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Application of Multivariable Linear Quadratic Gaussian Control and Generalized Predictive Control in a Hydropower Plant

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Abstract. *The hydroelectric energy is one of the most important renewable energy in the world. It does not encounter the problem of population displacement and is not as expensive as solar or wind energy. However, the hydro electrical generating units are usually isolated from the grid network; thus, they require control to maintain of constant the power for any working conditions. The simulation model of hydropower plant was constructed based on mathematical equations that summarize the behavior of the hydropower plant. The simulation model of power plant is useful in stability studies. This paper, presents the approach of Generalized Predictive Control (GPC) is applied to a multivariable model of the part turbine/generator of hydropower plant. In this study, the standard multivariable (GPC) algorithm is presented. It is then applied to achieve sets points tracking of the outputs of the plant. A Multi Input Multi Output (MIMO) model is used for control purposes. A comparative study is carried out using the named controller's multivariable Linear Quadratic Gaussian (LQG) and multivariable (GPC) Controls. The performance of the proposed controller is illustrated by a simulation example of hydropower plant. Encouraging results are obtained that motivate for further investigations.*

Keywords. *Generalized Predictive Control, Modeling, Linear Quadratic Gaussian Control, Multivariable Systems. Hydropower Plant*

1. Introduction

The Hydro-electric energy is most important renewable energy in the world. It provides energy to various loads. User load requires a uniform and uninterrupted supply of input energy. The load demand varies continuously. It affects the terminal voltage and real power output at the generator terminals [1-4]. The objective of the control strategy is to generate and deliver power in an interconnected system as economically

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and reliably as possible while maintaining the voltage and frequency within permissible limits. Hydropower plant is equipped with hydraulic turbine governor and excitation control. The errors in the terminal voltage and in the output active power, with respect to their respective references, represent the controller inputs and the generator-exciter voltage and governor-valve position represent the controller outputs. The control of real power output and the terminal voltage keeps the system in the steady state [5-10].

This paper presents the application of multivariable GPC control to achieve sets points tracking of the outputs of the plant. The GPC control is one of the most favorite predictive control methods, popular in industry and also at universities. It was first published in 1987 [11-12]. The authors wanted to find one universal method to control different systems. Multivariable GPC Control has been successfully implemented in many industrial applications, showing good performance and a certain degree of robustness. It is applicable [13] to the systems with non-minimal phase, unstable systems in open loop, systems with unknown or varying dead time, systems with unknown order and nonlinear systems approximated by linear models.

The basic idea of GPC control [14], [15] is to calculate a sequence of future control signals in such a way that it minimizes a multistage cost function defined over a prediction horizon. The index to be optimized is the expectation of a quadratic function measuring the distance between the predicted system output and some reference sequence over the horizon plus a quadratic function measuring the control effort. The predictive model is carried out based on the solving Diophantine equations.

The paper is organized as follows. Section II presents the system modeling. Section III describes the designed multivariable LQG Controller. Section IV is devoted the description of multivariable GPC algorithm. In section V, the effectiveness and superiority of the proposed algorithm, is demonstrated by simulation example. Some concluding remarks end the paper.

2. System Modeling

The block diagram of the sample controlled power system is shown in figure 1 that comprises a hydraulic turbine driving a synchronous generator which is connected to an infinite bus via a step-up transformer and a transmission line. The output real power P_t and terminal voltage V_t at the generator terminals are measured and fed to the controller. The outputs of the controller (system control inputs) are fed into the generator-exciter and governor-valve. In the simulation studies described here, the nonlinear equations of the synchronous generator are represented by a third-order nonlinear model based on park's equations. The hydraulic turbine, governor valve and exciter are each represented by a first order model. The model equations are as follows [16-26]:

