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Integrated Active Fault Tolerant Control Approach Based LMI

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Abstract. *In this work we study a new approach of fault tolerant control schema.The suggested strategy is an active Fault Tolerant Control (FTC) strategy that reconfigures the controller on-line taking into account changes due to the faults where both the block of fault detection and the block of the controller are formulated as a Linear Matrix Inequality (LMI) problem. The problem of fault estimation is based on a robust adaptive observer. The control problem is formulated as LMI problem with an approach to design an Admissible Model Matching (AMM) Fault Tolerant Control (FTC). Finally we give a definition and solution of this approach applied to a flight problem control.*

Keyword.*Fast adaptive observer fault estimation, Fault tolerant controller, LMI.*

1 Introduction

Fault Tolerant Control (FTC) has been considered as an important research topic in the control applications during last years. Due to its intimate relationship to the robust control theory, more attention is done upon this topic [11].

The aim of fault tolerant control system (FTCS) is to keep plant available by the ability to achieve the objectives assigned and preserve stability conditions in the presence of component and/or instrument faults, and to accept reduced performance when critical fault occurs [7]. Accommodation capability of a control system depends on many factors such as severity of fault, the robustness of the nominal system and mechanisms that introduce redundancy in sensors and/or actuators [9].

FTCS can be classified into two types: passive fault tolerant control system (PFTCS) and active fault tolerant control system (AFTCS). In the passive approaches, the same controller is used for the normal case as well as for the faulty cases where, a presumed set of process component faults are considered in the design stage controllers are fixed and are designed to be robust against a class of faults. In contrast to PFTCS, AFTCS consists of :

1. FDD scheme with high sensitivity to faults and should be able to provide precise and the most updated information about the system as soon as possible

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after the fault occurrence to provide as precisely as possible, the information about a fault. Many FDD approaches are developed and there are quantitative model-based approaches, qualitative model based approaches and knowledge based approaches [10].

2. Reconfiguration blockk to design a new control scheme to compensate the fault induced changes in the system so that the stability and acceptable closed-loop system performance can be maintained [17].

In this paper an integrated active fault tolerant control system is used where both the model-based FDD problem and the reconfigurable state feedback controller problem are formulated in one Linear Matrix Inequality problem. The solution of this LMI incorporates the possible error in the FDD algorithm into the control loop performance.

This paper is organized as follows: Section 2 provides the integrated AFTC principle with the rules to design the controller and the FDD block. Section 3 presents the simulation results. Finally, concluding remarks are made in section 4 followed by the list of references.

2 The integrated Active Fault tolerant Control Formulation

The standard active Fault tolerant Control Schema is composed by two principle modules: the module of the fault detection and estimation, and the module of the reconfigurable controller where are designed separately. Hereafter, we propose a new approach to design both the first step and the second step of fault tolerance in one problem. The figure 1 recapitulates this approach. Our idea consists in

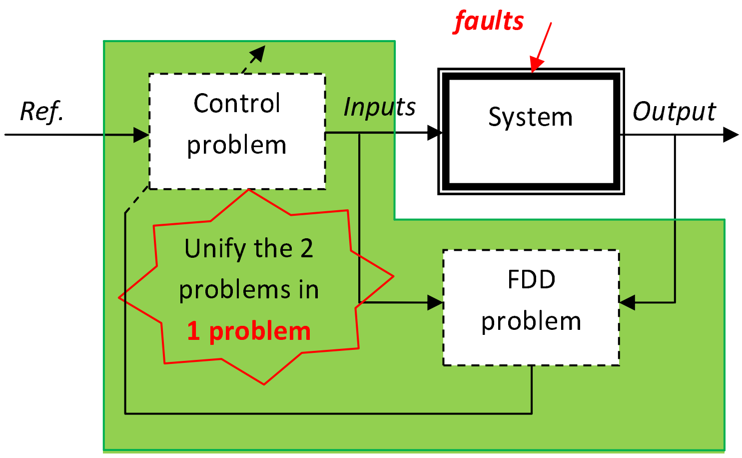

Fig. 1. The Integrated AFTC principle.

unifying the two problems which are often designed separately in standard FTC schema [4], in one problem to design the controller and the diagnosis observer.

