International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 7, N° 1, April 2013, pp. 1816–1833.

On Decentralized Control of a Multimachine Power System: A Comparative study

S. Elloumi(1), B. Abidi(1), N. Benhadj Braiek(1)
(1)Laboratory of Advanced Systems (L.A.S.)
Polytechnic School of Tunis, BP 743, 2078 La Marsa, Tunisia
salwa.elloumi@laposte.net

Abstract. *In this paper, our focus is to compare two decentralized control laws implemented to a three-machine power system which generators are strongly nonlinear interconnected. We present first a linear decentralized control, which gains depend on the system nonlinearity, and confirm, via simulations, its ability to enhance the system transient stability. The second technique is a nonlinear decentralized optimal control based on a successive approximation approach where the nonlinear decentralized controllers are determined by the transformation of each high order coupling nonlinear two-point boundary value (TPBV) problem into a sequence of linear decoupling TPBV problems. We develop algorithmically and implement this decentralized optimal control to the three-machine power system. We prove, via advanced simulations, that this approach brings better performances than the linear decentralized controller, improving effectively transient stability of these power systems in a few steps.*

Keywords. *Power system, Decentralized control, Successive approximation approach.*

1 Introduction

Power systems are classified as large-scale, distributed and highly non linear systems. They present generally fast transients which have been recently focused by many researches in order to improve the overall system transient stability ([1]-[3]). Centralized controllers are obviously not adapted to control such distributed systems because the global information of the entire system is not fully available in a centralized way to allow coordinating the control activity of the overall system. Furthermore, centralized controllers are technically and economically very difficult to design and implement for power systems modeled as complex and large-scale systems. As a logical alternative, decentralized control schemes are proposed, especially since power systems can be characterized by an interconnection of many subsystems. In fact, global central controllers can be substituted by local (decentralized) controllers designed especially for each sub-system. The main goal of decentralized control is to find some feedback laws to adapt the interactions from the other subsystems where no state information is transferred.

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

The advantage of this aspect in controller design is to reduce complexity, and minimize the amount of information transmission, which leads to better feasibility for the control implementation. Unlike centralized control, decentralized control cannot have access to the entire state information. Therefore, interconnections between subsystems need to be analyzed, so that their influence on the system performance can be properly addressed by the control. It is to be noted that the research on decentralized control has been prolific, founding applications in large space structures, power systems, communication networks, etc. ([8]-[16]-[26]-[27]-[28]). A wide variety of properties of the decentralized control systems are extensively studied in the literature and different design techniques are proposed accordingly ([17]-[20]).

In addition to the decentralized control, it is useful to reduce computation through modeling and decomposition techniques for complex systems. However, the simplified computation brings often a conservative result. To find a simpler analysis method or control strategy, we are always in front of high order, coupling and nonlinear TPBV problems which are generally impossible to resolve analytically [21]. This has inspired researches to look for some approaches to approximately obtain the solution to the nonlinear TPBV problem as well as obtain a suboptimal feedback control for nonlinear interconnected large-scale dynamic systems([24]-[25]).

Tang and Sun proposed in [22] a method of driving an optimal control for nonlinear interconnected large scale systems using a successive approximation approach, with respect to quadratic performance indexes. This technique transforms a high order coupling nonlinear TPBV problem into a sequence of linear decoupling TPBV problems.

In this paper, we are intended to use this result in order to develop algorithmically and implement a nonlinear decentralized optimal control for a nonlinear multimachine power system. A suboptimal control law can be obtained by using a finite iterative result of the optimal control sequence [22]. This technique is then compared with a linear decentralized control for the same multimachine power system. The paper is organized as follows: first we introduce in section II the description of the studied systems. Then, in section III, we present the decentralized linear control technique for nonlinear interconnected systems. Section IV focuses on the development of the algorithm for the optimal non linear decentralized control. In Section V, numerical simulations on a power system with three interconnected machines and a comparison study are given to highlight the efficiency of the proposed approaches. Finally, some conclusions are provided in section VI.

2 Description of the studied system

A nonlinear interconnected large-scale system decomposed into N subsystems, can be presented as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + f_i(t, x(t)), \quad t > t_0 \\ x_i(t_0) = x_{i0}, \quad i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where:

- $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector of the i^{th} subsystem,
- $u_i \in \mathbb{R}^{m_i}$ is the control vector of the i^{th} subsystem,
- A_i and B_i are respectively the state matrix and the input matrix of each subsystem; (A_i, B_i) are assumed to be controllable,
- $x = [x_1, x_2, \ldots, x_N^T]^T$,
- $n_1 + n_2 + \ldots + n_N = N$
- $f_i : C^1(\mathbb{R}^n) \to U_i \subset \mathbb{R}^{ni}$, and $f_i(t, x(t)) = f_i(x)$ expresses the interconnection terms vectors characterizing the nonlinearity of the i^{th} subsystem.

Assume that the nonlinear terms $f_i(x)$ satisfy the Lipshitz conditions described by:

$$\|f_i(x)\| \leq c\|x\|, \|f_i(x) - f_i(y)\| \leq h\|x - y\|, \forall x, y \in \mathbb{R}^n \tag{2}$$

where $\|.\|$ is the norm of vectors, c and h are known positive constants.
The control law u_i of each subsystem is a decentralized control. That is, it depends only on the information provided by the i^{th} subsystem, and does not need information exchange with the other subsystems.
We present in the following two approaches to calculate decentralized control for the studied interconnected non linear systems.

