

Lemma 2. [19]

For given matrices U, V, W and $\Upsilon = \Upsilon^T > 0$ with appropriate dimensions, then

$$\Upsilon + UV(t)W + W^T V^T(t) U^T < 0$$

for all $V^T(t)V(t) \leq I$, if and only if there exist a positive scalar ε such that

$$\Upsilon + \varepsilon^{-1} U U^T + \varepsilon W^T W < 0$$

Lemma 3. [5]

For given matrices S_1, S_2, S_3 satisfying $S_1 = S_1^T$, $S_3 = S_3^T > 0$, then

$$S=\begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix}>0 \quad \Leftrightarrow \quad S_1 - S_2 S_3^{-1} S_2^T > 0$$

$$S=\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix}<0 \quad \Leftrightarrow \quad S_1 + S_2 S_3^{-1} S_2^T < 0$$

Lemma 4. [20]

Let $f(x), y_1(x),..., y_k(x)$ be some non-negative functional or functions, and define the following conditions

(a) $f(x) \geq 0,$

(b) $\tau_1 \geq 0,..., \tau_k \geq 0,$

such that $f(x) - \sum_{j=1}^{k} \tau_j y_j(k) \geq 0.$

Then (b) implies (a).

3. Exponential stability of neutral systems with time varying delay and nonlinear perturbations

This section performs stability analysis of neutral systems with time varying delay and nonlinear perturbations described by (1)-(3). Theorem 1 provides a delay dependent stability criterion in terms LMIs optimization approaches.

Theorem 1.

For given scalars $\alpha > 0$ and $h > 0$, the nominal neutral system (1) is exponentially stable with decay rate α, if there exist positive definite symmetric $n \times n$ matrices P, W_{11}, W_{22}, X_{11}, X_{22}, Z_{11}, Z_{22}, R, H_1, H_2, H_3 and any $n \times n$ matrices W_{12}, X_{12}, Z_{12} such that the following LMIs hold:

$$\Omega = \begin{bmatrix} \Xi & \Omega_1 \\ * & \Omega_2 \end{bmatrix} < 0 \qquad (8)$$

$$W = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} > 0,\ X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0,\ Z = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} > 0 \qquad (9)$$

with

$$\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19}\\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & 0 & 0 & 0\\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \Xi_{55} & \Xi_{56} & 0 & 0 & 0\\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0\\ * & * & * & * & * & * & \Xi_{77} & 0 & 0\\ * & * & * & * & * & * & * & \Xi_{88} & 0\\ * & * & * & * & * & * & * & * & \Xi_{99}\end{bmatrix}, \tag{10}$$

$$\Xi_{11}=2\alpha P+PA+A^TP-\frac{3h}{2}e^{-2\alpha h}R+\frac{h^4}{4}X_{11}+\frac{h^4}{4}X_{12}A+\frac{h^4}{4}A^TX_{12}^T-h^2e^{-2\alpha h}X_{22}+h^2Z_{11}$$
$$-e^{-2\alpha h}Z_{22}+h^2Z_{12}A+h^2A^TZ_{12}^T+W_{11}+W_{12}A+A^TW_{12}^T+\beta_1^2H_1,$$

$$\Xi_{12}=PA_h+e^{-2\alpha h}Z_{22}+\frac{h^4}{4}X_{12}A_h+h^2Z_{12}A_h+W_{12}A_h,$$

$$\Xi_{22}=-e^{-2\alpha h}Z_{22}-(1-d)e^{-2\alpha h}W_{11}+\beta_2^2H_2,\ \Xi_{13}=PC+\frac{h^4}{4}X_{12}C+W_{12}C+h^2Z_{12}C,$$

$$\Xi_{23}=-(1-d)e^{-2\alpha h}W_{12},\ \Xi_{33}=-(1-d)e^{-2\alpha h}W_{22}+\beta_3^2H_3,\ \Xi_{14}=-e^{-2\alpha h}Z_{12}^T,$$
$$\Xi_{24}=e^{-2\alpha h}Z_{12}^T,\ \Xi_{44}=-e^{-2\alpha h}Z_{11},\ \Xi_{15}=he^{-2\alpha h}X_{22},\ \Xi_{55}=-e^{-2\alpha h}X_{22},$$
$$\Xi_{16}=\frac{3}{h}e^{-2\alpha h}R-he^{-2\alpha h}X_{12}^T,\ \Xi_{56}=he^{-2\alpha h}X_{12}^T,\ \Xi_{66}=-e^{-2\alpha h}X_{11}-\frac{6}{h^3}e^{-2\alpha h}R,$$