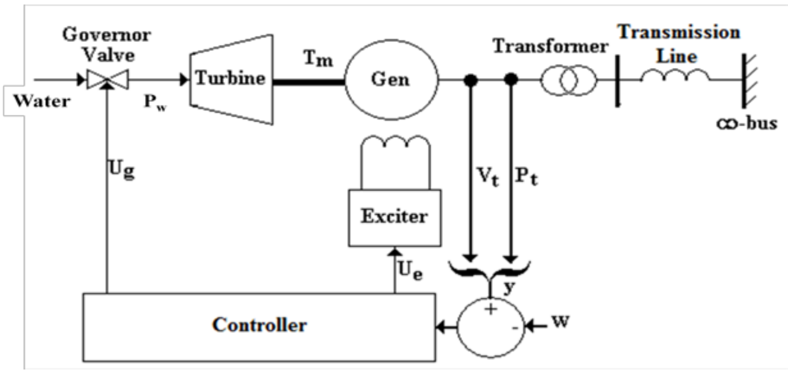

Fig. 1. Controlled sample hydropower system

The Mechanical equations

The rotor speed of the generator is given by:

$$\dot{\delta}(t) = \omega(t) \tag{1}$$

The mechanical equation of the motion is as follows:

$$\frac{H}{\pi f_0}\frac{d\omega(t)}{dt} + D\omega = P_m - P_t$$

i.e

$$M\frac{d\omega(t)}{dt} + D\omega = P_m - P_t \tag{2}$$

Where,

$$M = \frac{H}{\pi f_0} \text{ and } f_0 = \frac{\omega_0}{2\pi}$$

The electrical generator dynamics equations

$$\frac{dE_q'(t)}{dt} = \frac{1}{T_{do}'}(E_{fd}(t) - E_q(t)) \tag{3}$$

The electrical equations (assumed $x_d' = x_q$)

$$E_q(t) = E_q'(t) + (x_d - x_q)I_d(t) \tag{4}$$

$$P_t(t) = E_q(t)I_q(t) \tag{5}$$

$$I_d(t) = \frac{E_q'(t) - V_s\cos\delta(t)}{x_{ds}'} \tag{6}$$

$$I_q(t) = \frac{V_s\sin\delta(t)}{x_{ds}'} \tag{7}$$

$$E_q'(t) = x_{ad}I_f(t) \tag{8}$$

$$V_t(t) = \left[\left(V_d(t)\right)^2 + \left(V_q(t)\right)^2\right]^{\frac{1}{2}} \tag{9}$$

$$V_d(t) = E_q'(t) - x_d'I_d(t) \tag{10}$$

$$V_q(t) = x_d'I_q(t) \tag{11}$$

Where,

$$x_{ds} = x_d + x_T + x_L \quad x'_{ds} = x'_d + x_T + x_L \quad x_s = x_T + x_L$$

More details about power system modelling can be seen in [20-23]

Using the above equations, we can express $P_t(t)$ as

$$P_t(t) = \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t)\sin\delta(t) - \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2} \sin\delta(t)\cos\delta(t) \tag{12}$$

In terms of the state variables δ and $\omega(t) = \dot{\delta}(t)$, the equation (2) becomes

$$\frac{d\omega(t)}{dt} = \left(P_m - \frac{V_s x_{ds}}{(x'_{ds})^2} E'_q(t)\sin\delta(t) + \frac{(x_d - x_q)(V_s)^2}{(x'_{ds})^2} \sin\delta(t)\cos\delta(t) - D\omega(t)\right)\frac{\omega_0}{2H} \tag{13}$$

In terms of the state variables $E'_q(t)$ and $E_f(t)$ the equation (3) becomes

$$\frac{dE'_q(t)}{dt} = \frac{\omega_0 r_{fd}}{x_{ad}} E_{fd}(t) + \frac{x_{ds}}{x'_{ds} T'_{do}} E'_q(t) + \frac{(x_d - x_q)V_s}{x'_{ds} T'_{do}} \tag{14}$$