3 Decentralized linear control for the nonlinear interconnected systems

This section aims mainly to find a decentralized control for nonlinear interconnected system model described in (1) and verifying constraint (2). We suppose that the term of interconnection $f_i(x)$ can be expressed as follows:

$$f_i(x) = \sum_{j=1, j\neq i}^{N} G_{ij} g_{ij}(x_i, x_j), i = 1, \cdots, N \tag{3}$$

where G_{ij} is a matrix of appropriate dimension and $g_{ij}(x_i, x_j)$ is a nonlinear function, satisfying the following inequality:

$$\|g_{ij}(x_i, x_j)\| \leq \|W_i x_i(t)\| + \|W_{ij} x_j(t)\| \tag{4}$$

for all $x_i \in \mathbb{R}^{n_i}$ and $x_j \in \mathbb{R}^{n_j}$, where W_i and W_{ij} are two known and constant matrices.
The decentralized control is in a linear form; calculating control gains depends on the nonlinearity term of the power system.
In [23] the following theorem is demonstrated:

Theorem 1 *For each $i = 1, 2, \ldots, N$, if there are $R_i > 0$ and defined positive matrix $Q_i(n_i \times n_i)$ such that there is a positive defined matrix $P_i(n_i \times n_i)$ which* *is the unique solution of the following modified algebraic Riccati equation:*

$$A_i^T P_i + P_i A_i + P_i(\sum_{j=1, j\neq i}^{N} p_{ij} G_{ij} G_{ij}^T) P_i - P_i B_i^T R_i^{-1} B_i P_i + \sum_{j=1, j\neq i}^{N} p_{ij} (W_i^T W_i + W_{ji}^T W_{ji}) + Q_i = 0 \tag{5}$$

Then the linear decentralized control law defined by:

$$u_i(t) = -K_i x_i(t), \quad K_i = R_i^{-1} B_i^T P_i \tag{6}$$

stabilizes the global system (21)

4 Decentralized suboptimal control for interconnected systems

We consider the interconnected nonlinear system described by (1) and verifying constraint (2). The goal is to find an optimal control law that minimizes the quadratic cost function:

$$J_i = \frac{1}{2} \sum_{i=1}^{N} \left(x_i^T(t_f) F_i x_i(t_f) + \int_{t_0}^{t_f} (x_i^T Q_i x_i + u_i^T R_i u_i) dt \right) \tag{7}$$

Where Q_i and R_i are respectively the state and input weighting matrices that satisfy the general conditions of a linear-quadratic regulator.
if we apply the maximum principle to (1) and (7), then we can deduce that the necessary condition of the optimal control problem can be described as follows:

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) - S_i \lambda_i(t) + f_i(x), \quad t_0 < t \leq t_f \\ -\dot{\lambda}_i(t) = Q_i x_i(t) + A_i^T \lambda_i(t) + \sum_{j=1}^{N} \sigma_{ij} \lambda_j(t), t_0 \leq t < t_f \\ x_i(t_0) = x_{i0}, \quad \lambda_i(t_f) = F_i x_i(t_f), \quad i = 1, 2, \ldots, N \end{cases} \tag{8}$$

where $S_i = B_i R_i^{-1} B_i^T$, $\sigma_{ij} = \dfrac{\partial f_j}{\partial x_i}$ and λ_i is the adjoint vector introduced in the system Hamiltonian.
The resolution of the differential system (8) leads to the following decentralized optimal control law:

$$u_i^\star(t) = -R_i^{-1} B_i^T \lambda_i(t); \quad t_0 < t \leq t_f; \quad i = 1, 2, \ldots, N \tag{9}$$

The nonlinear interconnected n^{th} order large-scale TPBV problems described by (8) can be decomposed into N sub-problems, which are very difficult to solve for general nonlinear interconnected function vectors f_i. The goal is to develop some approximate approaches for solving the nonlinear large-scale interconnected TPBV problems in (8). Let's consider now the following sequence describing the linear TPBV problems:

$$\begin{cases} \dot{x}_i^{(k)}(t) = A_i x_i^{(k)}(t) - S_i \lambda_i^{(k)}(t) + f_i(x^{(k-1)}), \quad t_0 < t \le t_f \\ -\dot{\lambda}_i^{k}(t) = Q_i x_i^{(k)}(t) + A_i^T \lambda_i^{(k)}(t) + \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t), \quad t_0 \le t < t_f \\ x_i^{(k)}(t_0) = x_{i0}, \quad \lambda_i^{(k)}(t_f) = F_i x_i(t_f), \\ i = 1, 2, \ldots, N, k = 1, 2, \ldots \end{cases} \tag{10}$$

where $f_i(x^{(0)}) = 0$, $\lambda_i^{(0)}(t) = 0$, $\sigma_{ij}^{(k)} = \left[\dfrac{\partial f_j}{\partial x_i}\right]_{x_i = x_i^k}$.
The control vector sequence is defined by:

$$u_i^{(k)}(t) = -R_i^{-1} B_i^T \lambda_i^{(k)}(t), \quad t_0 < t \le t_f, i = 1, 2, \ldots, N, k = 1, 2, \ldots \tag{11}$$

Note that $f_i(x^{(k-1)})$, $\lambda_j^{(k-1)}$ and $\sigma_{ij}^{(k-1)}$ are known. Therefore, (10) is a linear nonhomogeneous TPBV problem.
Let:

$$\lambda_i^{(k)}(t) = P_i(t) x_i^{(k)}(t) + g_i^{(k)}(t), \quad i = 1, 2, \ldots, N \tag{12}$$

Where $P_i(t)$ is the unique semi-positive definite matrix of the following Riccati matrix differential equation:

$$\dot{P}_i(t) + P_i(t) A_i + A_i^T P_i(t) - P_i(t) S_i P_i(t) + Q_i = 0 \tag{13}$$