$$\Xi_{17}=PF_1+\frac{h^4}{4}X_{12}F_1+h^2Z_{12}F_1,\ \Xi_{77}=-H_1,\ \Xi_{18}=PF_2+\frac{h^4}{4}X_{12}F_2+h^2Z_{12}F_2,$$

$$\Xi_{88}=-H_2,\ \Xi_{19}=PF_3+\frac{h^4}{4}X_{12}F_3+h^2Z_{12}F_3,\ \Xi_{99}=-H_3,$$

$$e_1=\begin{bmatrix}A & A_h & C & 0 & 0 & 0 & F_1 & F_2 & F_3\end{bmatrix}^T,\ e_2=[W_{22}\quad hZ_{22}\quad \frac{h^2}{2}X_{22}\quad R],$$

$$\Omega_1=e_1e_2,\ \Omega_2=diag\{-W_{22},-Z_{22},-X_{22},-\frac{6}{h^3}R\}.$$

Proof.

Let construct the following Lyapunov–Krasovskii functional candidate

$$V = \sum_{i=1}^{5} V_i \tag{11}$$

with

$$V_1 = e^{2\alpha t} x^T(t) P x(t),$$

$$V_2 = \int_{t-h(t)}^{t} e^{2\alpha s} \xi^T(s) W \xi(s) ds,$$

$$V_3 = h \int_{-h}^{0} \int_{t+\theta}^{t} e^{2\alpha s} \xi^T(s) Z \xi(s) ds\, d\theta,$$

$$V_4 = \frac{h^2}{2} \int_{-h}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} e^{2\alpha s} \xi^T(s) X \xi(s) ds\, d\theta\, d\lambda,$$

$$V_5 = \int_{-h}^{0} \int_{\beta}^{0} \int_{\lambda}^{0} \int_{t+\theta}^{t} e^{2\alpha s} \dot{x}^T(s) R \dot{x}(s) ds\, d\theta\, d\lambda\, d\beta,$$

$$\xi^T(s) = \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix}.$$

Calculating the time derivative of V_1 along the solution of system (1) yields

$$\dot{V}_1 = e^{2\alpha t} \left\{ 2\alpha x^T(t) P x(t) + 2 x^T(t) P \dot{x}(t) \right\}, \tag{12}$$

By calculating $\dot{V}_2$, we have

$$\dot{V}_2 = e^{2\alpha t} \left\{ \xi^T(t) W \xi(t) - (1 - \dot{h}(t)) e^{-2\alpha h} \xi^T(t - h(t)) W \xi(t - h(t)) \right\}$$

$$\dot{V}_2 \le e^{2\alpha t} \left\{ \xi^T(t) W \xi(t) - (1 - h_d) e^{-2\alpha h} \xi^T(t - h(t)) W \xi(t - h(t)) \right\} \tag{13}$$

The time derivative of V_3 is

$$\dot{V}_3 = h^2 e^{2\alpha t} \xi^T(t) Z \xi(t) - h \int_{t-h}^{t} e^{2\alpha s} \xi^T(s) Z \xi(s) ds$$

$$\dot{V}_3 \le e^{2\alpha t} \{ h^2 \xi^T(t) Z \xi(t) - h(t) e^{-2\alpha h} \int_{t-h(t)}^{t} \xi^T(s) Z \xi(s) ds \}$$

By applying lemma 1, the following inequality holds

$$\dot{V}_3 \le e^{2\alpha t}\{\xi^T(t)Z\xi(t) - e^{-2\alpha h}\int_{t-h(t)}^{t}\xi^T(s)ds\ \ Z\int_{t-h(t)}^{t}\xi(s)ds\} \tag{14}$$