2.1 Problem 1: Control Design

The main idea of AMM FTC approach proposed in [13] is that instead of looking for a controller that provides an exact (or best) matching to a given single behavior after the fault appearance, a family of closed-loop behaviors that are acceptable is specified. In order to recall the principle of Admissible Model Matching,consider the following LTI system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, A is an $(n \times n)$ and B is an $(n \times m)$ constant matrices. There exists a stabilizing state feedback gain K for (1) and a classical state feedback control law is considered [9]:

$$u(t) = -Kx(t) \tag{2}$$

In the model matching problem, (1) and (2) result in the closed loop behavior that follows the reference model:

$$\dot{x}(t) = (A - BK)x(t) = Mx(t)$$

where M is chosen to be stable. In the AMM approach, a set of system matrices that are acceptable is considered and the FTC controller tries to provide a closed-loop behavior inside the set [13]. The triple $(A, B, K) \in \mathcal{M}_a$ is called admissible if and only if:

$$\mathcal{M}_a = \{(A, B, K) : \Phi_M(A, B, K) \preceq 0\}$$

where $\Phi_M(A, B, K)$ are the set of constraints that guarantee $(A, B, K) \in \mathcal{M}_a$. The set of reference models $\mathcal{M}_a$ is defined of-line by the designer.

Admissible model matching method proposed in [14] is considered to design a fault tolerant controller gain such that the poles of the closed-loop system are inside a pre-established region even in faulty case. This algorithm will be combined with an additional LMI constraint that enforce the applied load to respect as possible to predefined level with a priority to the actuators based on its criticalities. The set of admissible behaviors $\mathcal{M}_a$ can be proposed as:

$$\mathcal{M}_a = \{(A, B_f, K_f) : \Lambda(A - B_f K_f) \in \mathcal{D}_\alpha\} \tag{3}$$

where $\Lambda(.)$ is the set of the eigenvalues of the matrix (.). $\mathcal{D}_\alpha$ is a desired region included in the unit circle with an affix $(-q, 0)$ and a radius r such that $(q + r) < 1$ is fixed. These two scalars q and r are used to determine a specific region included in the unit circle. According to [1], (3) can be rewritten as follows:

$$\begin{bmatrix} -rP & qP + PM^T \\ qM + MP & -rM \end{bmatrix} \prec 0 \tag{4}$$

The applied load of the control input $u(t) = -Kx(t)$ evaluated as the norm $\|u(t)\|$ can be enforced to respect an upper bound such that for every initial condition $x(0)$ with $\|x(0)\| \leq 1$, the resulting control satisfies $\|u(t)\| \leq u_{\max}$ for all $t \geq 0$, when the initial condition is known , an upper bound on the norm of the control input $u(t) = -Kx(t)$ can be found where [8],

$$\max_{t\geq 0} \|u(t)\| = \max_{t\geq 0} \left\|SP^{-1}x(t)\right\| \leq \max_{x\in\xi} \left\|SP^{-1}\right\| = \Lambda_{\max}(P^{-1/2}S^TSP^{-1/2}) \quad (5)$$

and

$$\xi = \left\{x \in \mathbb{R}^n, x^nP^{-1}x \leq 1\right\} \quad (6)$$

with $\Lambda_{\max}(.) = \max(\Lambda(.))$ is the maximum eigenvalue of the matrix (.).

Therefore the constraint $\|u(t)\| \leq u_{\max}$ is enforced at all times if the following LMI condition is satisfied.

$$(P^{-1/2}S^TSP^{-1/2}) \leq u_{\max}^2 \quad (7)$$

then,

$$S^TS \leq u_{\max}^2 P \quad (8)$$

Or equivalent to,

$$P - u_{\max}^{-2}S^TS \geq 0 \quad (9)$$

By using the Shur complement of this constraint [12],

$$\begin{pmatrix} u_{\max}^2 I & S \\ S^T & P \end{pmatrix} \succ 0 \quad (10)$$

holds for a given $u_{\max} \succ 0$ where $P \geq 0, \left\{x(0)^TP^{-1}x(0) \leq 1\right\}$, and $K = SP$, such that S satisfies the stabilizing condition

$$-AP - PA^T - BS - S^TB^T > 0 \quad (11)$$

The equation (4) is equivalent to the next equation :

$$\begin{bmatrix} -rP & qP + PA^T - S^TB^T \\ qP + AP - BS & -rP \end{bmatrix} \prec 0 \quad (12)$$

2.2 Problem 2: FDD Design

In the following, an adaptive observer [17] is proposed, in order to detect and estimate faults. Let consider the following state space model representation with actuator fault consideration:

$$\begin{cases} \dot{x} = Ax + Bu + Ef_a \\ y = Cx \end{cases} \quad (13)$$

where x is the state vector $\in \mathbb{R}^n$, u is the control input vector $\in \mathbb{R}^m$, y is the output vector $\in \mathbb{R}^p$, f_a represents the actuator fault $\in \mathbb{R}^r$, A, B ,C, E are known constant real matrices of appropriate dimensions, the matrix E is of full column rank and the pair (A, C) is observable.