Substituting (12) into (10), we get a sequence of adjoint vector differential equations:

$$\begin{cases} \dot{g}_i^{(k)}(t) = (P_i(t) S_i - A_i^T) g_i^{(k)}(t) - P_i(t) f_i(x^{(k-1)}) - \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t), t_0 \le t < t_f \\ g_i^{(k)}(t_f) = 0, \quad i = 1, 2, \ldots, N, k = 1, 2, \ldots \end{cases} \tag{14}$$

From development above, we can deduce the model for the studied system as follows:

$$\begin{cases} \dot{x}_i^{(k)}(t) = A_i x_i^{(k)}(t) - S_i \lambda_i^{(k)}(t) + f_i(x^{(k-1)}) \\ \dot{g}_i^{(k)}(t) = (P_i(t) S_i - A_i^T) g_i^{(k)}(t) - P_i(t) f_i(x^{(k-1)}) - \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t) \\ -\dot{\lambda}_i^{(k)}(t) = Q_i x_i^{(k)}(t) + A_i^T \lambda_i^{(k)}(t) + \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t) \\ x_i^{(k)}(t_0) = x_{i0}, \quad \lambda_i^{(k)}(t_f) = F_i x_i(t_f), g_i^{(k)}(t_f) = 0 \\ t_0 \le t < t_f, i = 1, 2, \ldots, N, k = 1, 2, \ldots \end{cases} \tag{15}$$

The sequence solution to TPBV problem sequence (10) uniformly converges to the solution of large-scale nonlinear interconnected TPBV problem, and TPBV problem sequence (15) becomes solvable and uniformly convergent [22]. According to (11), $u_i^{(k)}(t)$ is also uniformly converging to optimal control $u_i^\star(t)$. Therefore:

$$\begin{array}{l} u_i^\star(t) = \lim\limits_{k\to\infty} u_i^{(k)}(t) = -R_i^{-1}B_i^T[P_i(t)x_i(t) + \lim\limits_{k\to\infty} g_i^{(k)}(t)] \\ t_0 \le t \le t_f, i = 1, 2, \ldots, N \end{array} \tag{16}$$

Replacing $k \to \infty$ with $k = M$ in (16), we may obtain a suboptimal control law useful in practical application. And we consider the $g_i^{(M)}(t)$ approximately as its limit. According to cost functional (7), an M^{th} order suboptimal control law for large scale system (21) is obtained as follows:

$$\begin{array}{l} u_{iM}(t) = -R_i^{-1}B_i^T[P_i(t)x_i(t) + g_i^{(M)}(t)], \\ t_0 \le t \le t_f, i = 1, 2, \ldots, N \end{array} \tag{17}$$

In practical control systems, we may consider $t_f \to \infty$ when t_f is large enough. Therefore, this approach is also applicable for the case of $t_f \to \infty$. Cost functional (7) becomes:

$$J_i = \frac{1}{2}\sum_{i=1}^{N}\int_{t_0}^{t_f}(x_i^T Q_i x_i + u_i^T R_i u_i)dt \tag{18}$$

Therefore, the following algebraic Riccati matrix equation is used instead of the Riccati matrix differential equation (13):

$$P_iA_i + A_i^TP_i - P_iS_iP_i + Q_i = 0, \quad i = 1, 2, \ldots, N \tag{19}$$

where the solution P_i is a unique positive definite constant matrix.
The successive approximation approach to obtain suboptimal control law (17) can be performed within the following steps:

– **Step 1:** We extract the semi-positive definite matrices $P_i(t)$ from the algebraic Riccati equations in (25). We give a positive constant ϵ. Let $k = 1$, $M = 1$ and $x_i^{(0)}(t) = g_i^{(0)}(t) = g_i^{(1)}(t) = 0$
– **Step 2:** We obtain $x_i^{(1)}(t)$ from:

$$\begin{cases} \dot{x}_i^{(1)} = (A_i - S_iP_i(t))x_i^{(1)}(t) \quad t_0 \le t < t_f \\ x_i^{(1)}(t_0) = x_{i0}, \quad i = 1, 2, \ldots, N \end{cases} \tag{20}$$

and this gives $\lambda_i^{(1)}$, $f_i(x^{(1)})$ and $\sigma_{ij}^{(1)}$. We get $u_{i1}(t)$ from (17) and J_1 from (7). Let $k = k + 1$.
– **Step 3:** Letting $M = k$, we find $g_i^{(k)}$ from (15). $u_{iM}(t)$ is therefore obtained from (17). We calculate J_M according to (7)
– **Step 4:** If $|\dfrac{J_M - J_{M-1}}{J_M}| < \epsilon$, then we stop and extract the suboptimal control law $u_{iM}(t)$.
– **Step 5:** We deduce $f_i(x^{(k)})$ from (15). This will lead to find $\lambda_i^{(k)}$, $f_i(x^{(k)})$ and $\sigma_{ij}^{(k)}$. Letting $k = k + 1$, we go to step 3.

5 Application of the proposed decentralized approaches to an interconnected multimachine power system

The purpose of this section is to implement the decentralized control methods presented in the previous paragraphs, on a power system with three interconnected machines (Fig.1), in order to analyze the performances due to the application of each approach.