The time derivative of V_4 leads to

$$\dot{V}_4 = \frac{h^4}{4}e^{2\alpha t}\xi^T(t)X\xi(t) - \frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}e^{2\alpha s}\xi^T(s)X\xi(s)dsd\theta$$

$$\dot{V}_4 \le e^{2\alpha t}\{\frac{h^4}{4}\xi^T(t)X\xi(t) - \frac{h^2}{2}e^{-2\alpha h}\int_{-h}^{0}\int_{t+\theta}^{t}e^{2\alpha s}\xi^T(s)X\xi(s)dsd\theta\}$$

Utilizing lemma 1, the following inequality holds

$$\dot{V}_4 \le e^{2\alpha t}\{\frac{h^4}{4}\xi^T(t)X\xi(t) - e^{-2\alpha h}\int_{-h}^{0}\int_{t+\theta}^{t}\xi^T(s)ds\,d\theta\ X\ \int_{-h}^{0}\int_{t+\theta}^{t}\xi(s)ds\,d\theta\} \tag{15}$$

We can obtain $\dot{V}_5$ as follows

$$\dot{V}_5 = \frac{h^3}{6}e^{2\alpha t}\dot{x}^T(t)R\dot{x}(t) - \int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}e^{2\alpha\lambda}\dot{x}^T(\lambda)R\dot{x}(\lambda)d\lambda ds\,d\theta$$

$$\dot{V}_5 \le e^{2\alpha t}\{\frac{h^3}{6}\dot{x}^T(t)R\dot{x}(t) - e^{-2\alpha h}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\dot{x}^T(\lambda)R\dot{x}(\lambda)d\lambda ds\,d\theta\}$$

Based on the proposed inequality (c) in lemma 1, one can see that

$$-\int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\dot{x}^T(\lambda)R\dot{x}(\lambda)d\lambda ds\,d\theta \le -\frac{6}{h^3}\left[\int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\dot{x}(\lambda)d\lambda ds\,d\theta\right]^T \times R\left[\int_{-h}^{0}\int_{\theta}^{0}\int_{t+s}^{t}\dot{x}(\lambda)d\lambda ds\,d\theta\right] \tag{16}$$

So, $\dot{V}_5$ can be upper bounded as

$$\dot{V}_5 \le e^{2\alpha t}\{\frac{h^3}{6}\dot{x}^T(t)R\dot{x}(t) - \frac{6}{h^3}e^{-2\alpha h}\left[\frac{h^2}{2}x(t) - \int_{-h}^{0}\int_{t+s}^{t}x(\lambda)d\lambda ds\right]^T \times R\left[\frac{h^2}{2}x(t) - \int_{-h}^{0}\int_{t+s}^{t}x(\lambda)d\lambda ds\right]\} \tag{17}$$

Thus, an upper bound of $\dot{V}$ can be obtained as

$$\dot{V} \le \Phi \tag{18}$$

with

$$\Phi = e^{2\alpha t}\left\{\varsigma^T(t)\Xi\varsigma(t) + Y^T\Delta Y\right\},$$

Ξ is defined in (10),

$$\varsigma^T(t) = \left[x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(t-h(t)) \quad \int_{t-h(t)}^{t} x^T(s)ds \quad \int_{t-h}^{t} x^T(s)ds \quad \int_{-h}^{0}\int_{t+\beta}^{t} x^T(s)dsd\beta \quad f_1^T \quad f_2^T \quad f_3^T\right],$$

$$Y = \left[A \quad A_h \quad C \quad 0 \quad 0 \quad 0 \quad F_1 \quad F_2 \quad F_3\right],$$

$$\Delta = W_{22} + h^2 Z_{22} + \frac{h^4}{4}X_{22} + \frac{6}{h^3}R.$$

On another hand, from assumptions defined in (3), the following inequalities hold

$$\begin{aligned}
&\beta_1^2 x^T(t)H_1x(t) - f_1^T H_1 f_1 \geq 0\\
&\beta_2^2 x^T(t-h(t))H_2x(t-h(t)) - f_2^T H_2 f_2 \geq 0\\
&\beta_3^2 \dot{x}^T(t-h(t))H_3\dot{x}(t-h(t)) - f_3^T H_3 f_3 \geq 0
\end{aligned} \tag{19}$$

where H_i, $i = 1,2,3$, are positive definite matrices.

