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Robust Stabilization with Saturating Actuators of Neutral and State-Delayed Systems

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Abstract

The stabilization using saturated static memoryless state-feedback of systems subject to polytopic uncertainties is addressed here, for neutral systems (systems with time-varying delays in the states and their derivatives), is addressed here. In particular, robust stabilizing state-feedback controllers are given, expressed as Linear Matrix Inequalities that depend on the maximum allowable delays. These conditions are derived by using a Lyapunov-Krasovskii functional on the vertices of the polytopic descriptions of the actuator saturations and system uncertainties. The approach is then particularized to standard state-delayed systems, providing relevant results, that are less conservative than those in the literature, as shown using some examples.

Key words– Uncertainties, Actuator saturation, Linear Matrix Inequalities, Neutral Systems, Time-varying Delays, State-delays.

1 Introduction

The stabilization of neutral systems has been studied in the literature (Han *et al*, 2004; Haurani *et al*, 2003; Li *et al*, 2003; Mahmoud, 2000; Yu and Lien, 2007; El Haoussi *et al*, 2011), albeit not in the simultaneous presence of saturation and uncertainty. Thus, in this paper, we are interested in studying saturated neutral systems subject to uncertainty. The methodology followed in this paper follows the approach developed by El Haoussi *et al* (2011): for uncertain neutral systems, the results of El Haoussi *et al* (2011) cannot be applied, so a parallel approach, based on using Lyapunov-Krasovskii functionals is used here, in order to get a set of *LMIs* that depends only on the vertices of the polytopic uncertainty and the maximum value of the delays, which can be solved using dedicated solvers (Boyd *et al*., 1994). Thus, the major contribution of this paper is the derivation of delay-dependent methods for the stabilization of saturated neutral systems with polytopic uncertainties. To represent the saturated system, as in El Haoussi *et al* (2011) a polytopic model (Cao *et al*, 2002)

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is used, as it guarantees the local stability of the closed loop system when the initial states are taken within a given region of attraction.

These main results are then particularized to standard state-delayed systems (i.e., a delay affects only the states, not their derivatives). It is shown that, for these systems, the obtained results are also relevant, as they are less conservative than those in the literature. This is also illustrated by some numerical examples.

Notation: The following notations will be used throughout the paper: $\Re$ denotes the set of real numbers, $\Re^n$ denotes the n dimensional Euclidean space and $\Re^{m\times n}$ denotes the set of all $m \times n$ real matrices. The notation $X \geq Y$ (respectively $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi−definite (respectively positive definite). The symbol * stands for a symmetric block in matrix inequalities. $\overline{\lambda}(P)$ and $\underline{\lambda}(P)$ denote, respectively, the maximal and minimal eigenvalues of a matrix P. $\|.\|$ refers to either the Euclidean vector norm, or the induced matrix 2−norm. The symbol $\mathbf{C}^1([-d, 0], \Re^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d, 0]$ into $\Re^n$. I denotes the identity matrix of appropriate dimensions. For a matrix K, the i^{th} row is denoted by k_i. For any vector $u \in \Re^m$, the saturation function is defined by $sat(u) = [sat(u_1)\ sat(u_2) \ldots\ sat(u_m)]^T$, where
$sat(u_i) = sign(u_i)min\{|u_i|, \overline{u}_i\}$, with given bounds $\overline{u}_i > 0$. The convex hull of a set is the minimal convex set containing it: Thus, for a set of points $x_1, x_2, ..., x_n \in \Re^n$, its convex hull is $Co\{x_1, x_2, \ldots, x_n\} = \{\sum_{i=1}^n \alpha_i x_i;\ \alpha_i \geq 0,\ \sum_{i=1}^n \alpha_i = 1\}$.