Where,

$$T'_{do} = \frac{x_{ad}}{\omega_0 r_{fd}}$$

The governor valve equation is given by

$$\frac{P_w}{U_g} = \frac{K_v}{1+\tau_g s} \tag{15}$$

The exciter equation defined by

$$\frac{E_{fd}}{U_e} = \frac{1}{1+\tau_e s} \tag{16}$$

The turbine equation

$$\frac{P_m}{P_w} = \frac{1}{1+\tau_b s} \tag{17}$$

In terms of the state variables E_{fd}, P_w and P_m the equations (15)-(17) written as follow:

$$P_w \quad \frac{dP_w(t)}{dt} = \frac{-P_w}{\tau_g} + \frac{K_v}{\tau_g} U_g \tag{18}$$

$$\frac{dE_{fd}(t)}{dt} = \frac{-E_{fd}(t)}{\tau_e} + \frac{1}{\tau_e} U_e \tag{19}$$

$$\frac{dP_m}{dt} = \frac{-P_m}{\tau_b} + \frac{1}{\tau_b} P_w \tag{20}$$

Defining $x = [\delta \quad \dot{\delta} \quad E'_q \quad E_{fd} \quad P_w \quad P_m]^T$ the state variables vector then the equations above can be written in the key form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (x_6 - K_1 x_3 \sin x_1 - K_2 \sin x_1 \cos x_1 - D x_2)\frac{\omega_0}{2H} \\ x_3 = \frac{\omega_0 r_{fd}}{x_{ad}} x_4 + K_3 x_3 - K_4 \cos x_1 \\ \dot{x}_4 = \frac{-x_4}{\tau_e} + \frac{1}{\tau_e} U_e \\ \dot{x}_5 = \frac{-x_5}{\tau_g} + \frac{1}{\tau_g} U_g \\ \dot{x}_6 = \frac{-x_6}{\tau_b} + \frac{x_5}{\tau_b} \end{cases} \tag{21}$$

The output y1, y2 may be expressed in terms of these state variable by

$$y_1 = P_t = K_1 x_3 \sin x_1 + K_2 \sin x_1 \cos x_1 \tag{22}$$

$$y_2 = V_t = \left(V_d^2 + V_q^2\right)^{1/2} \tag{23}$$

Where,

$$V_d = K_5 \sin x_1 \tag{24}$$

$$V_q = K_6 x_3 + K_7 \cos x_1 \tag{25}$$

2.1. Linear model of synchronous generator

A linear Multi-Input Multi-output (MIMO) model of the generator system is required to design a controller for such system. It is derived from the system nonlinear model by linearizing the nonlinear equations (13) and (14) around a specific operating point. The linear state-space model is derived next where the variables shown represent small displacements around the selected operating point.

$$\begin{cases} \dot{x}(t) = F_X x(t) + F_U u(t) \\ y(t) = G_X x(t) + G_U u(t) \end{cases} \tag{26}$$

Where,

$$F_X = \left.\frac{\partial f}{\partial X^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \cdots & \frac{\partial f_1}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial X_1} & \cdots & \frac{\partial f_n}{\partial X_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$F_U = \left.\frac{\partial f}{\partial U^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial f_1}{\partial U_1} & \cdots & \frac{\partial f_1}{\partial U_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial U_1} & \cdots & \frac{\partial f_n}{\partial U_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$G_X = \left.\frac{\partial g}{\partial X^T}\right|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial g_1}{\partial X_1} & \cdots & \frac{\partial g_1}{\partial X_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p}{\partial X_1} & \cdots & \frac{\partial g_p}{\partial X_n} \end{bmatrix}_{\bar{X},\bar{U}}$$

$$G_U = \frac{\partial g}{\partial U^T}\bigg|_{\bar{X},\bar{U}} = \begin{bmatrix} \frac{\partial g_1}{\partial U_1} & \cdots & \frac{\partial g_1}{\partial U_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_p}{\partial U_1} & \cdots & \frac{\partial g_p}{\partial U_m} \end{bmatrix}_{\bar{X},\bar{U}}$$

F_X, F_U, G_X et G_U are the Jacobian matrices of partial derivatives of f and g respectively to X and U evaluated at the point $(\bar{X}, \bar{U})$.