From (18) and by applying lemma 4, $\dot{V}$ has a new upper bound as

$$\begin{aligned}
\dot{V} \leq \Phi + e^{2\alpha t}\{&\beta_1^2 x^T(t)H_1x(t) + \beta_2^2 x^T(t-h(t))H_2x(t-h(t))\\
&+ \beta_3^2\dot{x}^T(t-h(t))H_3\dot{x}(t-h(t)) - e^{2\alpha t}\sum_{i=1}^{3} f_i^T H_i f_i\}
\end{aligned} \tag{20}$$

Applying Schur's complement (lemma 3), $\dot{V} < 0$ is equivalent to LMI given in (8). Then, it follows from the Lyapunov–Krasovskii stability theory that if the conditions (8) and (9) defined in Theorem 1 are satisfied, system (1) is robustly asymptotically stable. This result leads to

$$V \leq V(0),\ t \geq 0 \tag{21}$$

where

$$\begin{aligned}
V(0) = {}& x^T(0)Px(0) + \int_{-h(0)}^{0} e^{2\alpha s}\xi^T(s)W\xi(s)ds + h\int_{-h}^{0}\int_{\theta}^{0} e^{2\alpha s}\xi^T(s)Z\xi(s)dsd\theta\\
&+ \frac{h^2}{2}\int_{-h}^{0}\int_{\lambda}^{0}\int_{\theta}^{0} e^{2\alpha s}\xi^T(s)X\xi(s)dsd\theta d\lambda + \int_{-h}^{0}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{\theta}^{0} e^{2\alpha s}\dot{x}^T(s)R\dot{x}(s)ds\,d\theta d\lambda d\beta
\end{aligned}$$

$$V(0) \le \lambda_{Max}(P)\|x(0)\|^2 + \lambda_{Max}(W)\int_{-h(0)}^{0}\|\xi(s)\|^2 ds + h\lambda_{Max}(Z)\int_{-h}^{0}\int_{\theta}^{0}\|\xi(s)\|^2 dsd\theta$$
$$+\frac{h^2}{2}\lambda_{Max}(X)\int_{-h}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\|\xi(s)\|^2 dsd\theta d\lambda + \lambda_{Max}(R)\int_{-h}^{0}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{\theta}^{0}\|\dot{x}(s)\|^2 ds\,d\theta d\lambda d\beta$$

$$V(0) \le \sigma \|\psi\|_h^2 \tag{22}$$

with

$$\sigma = \lambda_{Max}(P) + h\lambda_{Max}(W) + h^3\lambda_{Max}(Z) + h^5\lambda_{Max}(X) + h^4\lambda_{Max}(R),$$

Moreover, one can see that

$$V \ge V_1 \tag{23}$$

Then, the following inequality is easily obtained

$$e^{2\alpha t}\lambda_{\min}(P)\|x(t)\|^2 \le V \le \lambda \|\psi\|_h^2$$

which leads to

$$\|x(t)\| \le \sqrt{\frac{\sigma}{\lambda_{\min}(P)}}e^{-\alpha t}\|\psi\|_h \equiv \delta e^{-\alpha t}\|\psi\|_h \tag{24}$$

with $\delta = \sqrt{\frac{\sigma}{\lambda_{\min}(P)}} \ge 1.$

Thus, by definition 1, the system (1) is robust exponentially stable with exponential convergence rate α . The proof is completed.