2 Problem formulation and previous results

We consider here the following state-space linear systems, with uncertainty in the system matrices, and time-varying delays in the states and their derivatives:

$$\begin{aligned}&\dot{x}(t) - C\dot{x}(t - \tau(t)) = A_0 x(t) + A_1 x(t - h(t))\\ &+Bsat(u(t)),\end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control input, C, A_0, A_1 and B are unknown matrices of real numbers, contained in the following convex polytopic set:

$$\begin{bmatrix} A_0 & A_1 & B & C \end{bmatrix} = \sum_{k=1}^N \mu_k \begin{bmatrix} A_0^{(k)} & A_1^{(k)} & B^{(k)} & C^{(k)} \end{bmatrix}, \tag{2}$$

where $\mu_k \geq 0$ and $\sum_{k=1}^N \mu_k = 1$.

These systems are called uncertain *neutral* systems. It must be noticed that throughout the paper, following Han *et al* (2004), Haurani *et al* (2003), Li *et al* (2003), Yu and Lien (2007), and El Haoussi *et al* (2011), all the eigenvalues of C are assumed to be inside the unit circle, and the delays $\tau(t)$ and $h(t)$ are assumed to be unknown but bounded functions of time, continuously differentiable, with their respective rates of change bounded as follows:

$$0 \leq h(t) \leq h_m,\ 0 \leq \tau(t) < \infty,\ \dot{h}(t) \leq d_1,\ \dot{\tau}(t) \leq d_2, \tag{3}$$

where $h_m > 0$ is the maximum allowable state-delay, and the given positive bounds on the delay derivatives satisfy: $d_1 < 1$ and $d_2 < 1$. The initial condition of system (1) is given by

$$x(t + \theta) = \phi(\theta),\ \ \theta \in [-\bar{h}, 0], \tag{4}$$

where $\bar{h} = \max_{t\geq 0}\{\tau(t), h(t)\}$ and $\phi(.)$ is a vector of differentiable functions of initial values (i.e., $\phi \in \mathbf{C}^1[-\bar{h}; 0]$).
If the origin is asymptotically stable for all delays satisfying (3), then its domain of attraction is

$$\Psi = \{\phi \in \mathbf{C}^1[-\bar{h}; 0] : \lim_{t\to\infty} x(t) = 0\}. \tag{5}$$

Normally an estimation $\Xi_\delta \subset \Psi$ of the domain of attraction is used:

$$\Xi_\delta = \{\phi \in \mathbf{C}^1[-\bar{h}; 0] : \max_{[-\bar{h};0]} \|\phi\| \leq \delta\} \tag{6}$$

with the *stability radius* $\delta > 0$ a scalar to be determined.

Following El Haoussi *et al* (2011), controllers in this paper are assumed to correspond to symetrically saturated memoryless state-feedback:

$$u(t) = sat(Kx(t), \bar{u}). \tag{7}$$

A similar approach as the one proposed in (Cao *et al*, 2002) is used to represent the saturated system by a polytopic model. Let Θ be the set of all diagonal matrices in $\Re^{m\times n}$ with elements that are 1 or 0; each of these 2^m matrices is denoted D_i.

Lemma 2.1 *(Cao et al, 2002) Given K and H in $\Re^{m\times n}$, then*

$$sat(Kx, \bar{u}) \in Co\{D_iKx + D_i^- Hx, i = 1, \ldots, 2^m\} \tag{8}$$

for all $x \in \Re^n$ that satisfy $|h_ix| \leq \bar{u}_i$, $i = 1, \ldots, m$.

Therefore, if we consider any compact set $S_c \subset \Re^n$, for any $x \in S_c$ and H in $\Re^{m\times n}$ such that $|h_ix| \leq \bar{u}_i$, then the closed loop system of (1) and (7) may be written as follows:

$$\dot{x}(t) - C\dot{x}(t - \tau(t)) = \sum_{j=1}^{2^m} \lambda_j \widehat{A}_j x(t) + A_1x(t - h(t)), \tag{9}$$

where $\widehat{A}_j = B(D_jK + D_j^- H) + A_0$, $\sum_{j=1}^{2^m} \lambda_j = 1$ and $\lambda_j \geq 0$.