The adaptive fault diagnosis observer is constructed as:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + E\hat{f}_a - L(\hat{y} - y) \\ \hat{y} = C\hat{x} \end{cases} \tag{14}$$

where $\hat{x} \in \mathbb{R}^n$ is the observer state vector, $\hat{y} \in \mathbb{R}^p$ is the observer output and the $\hat{f}_a \in \mathbb{R}^r$ is an estimate of actuator fault.

Since it has been assumed that the pair (A, C) is observable, the observer gain matrix L can be selected such that $(A - LC)$ is a stable matrix.

Denote e_x the state error, e_y the output error and e_f the fault error.

$$\begin{cases} e_x = \hat{x} - x \\ e_y = \hat{y} - y \\ e_f = \hat{f}_a - f_a \end{cases} \tag{15}$$

Before presenting the main results, two assumptions are given:

- **Assumption 1**: $rank(CE) = r$.
- **Assumption 2**: the invariant zeros of (A, E, C) lie in open left half plane.

Considering a variable fault, the derivative of e_f is:

$$e_f = \dot{\hat{f}}_a - \dot{f}_a \tag{16}$$

Theorem 1. *If there exist symmetric positive definite matrices P and Q, an observer gain L, and a matrix F such that the following conditions hold,*

$$P(A - LC) + (A - LC)^T = -Q \tag{17}$$

$$E^T P = FC \tag{18}$$

then the adaptive fault estimation algorithm:

$$\dot{\hat{f}}_a = -\Gamma F(\dot{e}_y + \sigma e_y) \tag{19}$$

can establish $\lim_{t\to\infty} e_x = 0$*, where the symmetric positive definite matrix $\Gamma \in \mathbb{R}^r$ is the learning rate.*

Actuator fault estimate using the above method can be obtained:

$$\hat{f}_a = -\Gamma F(e_y + \int_{t_f}^{t} e_y(\tau)d\tau) \tag{20}$$

Theorem 2. *Under assumptions 1-2, given scalars $\sigma, \mu > 0$, if there exist symmetric positive definite matrices $P \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{r\times r}$, $L \in \mathbb{R}^{n\times p}$ and $F \in \mathbb{R}^{r\times p}$ such that* (11) *and the following condition hold*

$$\begin{bmatrix} PA + A^T P - PLC - C^T LP^T & -\frac{1}{\sigma}(A^T PE - C^T L^T P^T E) \\ -\frac{1}{\sigma}(A^T PE - C^T L^T P^T E)^T & -2\frac{1}{\sigma}E^T PE + \frac{1}{\sigma\mu}G \end{bmatrix} < 0 \tag{21}$$

To solve conditions in theorem 2 it is easy to solve the inequality (21) by LMI toolbox but the solving difficulty is added because of (18) that we transform into the following optimisation problem:

$$\textit{Minimise } \eta \textit{ subject to } (21) \textit{ and}$$

$$\begin{bmatrix} \eta I & E^T P - FC \\ (E^T P - FG)^T & \eta I \end{bmatrix} < 0 \tag{22}$$

3 Simulations and Results

The ADMIRE model has been used by several researchers (e.g. [8]) and within the Group of Aeronautical Research ans Technology in Europe (GARTEUR). The linear model used here has been obtained at a low speed flight condition of Mach 0.22 at an altitude of $3000m$ and is similar to the one in [2]. The states are $x = [\alpha\ \beta\ p\ q\ r]^T$ with controlled outputs $y = [\alpha\ \beta\ p]$; where α is the angle of attack (rad), β is the sideslip angle (rad), and p is the roll rate (rad/s), q defines the pitch rate (rad/s) and r is the yaw rate (rad/s). The control surfaces are $u = [u_c\ u_{re}\ u_{le}\ u_r]^T$, which represent the deflections of the canard, the right eleven, the left eleven and the rudder respectively.