4. Robust exponential stability of neutral systems with time varying delay and nonlinear perturbations

The robust stability for the perturbed and uncertain systems with time varying delays presents an important focus that described systems more physical than other ones. Thus, this section handles the case that system (1) is with norm-bounded uncertainties. The nonlinear uncertain neutral delay system (1) can be rewritten as the following state equation:

$$\begin{cases} \dot{x}(t) = \left(A + \Delta A(t)\right)x(t) + \left(A_h + \Delta A_h(t)\right)x(t-h(t)) + \left(C + \Delta C(t)\right)\dot{x}(t-h(t)) \\ \quad + \left(F_1 + \Delta F_1(t)\right) f_1(t, x(t)) + \left(F_2 + \Delta F_2(t)\right) f_2(t, x(t-h(t))) \\ \quad + \left(F_3 + \Delta F_3(t)\right) f_3(t, \dot{x}(t-h(t))) \\ x(t) = \phi(t), \quad \dot{x}(t) = \varphi(t), \ \forall t \in [-h, 0] \end{cases} \tag{25}$$

$\Delta A(t)$, $\Delta A_h(t)$, $\Delta C(t)$ and $\Delta F_i(t)$, $i = 1,2,3$, are the norm bounded matrix functions representing time varying uncertainties defined in the form of:

$$\begin{bmatrix}\Delta A(t) & \Delta A_h(t) & \Delta C(t) & \Delta F_1(t) & \Delta F_2(t) & \Delta F_3(t)\end{bmatrix} = D\Sigma(t)\begin{bmatrix}E_a & E_h & E_c & E_1 & E_2 & E_3\end{bmatrix} \tag{26}$$

where $\Sigma(t) \in \Re^{n\times m}$ is a real unknown time varying matrix satisfying

$$\Sigma^T(t)\Sigma(t) \le I, \quad \forall t > 0 \tag{27}$$

and D, E_a, E_h, E_c, E_i, $i = 1,2,3$, are constant real matrices with appropriate dimensions.

Based on Theorem 1 and under the assumptions (26) and (27), novel robust exponential stability criterion for system (25) is obtained as follow.

Theorem 2.

For given scalars $\alpha > 0$ and $h > 0$, the uncertain neutral system (25) is robustly exponentially stable with decay rate α, if there exist positive definite symmetric $n\times n$ matrices P, W_{11}, W_{22}, X_{11}, X_{22}, Z_{11}, Z_{22}, R, H_1, H_2, H_3 any $n\times n$ matrices W_{12}, X_{12}, Z_{12} and a positive scalar ε such that the following LMIs hold:

$$\Pi = \begin{bmatrix} \Omega & U & \varepsilon G^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{28}$$

$$W = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} > 0, X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, Z = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{22} \end{bmatrix} > 0.$$

with

Ω is given in (8),

$$U^T = D^T\left[\left(P+\frac{h^4}{4}X_{12}^T+h^2Z_{12}^T+W_{12}^T\right)\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ W_{22}\ hZ_{22}\ \ \frac{h^2}{2}X_{22}\ R\ 0\ 0\ 0\ 0\right], \tag{29}$$
$$G=\left[E_a\ \ E_h\ \ E_c\ \ 0\ 0\ 0\ E_1\ E_2\ E_3\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right].$$

Proof. Reconsidering the matrix inequality (8) given in Theorem 1, and replacing A, A_h, C, F_1, F_2, F_3 by $A+\Delta A(t)$, $A_h+\Delta A_h(t)$, $C+\Delta C(t)$, $F_1+\Delta F_1(t)$, $F_2+\Delta F_2(t)$, $F_3+\Delta F_3(t)$ respectively , the resulting matrix inequality can be rewritten as follows

$$\Omega+U\Sigma(t)G+G^T\Sigma^T(t)U^T<0 \tag{30}$$

where U and G are given in (29).

It follows from lemma 2, that there exist a scalar ε such that the inequality (30) is equivalent to

$$\Omega+\varepsilon^{-1}UU^T+\varepsilon G^TG<0 \tag{31}$$

Applying Schur's complement, we obtain the linear matrix inequality given in (28). Thus, the system (25) is robustly exponentially stable in the sense of the argument given in the proof of Theorem 1. This completes the proof.

4. Simulation results and comparison

In order to show the effectiveness of the proposed methods, in this section, numerical examples are considered.

Example 1.

Let consider the nominal neutral system with the following parameters

$$\dot{x}(t)=\begin{pmatrix}-0.9 & 0\\ 0.1 & -0.9\end{pmatrix}x(t)+\begin{pmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{pmatrix}x(t-h)+\begin{pmatrix}-0.2 & 0\\ 0.2 & -0.1\end{pmatrix}\dot{x}(t-h) \tag{32}$$

Table 1 lists the comparison of the upper bound on the allowable time delay by different methods. By applying Theorem 1 for system (32), it is clearly shown that the stability method proposed in this paper gives less conservative result than those obtained in [10, 12-15, 22].