Finally, for a positive scalar β and a positive definite symmetric matrix P_1, the ellipsoid D_e is defined as follows

$$D_e \equiv \{x(t) \in \Re^n;\ x^T(t)P_1x(t) \leq \beta^{-1}\}. \tag{10}$$

The following result was derived in El Haoussi *et al* (2011) to check the stability of neutral systems when the system matrices were perfectly known, and provides a preliminary result that guarantees the convergence to the origin of all the trajectories of system (1), starting from the domain Ξ_δ, included in the ellipsoid (10), when there are no uncertainties in the system. This basic result will be applied later to derive stabilization conditions for uncertain systems.

Theorem 2.1 *(El Haoussi et al, 2011) The system described by (9) is asymptotically stable if there exist $P_1 = P_1^T > 0, Q = Q^T > 0, R = R^T > 0, W = W^T > 0$ and appropriately dimensioned matrices $P_i, i = 2, ..., 6$ such that the following condition holds:*

$$\Gamma_j = \begin{pmatrix} \Gamma_{11(j)} & \Gamma_{21(j)}^T & \Gamma_{31}^T & -P_4^T & P_2^TC \\ \Gamma_{21(j)} & \Gamma_{22} & \Gamma_{32}^T & -P_5^T & P_3^TC \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & -P_6^T & 0 \\ -P_4 & -P_5 & -P_6 & -\frac{1}{h_m}R & 0 \\ C^TP_2 & C^TP_3 & 0 & 0 & (d_2-1)W \end{pmatrix}$$

$$< 0, \;\; j = 1, \ldots, 2^m, \tag{11}$$

where

$$|h_i x| \leq \bar{u}_i, \;\; \forall x \in D_e \tag{12}$$

$$\begin{cases} \Gamma_{11(j)} = P_2^T \widehat{A}_j + \widehat{A}_j^T P_2 + P_4 + P_4^T + Q \\ \Gamma_{21(j)} = P_1 + P_3^T \widehat{A}_j + P_5^T - P_2 \\ \Gamma_{22} = h_m R + W - P_3 - P_3^T \\ \Gamma_{31} = A_1^T P_2 - P_4 + P_6^T \\ \Gamma_{32} = A_1^T P_3 - P_5 \\ \Gamma_{33} = (d_1 - 1) Q - P_6 - P_6^T \end{cases} \tag{13}$$

This result gives a general solution for testing stability. In next section we provide a new result that permits a robust stabilizing controller to be calculated.

3 Main Results:

Some results are now derived, based on Theorem 2.1, to ensure robust stabilization of all systems in the uncertain set. These results will be later particularized for standard delayed systems.

3.1 Uncertain Saturated Neutral Systems

The following presents the main result in this paper.

Theorem 3.1 *The uncertain system (1)-(4) is robustly stabilizable via the feedback control law (7), if there exist matrices* $Q^{(k)} = Q^{(k)^T} > 0$, $R^{(k)} = R^{(k)^T} > 0$, $W^{(k)} = W^{(k)^T} > 0$ $($*k=1,...,N*$)$, $X_1 = X_1{}^T > 0$, X_2, $X_3 \in \Re^{n \times n}$, $U, G \in \Re^{m \times n}$, $\varepsilon_1, \varepsilon_2 \in \Re$, *and positive scalars* β *and* δ *that satisfy*