In this example, the actuator dynamics are neglected, and the approximate model can described by the following state space model representation [2]:

$$\begin{cases} \dot{x} = Ax + Bu + Ef_a \\ y = Cx \end{cases}$$

where

$$A = \begin{pmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5128 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.0069 & -0.0866 & -0.0866 & 0.0004 \\ 0 & 0.0119 & -0.0119 & 0.0287 \\ 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$E = B$$

In our exemple we solve the LMI problem with $\sigma = \mu = 1$, we obtain:

$$P = 10^4 \begin{pmatrix} 1.1110 & -0.0132 & 0.0217 & -0.0240 & -0.0105 \\ -0.0132 & 0.8405 & 0.0808 & -0.0377 & 0.1151 \\ 0.0217 & 0.0808 & 0.0476 & -0.0004 & -0.0201 \\ -0.0240 & -0.0377 & -0.0004 & 0.2234 & 0.0002 \\ -0.0105 & 0.1151 & -0.0201 & 0.0002 & 0.9027 \end{pmatrix}$$

$$L = \begin{pmatrix} 1.0785 & 0.0817 & -0.0220 & 0.4311 & 0.0080 \\ -0.0183 & 2.8831 & -0.3117 & 0.0419 & -0.0393 \\ -0.0370 & -7.3738 & 1.8115 & -0.1485 & 0.0134 \\ 2.0581 & 0.2746 & -0.0295 & 1.6943 & 0.0063 \\ 0.0150 & -0.4661 & 0.0597 & -0.0008 & 0.7857 \end{pmatrix}$$

$$F = 10^3 \begin{pmatrix} -0.3201 & -0.6249 & -0.0052 & 3.6911 & 0.0024 \\ -1.5472 & -3.1582 & -1.9687 & -2.8115 & -1.6609 \\ 0.2342 & 4.1424 & 1.9416 & -2.8361 & 1.6742 \\ 0.4146 & 0.4262 & 0.9086 & -0.0133 & -8.2296 \end{pmatrix}$$

$$G = 10^3 \begin{pmatrix} 2.3945 & -2.0455 & -1.8871 & 0.0230 \\ -2.0455 & 5.2245 & -1.7369 & 0.2219 \\ -1.8871 & -1.7369 & 4.8477 & -0.2838 \\ 0.0230 & 0.2219 & -0.2838 & 4.9462 \end{pmatrix}$$

$$K = \begin{pmatrix} 0.0001 & 0.0598 & -3.7157 & -0.0009 & -0.8164 \\ 0.0115 & 5.3188 & -330.6148 & -0.0763 & -72.6408 \\ -0.0116 & -5.3444 & 332.2032 & 0.0766 & 72.9898 \\ -0.0220 & -10.1517 & 631.0237 & 0.1456 & 138.6449 \end{pmatrix}$$

We consider a lost of actuator effectiveness as a fault to diagnosis [3]. In the following, we assume that a fault is occurred in the right eleven actuator. This actuator has lost 50% of its effectiveness. The simulations results are listed below in figures 2, 3, 4 and 5. Figure 2 shows the outputs of the system fault-free case and the faulty case with the diagnosis and control actions where the system is reconfigured perfectly. The control signals are regrouped in figure 3. Figures 4 and 5 perform, respectively, the estimated fault and effectiveness lost.

4 Conclusion

In this paper, a new approach to design an AMM FTC has been proposed based on LTI fault representation. The active AMM FTC uses only one block to represent the problem of fault detection and control where both are formulated in the same Linear Matrix Inequality (LMI) problem.

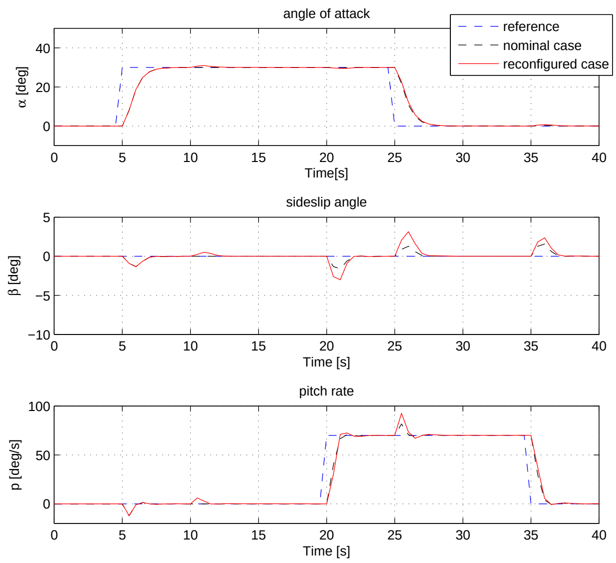

Fig. 2. The output responses of the system.