5.1 Multimachine Power System nonlinear Model

A large scale power system S with steam valve control can be described by the interconnection of N subsystems (or synchronous generators) S_i, $i = 1, \cdots, N$. The mathematical model of the subsystems is described by the following equations [23]:

$$\dot{x}(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1, j\neq i}^{N} p_{ij} G_{ij} g_{ij}(x_i, x_j), i = 1, \cdots, N \qquad (21)$$

where $x_i(t)$ is the state vector defined by: $x_i(t)^T = [\Delta\delta_i(t) \quad \omega_i(t) \quad \Delta P_{m_i}(t) \quad \Delta X_{e_i}(t)]$,
$\Delta\delta_i(t) = \delta_i(t) - \delta_{i0}$,
$\Delta P_{m_i}(t) = P_{m_i}(t) - P_{m_i0}$,
$\Delta X_{e_i}(t) = X_{e_i}(t) - X_{e_i0}$

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{D_i}{2H_i} & -\dfrac{\omega_0}{2H_i} & 0 \\ 0 & 0 & -\dfrac{1}{T_{m_i}} & \dfrac{K_{m_i}}{T_{m_i}} \\ 0 & -\dfrac{K_{e_i}}{T_{e_i} R_i \omega_0} & 0 & -\dfrac{1}{T_{e_i}} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{T_{e_i}} \end{bmatrix}, G_{ij} = \begin{bmatrix} 0 \\ -\dfrac{\omega_0 E'_{qi} E'_{qj} B_{ij}}{2H_i} \\ 0 \\ 0 \end{bmatrix},$$

$g_{ij}(x_i, x_j) = \sin(\delta_i(t) - \delta_j(t)) - \sin(\delta_{i0} - \delta_{j0})$
with:

- p_{ij} a constant of either 1 or 0(=0 means that S_i has no connection with S_j);
- δ_i the rotor angle for S_i, in radian;
- ω_i the relative speed for S_i, in radian/s;
- P_{m_i} the mechanical power for S_i, in *pu*;
- X_{e_i} the steam valve opening for S_i, in pu;
- H_i the inertia constant for S_i, in second;
- D_i the damping coefficient for S_i, in *pu*;
- T_{m_i} the time constant for S_i's turbine, in second;
- K_{m_i} the gain of S_i's turbine;
- T_{e_i} the time constant of S_i's speed governor, in second;
- K_{e_i} the gain of S_i's speed governor;
- R_i the regulation constant of S_i, in *pu*;
- B_{ij} the nodal susceptance between S_i and S_j, in *pu*;
- ω_0 the synchronous machine speed, in radian/s;
- E'_{qi} the internal transient voltage for S_i, in *pu*, which is assumed to be constant;
- E'_{qj} the internal transient voltage for S_j, in *pu*, which is assumed to be constant;

δ_{i0}, P_{m_i} and X_{e_i} are the initial values of $\delta_i(t)$, $P_{m_i}(t)$ and $X_{e_i}(t)$ respectively.
The parameters of the power system with three interconnected machines are

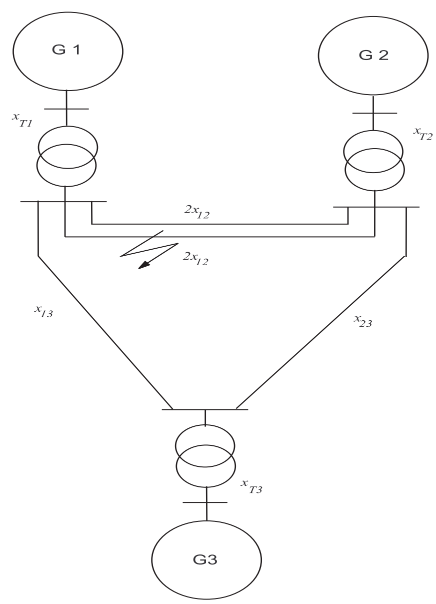

Fig. 1. three-machine example system

summarized in Table 1 [23].
Table 1: three-machine system parameters

	Machine 1	Machine 2	Machine 3
$x_d(pu)$	1.863	2.36	2.36
$x'_d(pu)$	0.257	0.319	0.319
$x_T(pu)$	0.129	0.11	0.11
$x_{ad}(pu)$	1.712	0.712	0.712
$T'_{d0}(pu)$	6.9	7.96	7.96
$H(s)$	4	5.1	5.1
$D(pu)$	5	3	3
$T_m(s)$	0.35	0.35	0.35
$T_e(s)$	0.1	0.1	0.1
R	0.05	0.05	0.05
K_m	1	1	1
K_e	1	1	1

According to model (21), the studied power system can be described by the following state equations:

$$\begin{cases} \dot{x_1}(t) = A_1x_1(t) + B_1u_1(t) + G_{12}g_{12}(x_1, x_2) + G_{13}g_{13}(x_1, x_3) \\ \dot{x_2}(t) = A_2x_2(t) + B_2u_2(t) + G_{21}g_{21}(x_2, x_1) + G_{23}g_{23}(x_2, x_3) \\ \dot{x_3}(t) = A_3x_3(t) + B_3u_3(t) + G_{31}g_{31}(x_3, x_1) + G_{32}g_{32}(x_3, x_2) \end{cases} \tag{22}$$

where:
- $G_{ij} = \begin{bmatrix} 0 \ \alpha_{ij} \ 0 \ 0 \end{bmatrix}^T$;
- α_{ij} represents the midpoints of $\dfrac{\omega_0 E'_{qi} E'_{qj} B_{ij}}{2H_i}$

In this case, the values of α_{ij} are taken as follows [23]:
$\alpha_{12} = \alpha_{13} = -27.49$, $\alpha_{21} = \alpha_{23} = \alpha_{31} = \alpha_{32} = -23.10$