$$\Sigma_{(j)} = \begin{pmatrix} \Sigma_{11} & * & * & * & * & * & * & * \\ \Sigma_{21(j)} & \Sigma_{22} & * & * & * & * & * & * \\ -\varepsilon_1 Q^{(k)} A_1^{(k)^T} & (1-\varepsilon_2) Q^{(k)} A_1^{(k)^T} & (d_1-1) Q^{(k)} & * & * & * & * & * \\ -\varepsilon_1 R^{(k)} A_1^{(k)^T} & -\varepsilon_2 R^{(k)} A_1^{(k)^T} & 0 & -\frac{1}{h_m} R^{(k)} & * & * & * & * \\ 0 & W^{(k)} C^{(k)^T} & 0 & 0 & (d_2-1) W^{(k)} & * & * & * \\ h_m X_2 & h_m X_3 & 0 & 0 & 0 & -h_m R^{(k)} & * & * \\ X_2 & X_3 & 0 & 0 & 0 & 0 & -W^{(k)} & * \\ X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -Q^{(k)} \end{pmatrix}$$

$$< 0, \; j = 1, \ldots, 2^m \tag{14}$$

$$\begin{pmatrix} \beta & * \\ g_i^T & \bar{u}_i^2 X_1 \end{pmatrix} \geq 0, i = 1, \ldots, m \tag{15}$$

$$\delta^2 \max\left\{\overline{\lambda}(X_1^{-1})+2\tfrac{h_m}{1-d_1}\overline{\lambda}(Q^{(k)})^{-1};\ 2h_m^2\overline{\lambda}(Q^{(k)})^{-1} + \tfrac{1}{1-d_2}\overline{\lambda}(W^{(k)})^{-1}+h_m\overline{\lambda}(R^{(k)})^{-1}\right\}\leq\beta^{-1} \tag{16}$$

where

$$\Sigma_{11}=X_2+X_2^T+\varepsilon_1(X_1 {A_1^{(k)}}^T+A_1^{(k)}X_1)$$
$$\Sigma_{21(j)}=X_3^T-X_2+(A_0^{(k)}+\varepsilon_2 A_1^{(k)})X_1+B^{(k)}(D_j U+D_j^- G)$$
$$\Sigma_{22}=-X_3^T-X_3$$

Proof 1 *See Appendix.*

Remark 3.1 *The two slack variables ε_1 and ε_2 can be used to optimize the control law, by enlarging the stability radius or the maximum allowable delay.*

Remark 3.2 *A practical procedure to design controllers can be derived from the previous result by applying a simple numerical optimization algorithm to optimize a performance index (for example, enlarging the bound h_m on the time varying delay), using ε_1 and ε_2 as slack variables: see El Haoussi et al (2011) for a parallel algoritm.*

3.2 Uncertain Saturated State-delayed Systems

The previous results can be easily extended to standard State-delayed systems (i.e., neutral systems with $C^{(k)} = 0$), such as those studied by Liu (2012), defined by

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h(t)) + Bsat(u(t), \bar{u}), \tag{17}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, and the uncertain A_0, A_1 and B are contained in

$$\begin{bmatrix} A_0 & A_1 & B \end{bmatrix} = \sum_{k=1}^{N} \mu_k \begin{bmatrix} A_0^{(k)} & A_1^{(k)} & B^{(k)} \end{bmatrix}, \tag{18}$$

where $\mu_k \geq 0$ and $\sum_{k=1}^{N} \mu_k = 1$.

In this case, it is easy to see that (14) reduces to

$$\Sigma_{(j)}=\begin{pmatrix} \begin{pmatrix} \Sigma_{11} & * \\ \Sigma_{21(j)} & \Sigma_{22} \end{pmatrix}+h_m\Pi & * & * & * \\ \begin{pmatrix} -\varepsilon_1 Q^{(k)} {A_1^{(k)}}^T & (1-\varepsilon_2)Q^{(k)} {A_1^{(k)}}^T \end{pmatrix} & (d_1-1)Q^{(k)} & * & * \\ \begin{pmatrix} h_m X_2 & h_m X_3 \end{pmatrix} & 0 & -h_m R^{(k)} & * \\ \begin{pmatrix} X_1 & 0 \end{pmatrix} & 0 & 0 & -Q^{(k)} \end{pmatrix}<0$$

with
$\Pi = \begin{pmatrix} \varepsilon_1 A_1^{(k)} R^{(k)} \\ \varepsilon_2 A_1^{(k)} R^{(k)} \end{pmatrix} R^{(k)-1} \begin{pmatrix} \varepsilon_1 R^{(k)} A_1^{(k)T} & \varepsilon_2 R^{(k)} A_1^{(k)T} \end{pmatrix}$