The linear state-space model defined by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{27}$$

Where,

$A = F_X \quad B = F_U \quad C = F_U \quad D = G_U$

The matrices A, B, C and D have the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ K_8 & \frac{-D\omega_0}{2H} & K_9 & 0 & 0 & \frac{\omega_0}{2H} \\ K_{10} & 0 & K_3 & \frac{\omega_0 r_{fd}}{x_{ad}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\tau_g} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\tau_b} & \frac{-1}{\tau_b} \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\tau_e} & 0 \\ 0 & \frac{K_g}{\tau_g} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} K_{11} & 0 & K_{12} & 0 & 0 & 0 \\ K_{13} & 0 & K_{14} & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where,

$x = [\delta \quad \dot{\delta} \quad E_q' \quad E_{fd} \quad P_w \quad P_m]^T$ State variables vector

$u = [U_e \quad U_g]^T$ Control input vector

$y = [P_t \quad V_t]^T$ Measurement vector

$P_t = K_{11}x_1 + K_{12}x_3$ Output Power

$V_t = K_{13}x_1 + K_{14}x_3$ Terminal voltage

Expressions for parameters K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13} and K_{14} are given in Appendix 2.

2.2. State space to transform function conversion

Consider the state equation (27). We may take its Laplace transform and rearrange it as follows:

$sX(s) = AX(s) + BU(s) \rightarrow (sI - A)X(s) = BU(s)$ (28)

If we combine this with the transform of the output equation: $Y(s) = CX(s) + DU(s)$, we get $Y(s) = C(sI - A)^{-1}BU(s) + DU(s)$

Or, equivalently

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D \tag{29}$$

In the Control Systems Toolbox, the command $[\mathrm{num}, \mathrm{den}] = \mathrm{ss2tf}(A, B, C, D, i)$ converts the state equation to a transfer function for i^{th} input.

3. Multivariable Linear Quadratic Gaussian Control Algorithm

Let us, consider the following $m \times m$ process model [27]:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\xi(t) \tag{30}$$

Where,

$$A(q^{-1}) = I_m + A_1 q^{-1} \dots\dots\dots\dots.. + A_{na} q^{-na} \quad A_j \epsilon R^{m,m}$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} \dots\dots\dots\dots.. + B_{nb} q^{-nb} \quad B_j \epsilon R^{m,m}$$

$$C(q^{-1}) = C_0 + C_1 q^{-1} \dots\dots\dots\dots.. + C_{nc} q^{-nc} \quad C \epsilon R^{m,m}$$

$y(t)\epsilon R^m$ is the output vector
$u(t)\epsilon R^m$ is the input vector
$\xi(t)\epsilon R^m$ is a sequence of independent random vectors with zero mean value and finite covariance matrix
q^{-1} is the backward shift operator such that $q^{-1}f(t) = f(t-1)$
To this model, we can associate the companion block state representation in the observable form by:

$$\begin{cases} x(t+1) = A'x(t) + B'u(t) + G'\xi(t) \\ y(t) = C'x(t) + \xi(t) \end{cases} \tag{31}$$

Where:

$$A' = \begin{bmatrix} -A_1 & I' & 0' & \cdots & 0' \\ -A_2 & 0' & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0' \\ \vdots & \vdots & 0 & \ddots & I \\ -A_n & 0' & \cdots & \cdots & 0' \end{bmatrix} \quad B' = \begin{bmatrix} B_0 \\ \vdots \\ B_{n-1} \end{bmatrix}$$

$$G' = \begin{bmatrix} C_1 + A_1 \\ \vdots \\ C_n + A_n \end{bmatrix} \qquad C' = [I_m \quad 0' \quad \dots \quad 0']$$

$A'\epsilon R^{mn,mn}$, $B'\epsilon R^{mn,m}$, $G'\epsilon R^{m,m}$, $C'\epsilon R^{m,mn}$
$x(t)\epsilon R^{mn}$ is the system state
$n = \max(na, nb, nc)$

The problem is to found a control vector by state feedback that minimizing the following criterion:

$$J = \lim_{T\to\infty} \sum_{t=0}^{T}\{[y(t) - y^*(t)]^T Q[y(t) - y^*(t)] + u^T(t-1)\Lambda u(t-1)\} \quad (32)$$