Figure 1 shows the feasible area satisfying the conditions of Theorem1 for the variables α and *h*, respectively. In this figure, the feasible area are plotted for $0\le\alpha\le 1.5$ and $0<h\le 2$. We can see what effect the decay rate bound α has on *h* as regards stability.

Table 1. The maximal admissible bounds of time delay *h*

Methods	Maximum delay allowed
[15]	1.3718
[10]	1.6527
[13]	1.7844
[22]	1.7856
[12]	1.8266
[14]	1.9132
Theorem 1	1.9897

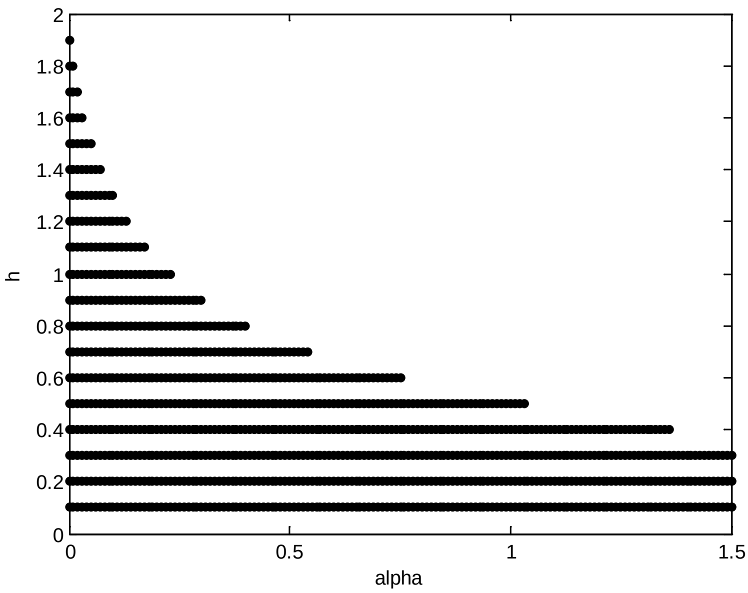

Fig.1. Feasible area for the stability conditions of Theorem 1 for system (32)

Figure 2 gives the simulation results of $x_1(t)$ and $x_2(t)$ when the initial condition is $\varphi(t)=\begin{bmatrix}0.1\\-0.1\end{bmatrix}$. One can see, from figure 2, that the states $x_1(t)$ and $x_2(t)$ asymptotically converge to zero.

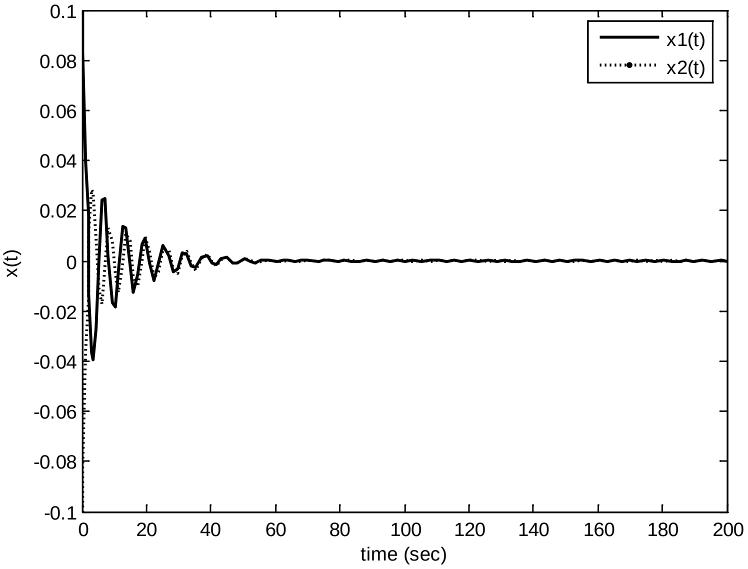

Fig.2. State responses of system (32)

Example 2.