By introducing a semi-positive definite matrix $Z=\begin{pmatrix} Z_{11} & * \\ Z_{21} & Z_{22} \end{pmatrix}$, we can write:

$$\Delta_j=\begin{pmatrix} \Sigma_{11}+h_m Z_{11} & * & * & * & * \\ \Sigma_{21(j)}+h_m Z_{21} & \Sigma_{22}+h_m Z_{22} & * & * & * \\ -\varepsilon_1 Q^{(k)} A_1^{(k)T} & (1-\varepsilon_2) Q^{(k)} A_1^{(k)T} & (d_1-1)Q^{(k)} & * & * \\ h_m X_2 & h_m X_3 & 0 & -h_m R^{(k)} & * \\ X_1 & 0 & 0 & 0 & -Q^{(k)} \end{pmatrix}$$

$$-\begin{pmatrix} h_m\left(\begin{pmatrix} Z_{11} & Z_{21}^T \\ Z_{21} & Z_{22} \end{pmatrix}-\Pi\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}<0.$$

Let

$$\begin{pmatrix} Z_{11} & * \\ Z_{21} & Z_{22} \end{pmatrix} - \Pi \geq 0.$$

Applying the Schur complement formula gives

$$\begin{pmatrix} R^{(k)} & \varepsilon_1 R^{(k)} A_1^{(k)T} & \varepsilon_2 R^{(k)} A_1^{(k)T} \\ \varepsilon_1 A_1^{(k)} R^{(k)} & Z_{11} & Z_{21}^T \\ \varepsilon_2 A_1^{(k)} R^{(k)} & Z_{21} & Z_{22} \end{pmatrix} \geq 0,$$

which makes it possible to state the following result derived from Theorem 3.1.

Corollary 3.1 *For a system described by (17), if there exist $Q^{(k)} = Q^{(k)T} > 0$, $R^{(k)} = R^{(k)T} > 0$, $X_1 = X_1^T > 0$, $X_2, X_3 \in \Re^{n\times n}$, $U, G \in \Re^{m\times n}$, $\varepsilon_1, \varepsilon_2 \in \Re$ and positive scalars β, and δ, that satisfy the following set of inequalities:*

$$\Omega_{(j)}=\begin{pmatrix} \Omega_{11} & * & * & * & * \\ \Omega_{21(j)} & \Omega_{22} & * & * & * \\ -\varepsilon_1 Q^{(k)} A_1^{(k)T} & (1-\varepsilon_2) Q^{(k)} A_1^{(k)T} & (d_1-1)Q^{(k)} & * & * \\ h_m X_2 & h_m X_3 & 0 & -h_m R^{(k)} & * \\ X_1 & 0 & 0 & 0 & -Q^{(k)} \end{pmatrix}<0,\ j=1,\ldots,2^m \qquad (19)$$

$$\begin{pmatrix} R^{(k)} & * & * \\ \varepsilon_1 A_1^{(k)} R^{(k)} & Z_{11} & * \\ \varepsilon_2 A_1^{(k)} R^{(k)} & Z_{21} & Z_{22} \end{pmatrix} \geq 0 \tag{20}$$

$$\begin{pmatrix} \beta & * \\ g_i^T & \bar{u}_i^2 X_1 \end{pmatrix} \geq 0,\ i = 1, \ldots, m \tag{21}$$