Where:
T is the control horizon
$y^*(t)$ is the reference vector sequence
Λ is a symmetric semi definite positive matrix
Q is a symmetric definite positive matrix
The solution is:

$$\Delta u(t) = -\Gamma(t)x(t) - v(t) + W(t) \quad (33)$$

Where,

$$\Gamma(t)=(B'^T RB' + \Lambda)^{-1} B'^T RA' \quad (34)$$

$$W(t) = (B'^T RB' + \Lambda)^{-1} B'^T RG'[y(t) - C'x(t)] \quad (35)$$

$$v(t) = \left(B'^T RB' + \Lambda\right)^{-1} B'^T[y^*(t+1) + (A' - B'\Gamma)^T y^*(t+2) + \cdots + ((A' - B'\Gamma)^T)^m y^*(t+l+1) + \cdots] \quad (36)$$

R(t) is the Riccati matrix

$$Y^{*T} = [y^*(t) \quad 0' \quad \ldots \quad 0'] \quad (37)$$

Remark
The solution is an explicit form of the state variables. But they are not available. Therefore a state observer is necessary.
The state observer is given by:

$$\hat{x}(t)=H\phi_e(t-1) \quad (38)$$

Where,

$$H = \begin{bmatrix} -A_1 & \cdots & -A_n & -B_0 & \cdots & -B_n & -C_1 & \cdots & -C_n \\ \vdots & \iddots & \vdots & \vdots & \iddots & \vdots & \vdots & \iddots & \vdots \\ -A_n & \cdots & 0 & -B_n & \cdots & 0 & -C_n & \cdots & 0 \end{bmatrix}$$

$$\phi_e^T(t) = [y(t) \ldots y(t-n+1)\ u(t) \ldots u(t-n+1)\ \xi(t) \ldots \xi(t-n+1)]$$

$H \epsilon R^{3mn}$
$\phi_e \epsilon R^{3mn}$

$$\xi(t) = \hat{y}(t)\text{-}C'x(t) \quad (39)$$

4. Multivariable Generalized Predictive Control Algorithm

When considering regulation about a particular operating point, even a non-linear plant generally admits a locally-linearized model [11-12] given by the equation (30).

The objective of the GPC control is the output y(t) to follow some reference signal y*(t) taking into account the control effort. This can be expressed in the following cost function:

$$J\left(h_i, h_p, h_c, t\right) = E\left\{\sum_{h_i}^{h_p}[y(t+j) - y^*(t+j)]^T R[y(t+j) - y^*(t+j)] + \sum_{h_i}^{h_c} \Delta u^T(t+j-1) Q \Delta u(t+j-1)\right\} \quad (40)$$

$$\Delta u(t+j-1) = 0 \text{ for } j > h_c$$

Where:
hp is the prediction horizon.
hi is the initial horizon.
hc is the control horizon.
y*(t) is the output reference.
R is the output weighting factor.
Q is the control weighting factor.
The control objective is to compute at each time t, control inputs that minimize the quadratic criterion $J(h_i, h_p, h_c, t)$ for this there are two cases:

Let us first build j-step ahead predictors with following Diophantine equation:

$$1_m = E^j(q^{-1})A(q^{-1})\Delta(q^{-1}) + q^{-j}F^j(q^{-1}) \quad (41)$$
$$j = 1 \ldots hp$$

Where:

$$E^j(q^{-1}) = 1_m + E_1 q^{-1} \ldots\ldots\ldots\ldots.. + E_{j-1} q^{-(j-1)} \quad E_j \epsilon R^{m,m}$$

$$F^j(q^{-1}) = F_0^j + F_1^j q^{-1} \ldots\ldots\ldots\ldots.. + F_{na}^j q^{-na} \quad F_j^j \epsilon R^{m,m}$$

The polynomial matrices $E^j(q^{-1})$ and $F^j(q^{-1})$ are uniquely defined by: $A(q^{-1}), \Delta(q^{-1})$ and j.
Using equation (30) and (40) we obtain:

$$y(t+j) = E^j(q^{-1})B(q^{-1})\Delta u(t+j-1) + F^j(q^{-1})y(t) + E^j(q^{-1})\xi(t+j) \quad (42)$$

The optimal predictor $y(t+j)$at time t is given by:

$$\hat{y}(t+j/t) = G^j(q^{-1})B(q^{-1})\Delta u(t+j-1) + F^j(q^{-1})y(t) \quad (43)$$