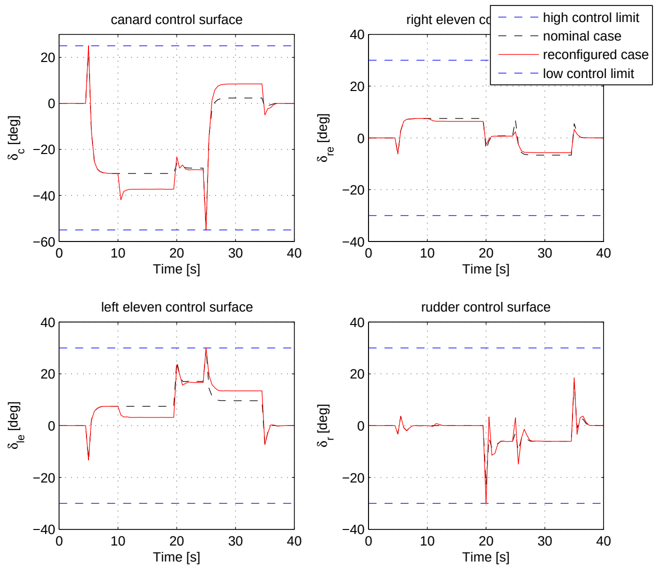

Fig. 3. The control inputs.

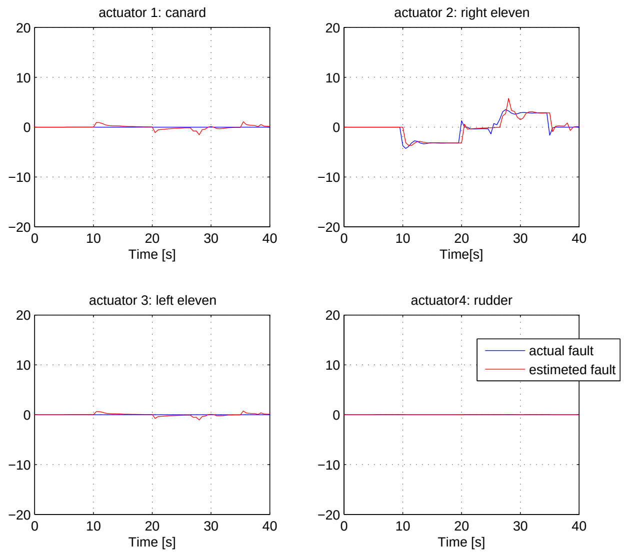

Fig. 4. The fault estimations.

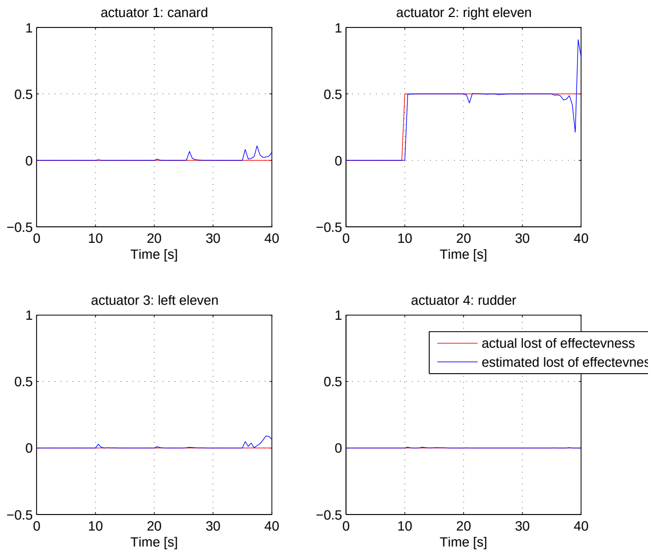

Fig. 5. The lost of effectiveness estimations.

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