5.2 Application of the decentralized linear control

The decentralized linear control law (6), applied to the three-machine power system, is defined by:

$$\begin{aligned} u_i(t) &= -K_i x_i(t) \\ &= -K_{\delta_i}[\delta_i(t) - \delta_{i0}] - K_{\omega_i}(t) - K_{P_i}[P_{m_i}(t) - P_{m_i0}] \\ &\quad -K_{X_i}[X_{e_i}(t) - X_{e_i0}] \end{aligned} \tag{23}$$

where:

$$K_i = R_i^{-1} B_i^T P_i \tag{24}$$

with P_i the symmetric positive solution of the modified Riccati equation (5) for $i = 1, 2, 3$.
The weighting matrices $R_i = 2$ and $Q_i = $diag$\{0.001, 0.001, 0, 01, 0.01\}$,
$i = 1, 2, 3$;
Based on the parameters above, we can derive the following decentralized control gains:

$$\begin{aligned} K_1 &= [-1.11 \ -1.16 \ -9.03 \ -3.32] \\ K_2 &= [-55.8 \ -33 \ -159.28 \ -106.62] \\ K_3 &= [-55.8 \ -33 \ -159.28 \ -106.62] \end{aligned}$$

Our goal is to prove the performance of the proposed linear decentralized controller for the following operation points of the power system variables: $\delta_{i0} = 24.6rad$, $P_{m_i0} = 1.1pu$, $X_{e_i0} = 1pu$.
Figure 2 shows the state variable evolution (of the controlled system) towards a perturbation on the rotor angle of the second generator. Figure 3 illustrates the corresponding control signal evolution. These simulations confirm the ability of the proposed linear decentralized control to enhance the system transient stability. But we can also notice clearly the limitation of this control: the transient regime is highly oscillatory and cannot dampen out rapidly the oscillations generated by perturbations.

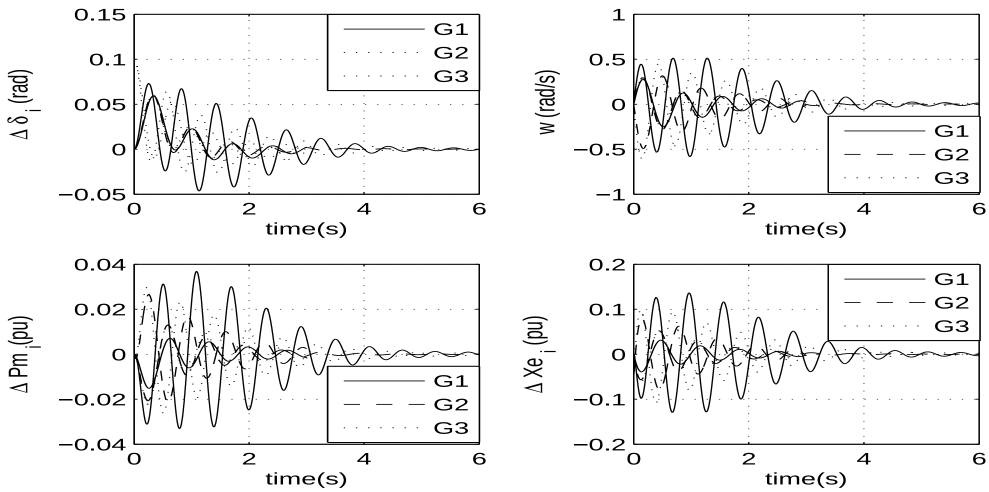

Fig. 2. State variable evolution towards a perturbation on δ_2

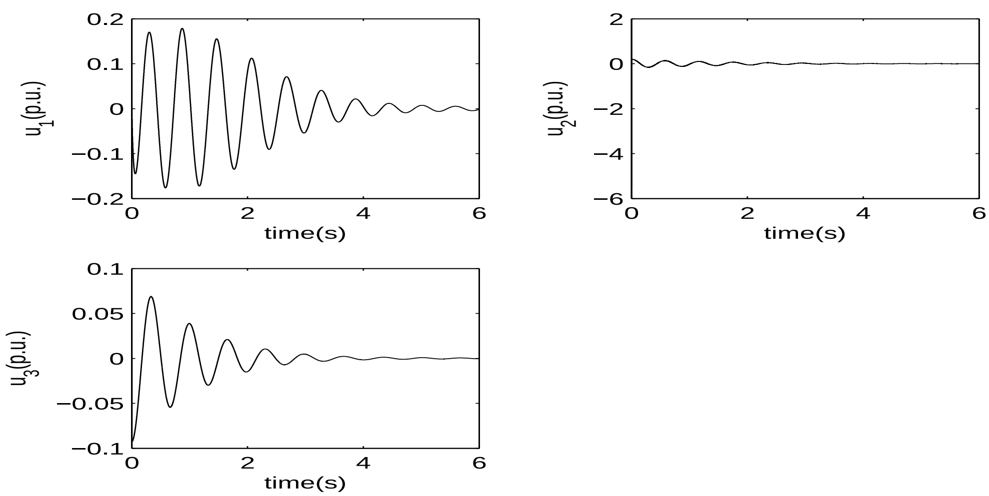

Fig. 3. The corresponding control signal evolution

5.3 Application of the decentralized nonlinear control

We propose to be within the same conditions for simulation in subsection 5.1, and apply therefore the nonlinear decentralized control using the successive ap-

proximation approach to the three machine power system choosing the same perturbation in the rotor angle of second machine. The weighting matrices are $R_i = 2$ and $Q_i = \text{diag}\{0.001, 0.001, 0, 01, 0.01\}$, $i = 1, 2, 3$;
First, we extract the semi-positive definite matrices $P_1(t)$, $P_2(t)$ and $P_3(t)$ from the algebraic Riccati matrix equations as follows:

$$P_i A_i + A_i^T P_i - P_i S_i P_i + Q_i = 0, \quad i = 1, 2, 3 \tag{25}$$

and let $k = 1$, $M = 1$ and $x_i^{(0)}(t) = g_i^{(0)}(t) = g_i^{(1)}(t) = 0$. Then we obtain $x_i^{(1)}$ from:

$$\begin{cases} \dot{x}_i^{(1)} = (A_i - S_i P_i(t)) x_i^{(1)}(t) & t_0 \le t < t_f \\ x_i^{(1)}(t_0) = x_{i0}, \quad i = 1, 2, 3 \end{cases} \tag{26}$$