Where:

$$G^j(q^{-1}) = E^j(q^{-1})B(q^{-1})$$

Defining $G^j(q^{-1}) = g_0^j + g_1^j q^{-1} \ldots\ldots\ldots\ldots.. + g_{j-1}^j q^{-(j-1)}$ then the equation above can be written in the key vector form:

$$\hat{Y} = G\Delta U_t + Y_0 \quad (44)$$

Where the vectors are all $h_p \times 1$:

$$\hat{Y} = \left[\hat{y}(t+1/t)\ \ \hat{y}(t+2/t) \ldots \ \hat{y}\left(t+h_p/t\right)\right]^T$$

$$\Delta U = [\Delta u(t)\ \ \Delta u(t+1) \ldots \ \Delta u(t+h_c-1)]^T$$

$$Y_0 = \left[Y_0(t+1)\ \ Y_0(t+2) \ldots \ Y_0\left(t+h_p\right)\right]^T$$

Note that: $G^j(q^{-1}) = \frac{B(q^{-1})[1-q^{-j}F^j(q^{-1})]}{A(q^{-1})}$ so that one way to computing G^j is simply to consider the Z-transform plant's step-response and to take the first j terms and therefore $g_j^i = g_i$ for j=0, 1, 2 …< i independent of the particular G polynomial [11]. The matrix G is then lower-triangular of dimension $mh_p \times mh_C$:

$$G = \begin{bmatrix} g_0 & 0 & \cdots & \cdots & 0 \\ g_1 & g_0 & \cdots & \cdots & 0 \\ . & .. & \cdots & \cdots & g_0 \\ \vdots & & & & \vdots \\ g_{h_p-1} & g_{h_p-2} & \cdots & \cdots & g_{h_p-h_c} \end{bmatrix}$$

From the definitions above of the vectors and with:

$$Y^* = \left[Y^*(t+1)\ \ Y^*(t+2) \ldots \ Y^*\left(t+h_p\right)\right]^T \tag{44}$$

The expectation of the cost-function of (4) can be written as follow:

$$J\left(h_i, h_p, h_c, t\right) = (G\Delta U_t + Y_0 - Y^*)^T \overline{R}(G\Delta U_t + Y_0 - Y^*) + \Delta U_t \overline{Q} \Delta U_t^T \tag{45}$$

The solution, ΔU_t minimizing the criterion can be explicitly found, using:

$$\frac{\partial J}{\partial \Delta U_t} = 0 \tag{46}$$

it follows that:

$$\Delta U_t^* = (G^T G + Q)^{-1} G^T R(Y_0 - Y^*) \tag{47}$$

Note that the first element ΔU_t^* of is $\Delta u(t)$ so that the current control u(t) is given by:

$$u(t) = u(t-1) + (G^T G + Q)^{-1} G^T R(Y_0 - Y^*) \tag{48}$$

5. Simulation and Discussion

To demonstrate the effectiveness of the above presented multivariable GPC Control algorithm, the result are presented and compared with those of the multivariable LQG control. The simulation results are obtained by using Matlab Toolbox.

Initial condition (operating point) for the non linear system:

$$x = [0.775 \quad 0 \quad 1.434 \quad -0.0016 \quad 0.8 \quad 0.8]^T$$

The hydropower plant model is as follow:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1)$$

Where:

$$A_1 = \begin{bmatrix} -1.826 & 0 \\ 0 & -1.54 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1.21 & 0 \\ 0 & 0.7693 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} -0.3479 & 0 \\ 0 & -0.1275 \end{bmatrix}$$
$$A_4 = \begin{bmatrix} 0.03653 & 0 \\ 0 & 0 \end{bmatrix}$$
$$B_0 = \begin{bmatrix} 1.367 & -0.03534 \\ -0.07828 & 1.218 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} -2.107 & -0.06115 \\ 0.1818 & -0.8947 \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 1.043 & 0.06876 \\ -0.1276 & 0.1182 \end{bmatrix}$$
$$B_3 = \begin{bmatrix} -0.1629 & -0.01589 \\ 0.02571 & 0.02204 \end{bmatrix}$$
$$B_4 = \begin{bmatrix} 0.003059 & 0 \\ 0 & 0 \end{bmatrix}$$

The objective of the hydropower plant control is to track a reference. The prediction controller parameters (h_p, h_c, h_i, Q, R and Λ) for GPC Controller and the weighting factors (Q and Λ) for LQG controller are chosen in order to get an acceptable tracking.