$$\delta^2 \max \Big\{ \overline{\lambda}(X_1^{-1}) + 2\frac{h_m}{1-d_1}\overline{\lambda}(Q^{(k)^{-1}}); \\ 2h_m^2 \overline{\lambda}(Q^{(k)})^{-1} + h_m \overline{\lambda}(R^{(k)})^{-1} \Big\} \leq \beta^{-1} \tag{22}$$

where

$$\Omega_{11}{=}X_2{+}X_2^T{+}\varepsilon_1(X_1 A_1^{(k)^T}{+}A_1^{(k)}X_1){+}h_m Z_{11}$$
$$\Omega_{21(j)}{=}X_3^T{-}X_2{+}(A_0^{(k)}{+}\varepsilon_2 A_1^{(k)})X_1{+}B(D_j U{+}D_j^- G){+}h_m Z_{21}$$
$$\Omega_{22}{=}{-}X_3^T{-}X_3{+}h_m Z_{22}$$

then, the uncertain saturated state-delayed system is asymptotically stable and the trajectories of $x(t)$ remain within the ellipsoid D_e when the feedback law (7) is used, with $K = UX_1^{-1}$.

4 Numerical Examples

4.1 Uncertain Saturated Neutral System

Suppose that a saturated uncertain neutral system described by (1)-(4), is subject to uncertainties described by

$$A_0 = \begin{pmatrix} 1+\rho & 1.5 \\ 0.3 & -2 \end{pmatrix},\ A_1 = \begin{pmatrix} 0 & -1+\rho \\ 0 & 0 \end{pmatrix},$$
$$B = \begin{pmatrix} 10+\rho \\ 1 \end{pmatrix},\ C = \begin{pmatrix} 0.2+\rho & 0 \\ 0 & 0.2 \end{pmatrix},$$
$$h(t) = h_m = 1,\ \bar{u} = 15, \tag{23}$$

where the uncertain parameter ρ is normalized: $|\rho| \leq 1$.
From Theorem 3.1, taking $\beta = 1$ and $d_1 = d_2 = 0.1$, the following stability radius is obtained: $\delta = 25.4112$, when the tuning parameters are $\varepsilon_1 = 0.0016$ and $\varepsilon_2 = 0.9997$. The corresponding state-feedback gain is:

$$K = \begin{pmatrix} -0.2016 & 0.0841 \end{pmatrix}.$$

This system was studied by Gomes da Silva *et al* (2011) when $\rho = 0$, obtaining asymptotic stability for $\delta \leq 70.74$. In contrast, when $\rho = 0$, from Theorem 3.1 we obtain a larger domain of attraction: $\delta = 80.21$, when $K = \begin{pmatrix} -0.4864 & -0.0124 \end{pmatrix}$, for the tuning parameters $\varepsilon_1 = 0.0061$ and $\varepsilon_2 = 0.9949$.

4.2 Saturated State-delay System

A comparison of the result in Corollary 3.1 with previous results in the literature for standard state-delay systems is now presented: in Cao *et al* (2002); Fridman *et al* (2003), Gomes da Silva *et al* (2011) and Tarbouriech and Gomes da Silva (2000), the system in the previous subsection was studied, albeit with system matrices perfectly known, and no delayed derivative of the state; that is, $\rho = 0$, $d_1 = 0.1$, $d_2 = 0$ and $C = 0$:

In Tarbouriech and Gomes da Silva (2000), stabilization via state-feedback was achieved for all initial conditions with $\delta \leq 42.33$, when the origin of the saturated system is asymptotically stable and the unsaturated system is $\alpha-$ stable with $\alpha = 1$. If we only need the saturated system to be asymptotically stable (that is, $\alpha = 0$), it can be seen that the domain of attraction is enlarged to $\delta \leq 58.39$.

In Cao *et al* (2002), stabilization by a saturated memoryless state feedback law was obtained for initial conditions with $\delta \leq 67.06$. Following Fridman *et al* (2003) and Gomes da Silva *et al* (2011), this domain can be still enlarged to stability radius of $\delta = 79.43$ and 83.55, respectively.