Then we get u_{i1} from (17) and J_1 from (7). The next step is to increment k into $k+1$ to obtain $g_i^{(2)}$ for $M = 2$ and calculate $u_{i2}(t)$.
After the third iteration of the control, we can deduce the cost functionals of composite system (22). $J_1 = 0.9934$, $J_2 = 0.967$ and $J_3 = 0.7712$. As expected: $J_1 > J_2 > J_3$. If $\epsilon = 0.15$, $|(J_3 - J_2)/J_3| = 0.102 < \epsilon$. Therefore, the control precision can be obtained after 3 iterations. When $k = 1, 2, 3$ the simulation curves of state variable evolution and the corresponding control signals are shown in Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 and Figure 9. It is clearly proven that within simulations advancement through iterations, the precision is getting progressively better. Simulation results presented in Figure 8 and Figure 9 demonstrate the ability of the decentralized nonlinear control to mitigate rapidly the effect of the occurred fault location, neutralizing the oscillations and improving rapidly transient stability of the multimachine power system even though it presents strong nonlinear interconnections between its generators.

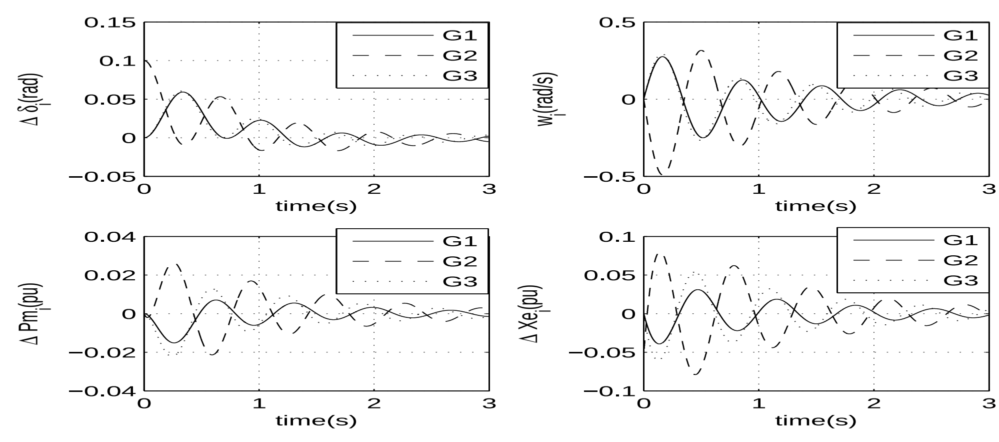

Fig. 4. State variable evolution (k=1)

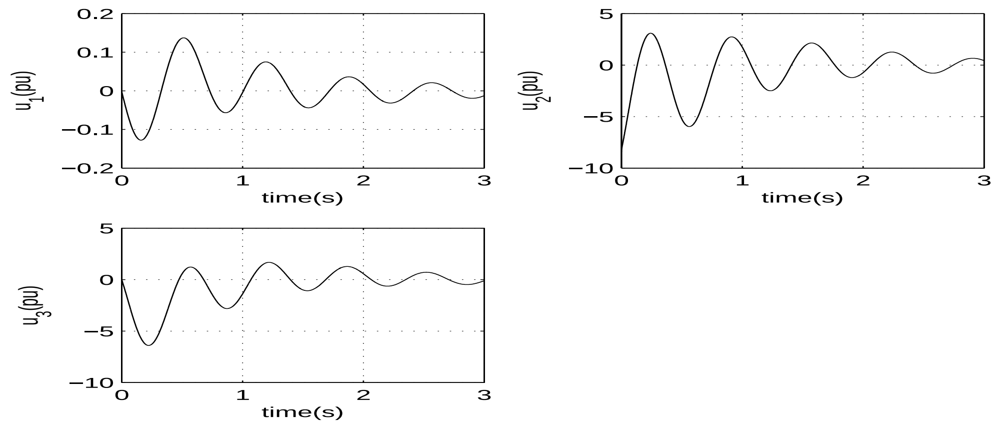

Fig. 5. Control signal evolution (k=1)

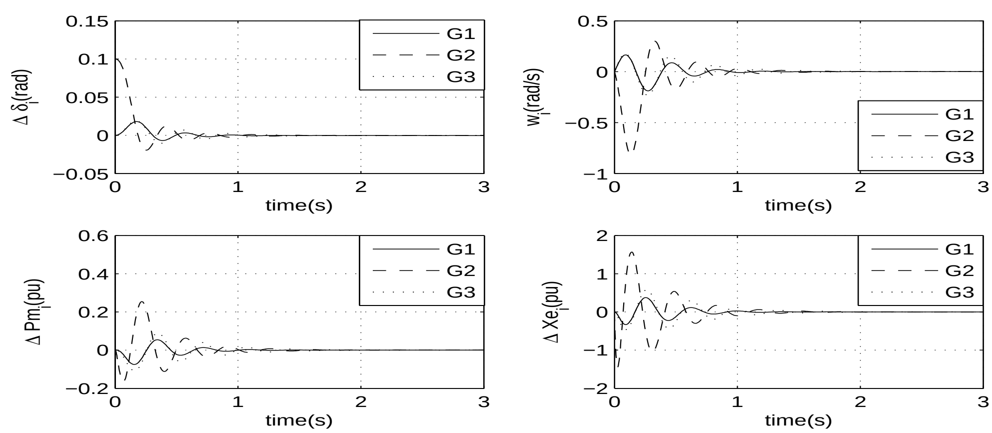

Fig. 6. State variable evolution (k=2)