To show the reduced conservatism of the asymptotic stability condition proposed in Theorem 1, let consider the neutral system with nonlinear perturbations (1) studied in [7, 17] with parameters as follows:

$$A=\begin{pmatrix}-1.2 & 0.1\\ -0.1 & -1\end{pmatrix}, A_h=\begin{pmatrix}-0.6 & 0.7\\ -1 & -0.8\end{pmatrix}, C=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix} \tag{33}$$
$$\beta_1=0.1,\quad \beta_2=0.1,\quad \beta_3\geq 0,\quad F_1=F_2=F_3=I$$

Case 1. For $d=0.5$ and different values of β_3, the maximum upper bounds of the time delay is obtained by applying the methods in [8, 16] and Theorem 1. From Table 2, it can be seen that our results are less conservative.

Case 2. For $\beta_3=0$ and under different values of d and α, Table 3 gives out the maximal allowable delay h for the robust exponential stability of system (33) by application of the Theorem 1.

From this example, one can see that as d increases, h decreases. In addition, it should be pointed out the effect of the decay rate α.

Table 2. Upper bounds of time delays h for different values of β_3

β_3	0	0.1	0.2	0.3
[8]	0.7434	0.5131	0.3112	0.1398
[16]	0.8084	0.5969	0.4151	0.2618
Theorem 1	0.8271	0.6420	0.4962	0.3761

Table 3. Upper bounds of time delays *h* for different values of *d* and *α*

	d = 0.1	d = 0.5	d = 0.9
α = 0	1.4316	0.8271	0.4713
α = 0.5	0.5066	0.4329	0.3447
α = 0.9	0.3948	0.3435	0.2808

Example 3.

Consider the following neutral system (34) with

$$\dot{x}(t) = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix} x(t) + \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix} x(t-h) + \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix} \dot{x}(t-h) \qquad (34)$$

For $d = 0$, figure 3 gives out the maximum allowable delay *h* for different values of *c* by using Theorem 1. The feasible area are plotted for $0 < h \le 5$ and $0 < c \le 0.9$. We can see from figure 3 that the variation of the c has a remarkable effect on the upper bound delay, *h* decreases as *c* increases.

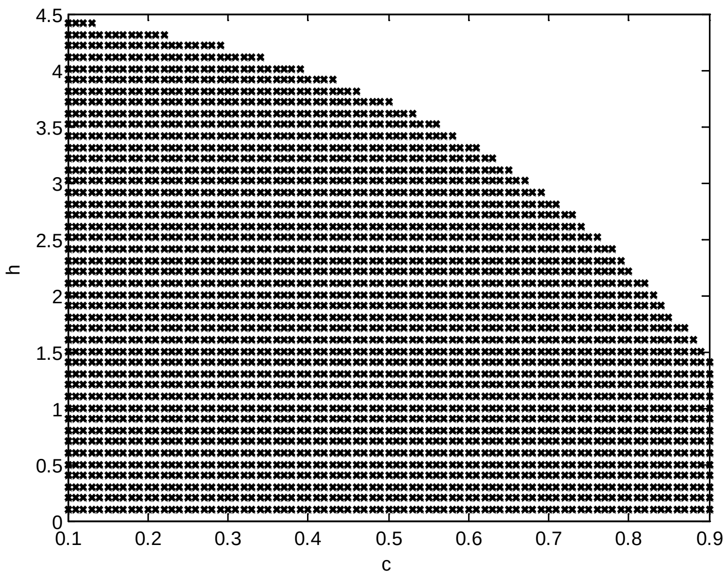

Fig 3. Feasibility fields from for $0 < h \leq 5$ and $0 < c \leq 0.9$ for system (34)

5. Conclusion

In this paper, by introducing new type of Lyapunov-Krasovskii functional, the problem of robust exponential stability for uncertain neutral systems with time varying delays and nonlinear perturbations is studied. The newly proposed delay dependent exponential stability criterion, which is less conservative than the previous relevant ones, can be easily checked by solving a set of LMIs. Numerical examples are given to demonstrate the effectiveness and the merits of the proposed methods than those in the literature.

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