In contrast, the application of Corollary 3.1 in the present paper, gives a larger stability region: when $\varepsilon_1 = 0.0064$, $\varepsilon_2 = 0.9936$ and $\beta = 1$, the stability radius obtained is $\delta = 96.16$, with the state-feedback gain:

$$K = \begin{pmatrix} -10.2107 & 0.9563 \end{pmatrix}. \tag{24}$$

Then, it is clear that for this specific example from the literature the obtained results are less conservative than previous results in yczlth; Fridman *et al* (2003), Gomes da Silva *et al* (2011) and Tarbouriech and Gomes da Silva, (2000).

5 Conclusions

This paper has presented a new approach for delay-dependent stabilization of neutral systems with polytopic uncertainties, saturating actuators and time-varying delays. The derived conditions are given as LMIs that depend on the tuning parameters ε_1 and ε_2, which can be used to optimize robustness or performance (increasing, for example, the size of the domain of attraction or the maximum allowable delay). The results have also been particularized for standard state-delayed systems. The proposed conditions have being shown to be less conservative than those previously proposed in the literature by numerical examples, that have also illustrated the feasibility of the proposed approach.

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Proof of Theorem 3.1

From the requirement that $P_1 = P_1^T > 0$, if condition (11) is satisfied, then $-P_3 - P_3^T$ must be negative definite. Thus, it follows that $\widetilde{P}$ is nonsingular, where

$$\widetilde{P}^{-1} = X = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix}^{-1} = \begin{pmatrix} X_1 & 0 \\ X_2 & X_3 \end{pmatrix}. \tag{25}$$

Then, multiplying (11) on the left by diag$\{X^T, I, I, I\}$, on the right by diag$\{X, I, I, I\}$, followed by letting $Q = \sum_{k=1}^{N} \mu_k Q^{(k)},\ R = \sum_{k=1}^{N} \mu_k R^{(k)},\ W = \sum_{k=1}^{N} \mu_k W^{(k)}, P_4 = \sum_{k=1}^{N} \mu_k P_4^{(k)}$, and $P_5 = \sum_{k=1}^{N} \mu_k P_5^{(k)}$, then introducing the change of variables

$$\begin{aligned} & X_1 = P_1^{-1},\ \overline{Q}^{(k)} = {Q^{(k)}}^{-1},\ \overline{R}^{(k)} = {R^{(k)}}^{-1}, \\ & \overline{W}^{(k)} = {W^{(k)}}^{-1},\ U = KX_1,\ G = HX_1, \\ & \begin{pmatrix} N_1^{(k)} \\ N_2^{(k)} \end{pmatrix} = \begin{pmatrix} X_1 P_4^T + X_2^T P_5^T \\ X_3^T P_5^T \end{pmatrix} X_1, \end{aligned} \tag{26}$$

and finally using the Schur complement (Boyd *et al*., 1994), some conditions are obtained that are bilinear due to cross products of P_6 with P_1, P_2 and P_3. To avoid such terms, we first select $P_6 = 0$, which leads to:

$$\begin{pmatrix} \Pi_{11} & * & * & * & * & * & * & * \\ \Pi_{21(j)} & -X_3 - X_3^T & * & * & * & * & * & * \\ -X_1^{-1}N_1^{(k)T} & {A_1^{(k)}}^T - X_1^{-1}N_2^{(k)T} & (d_1-1){\overline{Q}^{(k)}}^{-1} & * & * & * & * & * \\ -X_1^{-1}N_1^{(k)T} & -X_1^{-1}N_2^{(k)T} & 0 & -\frac{1}{h_m}{\overline{R}^{(k)}}^{-1} & * & * & * & * \\ 0 & \overline{W}^{(k)}C^T & 0 & 0 & (d_2-1)\overline{W}^{(k)} & * & * & * \\ h_m X_2 & h_m X_3 & 0 & 0 & 0 & -h_m\overline{R}^{(k)} & * & * \\ X_2 & X_3 & 0 & 0 & 0 & 0 & -\overline{W}^{(k)} & * \\ X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{Q}^{(k)} \end{pmatrix}$$