6 Conclusion

This paper is interested in the implementation of two different decentralized controls to a 3-machine large-scale power system which generators are strongly

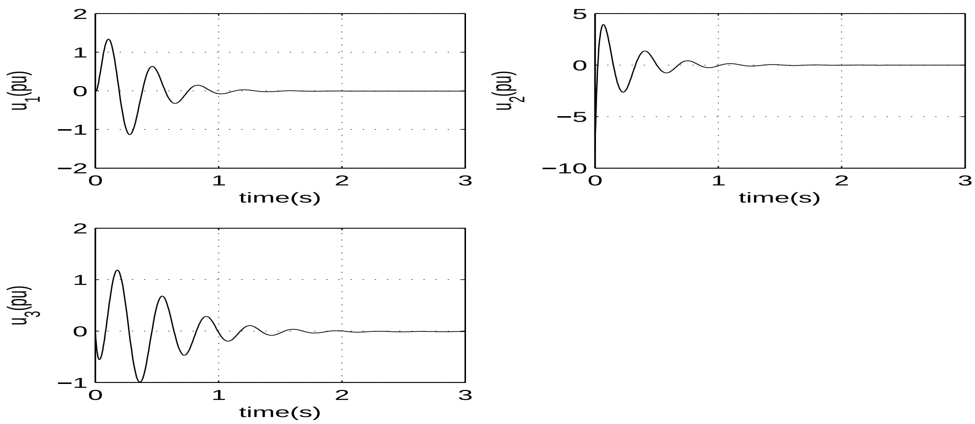

Fig. 7. Control signal evolution (k=2)

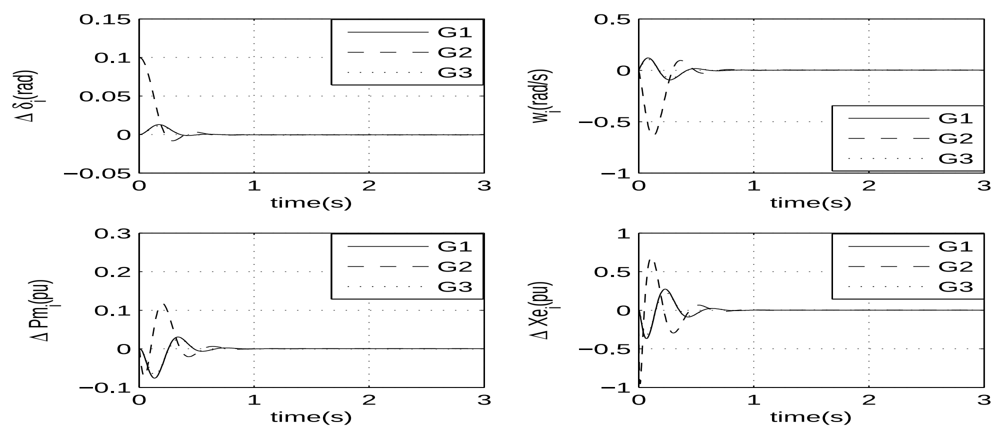

Fig. 8. State variable evolution (k=3)

nonlinear interconnected. First we applied the conventional linear decentralized control law obtained by solving a modified algebraic Riccati equation that depends on the nonlinearity of the large-scale system. The second approach required to develop algorithmically and apply a nonlinear decentralized optimal control scheme which control laws are calculated using a successive approximation approach. A suboptimal control law has been obtained by using a finite iterative result of optimal control law sequence. Simulation results demonstrate that the nonlinear decentralized controller provides better performance, and out-

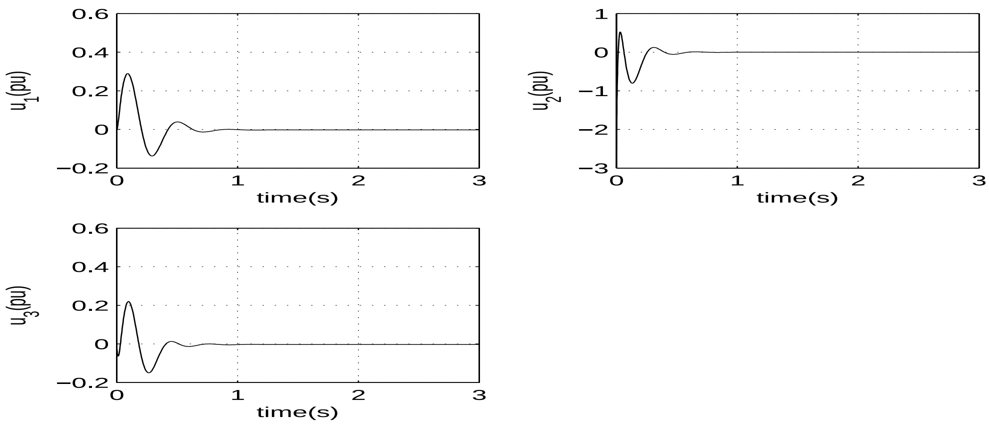

Fig. 9. Control signal evolution (k=3)

performs the optimized conventional linear decentralized control. In fact, the nonlinear decentralized controller based on successive approximation approach, proved more efficiency in damping out oscillations of the power system within few iterative sequences, improving significantly the system transient stability.

Bibliography

[1] Elloumi, S., Benhadj Braiek, N.: A Decentralized stabilization approach of a class of nonlinear polynomial interconnected systems- application for a large scale power system. Nonlinear Dynamics and Systems Theory, **12**(2)(2012) 159–172.

[2] Jiang, L., Wu, Q. H., Wen, J. Y.: Decentralized nonlinear adaptive control for multimachine power systems via high-gain perturbation observer. IEEE Trans. on Circuits and SystemsI: Regular Papers, **51**(10)(2004) 2052–2059.

[3] Gupta, F., Bandyopadhyay, B., Kuhni, A.M., Manjunath, T.C.: Design of decentralized power system stabilizer for multi-machine power system using periodic output feedback technique. Seventh lnternational Conference on Control, Automation, Robotics And Vision (ICARCV'OZ), 2002, Singapore.

[4] Kunder, P.: Power System Stability and Control. McGraw-Hill Ed., 1994.

[5] Machowski, J., Bialek, J., Bumby, J.: Power System Dynamics: Stability and Control. John Wiley and Sons Editions, 2008.

[6] Kundur, P., Paserba, J. , Ajjarapu, V., Anderson, G., Bose, A., Canizares, C., Hatziargyriou, N., Hill, D., Stankovic, A., Taylor, C., Van Cutsem, T., Vittal, V.: Definition and classification of power system stability IEEE/CIGRE joint task force on stability terms and definitions. IEEE Transactions on Power Systems, **19**(3) (2004) 1387 - 1401

[7] Pai, M.A.: Energy Function Analysis for Power System Stability. Kluwer Academic Publisher, 1989.

[8] Li, H., Wang, Y.: Nonlinear robust decentralized control of multimachine power systems. in Proceedings of 7th International Power Engineering Conference, (2005) 1–6.

[9] Khatir, M. E., Davison, E. J.: Decentralized control of a large platoon of vehicles operating on a plane with steering dynamics. in Proceedings of 2005 American Control Conference, Portland, OR, (2005) 2159–2165.

[10] Fay, A., Fischer, I.: Decentralized control strategies for transportation systems. International Conference on Control and Automation, Budapest, Hungary, (2005) 898–903. 8

[11] Movsichoff, B. A., Lagoa, C. M., Che, H.: Decentralized optimal traffic engineering in connectionless networks. IEEE Journal on Selected Areas in Communications, **23**(2) (2005) 293–303.

[12] Yan, X.G., Lam, J., Dai, G.Z.: Decentralized robust control for nonlinear large-scale systems with similarity. Computers and Electrical Engineering, **25**(3)(1999) 169–179.

[13] Lavaei, J., Aghdam, A.G.: Decentralized control design for interconnected systems based on a centralized reference controller. Proceed-

ings of the 45th IEEE Conference on Decision and Control, Manchester Grand Hyatt Hotel San Diego, CA, USA, December (2006) 13-15.

[14] Liu, W., Sarangapani, J., Venayagamoorthy, G. K., Liu, L., Wunsch II, D. C., Crow, M. L., Cartes, D. A. : Decentralized neural network-based excitation control of large-scale power systems. International Journal of Control, Automation, and Systems, **5**(5)(2007) 526-538.

[15] Yan, X.G., Dai, G.Z.: Decentralized output feedback robust control for nonlinear large-scale systems. Automatica, , **34**(11)(1998) 1469–1472.

[16] Guo, Y., Hill, D.J., Wang, Y.: Nonlinear decentralized control for large scale power systems. Automatica, **36**(9)(2000) 1275–1289.

[17] Lavaei, J., Aghdam, A. G.: Elimination of fixed modes by means of high-performance constrained periodic control. in Proceedings of 45th IEEE Conference on Decision and Control, San Diego, CA, pp.4441-4447, 2006.

[18] Lavaei, J., Aghdam, A. G.: Simultaneous LQ control of a set of LTI systems using constrained generalized sampled-data hold functions. Automatica, **43**(2) (2007) 274–280.

[19] Duan, Z., Wang, J. Z., Huang, L.: Special decentralized control problems and effectiveness of parameter-dependent Lyapunov function method. in Proceedings of 2005 American Control Conference, **3**(2005) 1697–1702.

[20] Lavaei, J., Aghdam, A. G.: Optimal periodic feedback design for continuous-time LTI systems with constrained control structure. International Journal of Control, **80**(2)(2007) 220–230.

[21] Lu, P.F., G.Y. Tang, X.B. Jia, Y. Tao, Successive approximation approach of suboptimal control for nonlinear time-delay systems, Control and Decision, **19**(2)(2004) 230–234.

[22] Tang, G.Y., Sun, L.: Optimal control for nonlinear interconnected large-scale systems: A successive approximation approach. ACTA Automatica Sinica, **31**(2)(2005) 248–254.

[23] Wang, Y., Hill, D.J., Guo, G.: Robust decentralized control for multimachine power systems. IEEE Transaction On Circuits and Systems, **45**(3)(1998) 271–279.

[24] Kim Y.-J. and Lim M. T. , " Parallel optimal control for weakly coupled nonlinear systems using successive Galerkin approximation", *IEEE Transactions on Automatic Control*, **53**(6)(2008), 1542-1547.

[25] Lawton J. , Beard R. W. and McLain T. , "Successive Galerkin approximation of nonlinear optimal attitude control", *Proceedings of the American Control Conference*, **6**(1999), 4373-4377.

[26] Lefebvre, S., Richter, S. et Decarlo, R. "Decentralized control of linear interconnected multivariable systems *19th IEEE Conference on Decision and Control including the Symposium on Adaptive Processes*, **19**(1980), 525–529.

[27] Jung. K., Kim. K.Y., Yoon. T. W. et Jang. G. "Decentralized control for multi-machine power systems with nonlinear interconnections and disturbances". *Conference ICCAS'02*, Korea, 16–19 (2002)

[28] Jiang. H., Cai. H., Dorsay. J. et Qu. Z. "Toward a globally robust decentralized control for large scale power systems". *IEEE Transactions on Control Systems and Technologies*, (1997) 309–319.