$$< 0 \; j = 1, \ldots, 2^m \tag{27}$$

with $\Pi_{11} = X_2 + X_2^T + N_1^{(k)} + N_1^{(k)T}$ and $\Pi_{21(j)} = X_3^T - X_2 + N_2^{(k)} + A_0^{(k)}X_1 + B^{(k)}(D_jU + D_j^- G)$.

This condition (27) still cannot be solved directly, due to the cross products $X_1^{-1}N_1^{(k)T}$ and $X_1^{-1}N_2^{(k)T}$. To overcome this, we select

$$N_1^{(k)} = \varepsilon_1 A_1^{(k)} X_1, \;\; N_2^{(k)} = \varepsilon_2 A_1^{(k)} X_1, \tag{28}$$

where ε_1 and ε_2 are decision variables. Substituting (28) into (27), the condition in (14) is obtained. Moreover, the satisfaction of the LMIs (15) guarantees that $|h_i x| \le \bar{u}_i$,$\forall x \in D_e$, $i = 1, \ldots, m$. This can be proven in the same manner as in yczlth and Tarbouriech and Gomes da Silva (2000). Furthermore, following Xu *et al* (2003), the following Lyapunov functional

$$\begin{aligned} V(t) &= x^T(t)P_1x(t) + \int_{t-h(t)}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{-h_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \\ &\quad + \int_{t-\tau(t)}^{t} \dot{x}^T(s)W\dot{x}(s)ds, \end{aligned} \tag{29}$$

where $P_1 = P_1^T > 0, Q = Q^T > 0, R = R^T > 0$ and $W = W^T > 0$ can be used, with $Q = \sum_{k=1}^{N}\mu_k Q^{(k)},\; R = \sum_{k=1}^{N}\mu_k R^{(k)}$, and $W = \sum_{k=1}^{N}\mu_k W^{(k)}$, satisfies

$$\pi_1\|\Delta\phi\|^2 \le V(\phi) \le \pi_2 \max_{[-\bar{h},0]} \|\phi\|^2, \tag{30}$$

with $\pi_1 = \underline{\lambda}(X_1^{-1})$ and

$$\pi_2{=}\max\left\{\overline{\lambda}(X_1^{-1}){+}2\tfrac{h_m}{1-d_1}\overline{\lambda}(\overline{Q}^{(k)})^{-1};\right.$$
$$\left.2h_m^2\overline{\lambda}(\overline{Q}^{(k)})^{-1}{+}\tfrac{1}{1-d_2}\overline{\lambda}(\overline{W}^{(k)})^{-1}{+}h_m\overline{\lambda}(\overline{R}^{(k)})^{-1}\right\}. \tag{31}$$

From $\dot{V}(t) < 0$ it follows that $V(t) < V(\phi)$, and therefore

$$x^T(t)X_1^{-1}x(t){\le}V(t){<}V(\phi){\le}\max_{\theta\in[-\bar{h},0]}\|\phi(\theta)\|^2\pi_2{\le}\beta^{-1}. \tag{32}$$

Then, the inequality (16) guarantees that the trajectories of $x(t)$ remain within D_e for all initial functions $\phi \in \Xi_\delta$; moreover, $\dot{V}(t) < 0$ along the trajectories of (9), which implies that $\lim_{t\to\infty} x(t) = 0$. This completes the proof. To regain the notation in the statement of Theorem 3.1, it is only necessary to replace $\overline{Q}^{(k)}$ with $Q^{(k)}$, $\overline{R}^{(k)}$ with $R^{(k)}$, etc.