Parameters of the GPC controller

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$
$$h_p = 10 \quad h_C = 5 \; h_i = 1$$

Parameters of the LQG controller

$$\Lambda = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$
$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reference is chosen as a square wave.

Simulations were carried out to verify the advantages of using multivariable GPC control in this application.

In the figures above, it can be observed the comparative results between multivariable GPC Control and multivariable LQG control.

The output real power P_t, the exciter input voltage U_e, the terminal voltage V_t and governor valve position U_g, under GPC and LQG controls are shown, respectively, in Figures 2, 3, 4 and 5. Best performance is characterized by best tracking, robustness, lower or no over/undershoots less or no oscillations. Based on this, for P_t response given in figure 2, GPC shows the best response whereas LQG shows the worse one with a bigger non minimum phase undershoot, which is eliminated by using GPC control. For V_t response shown in figure 4, it can be observed that the GPC control produces the best response in terms of tracking, and overshoot, cancellation of oscillation. The LQG response has a very high overshoot, which is eliminated by using GPC control.

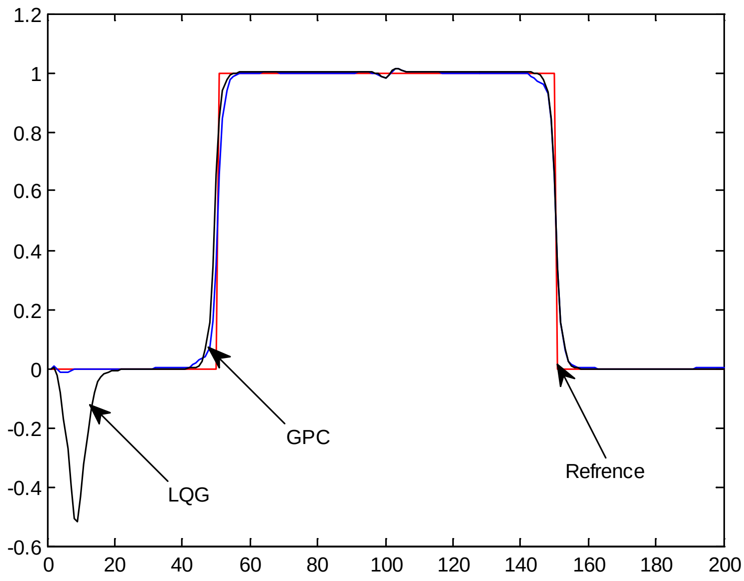

Fig. 2. Power output P_t

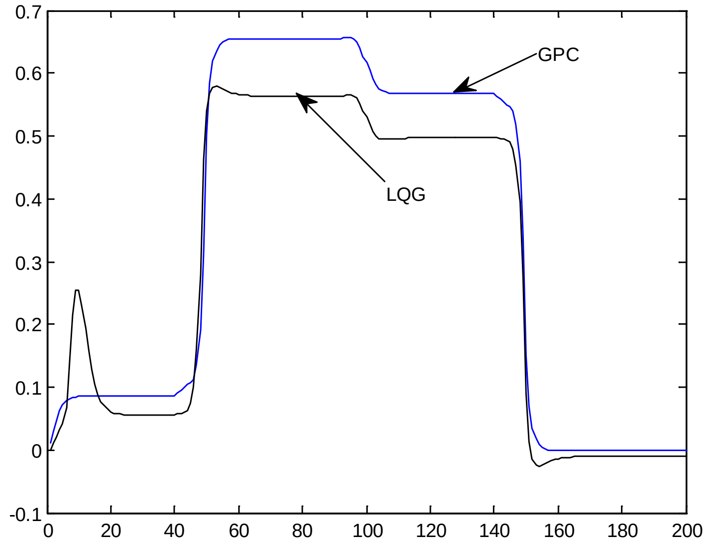

Fig. 3. Exciter Input U_e

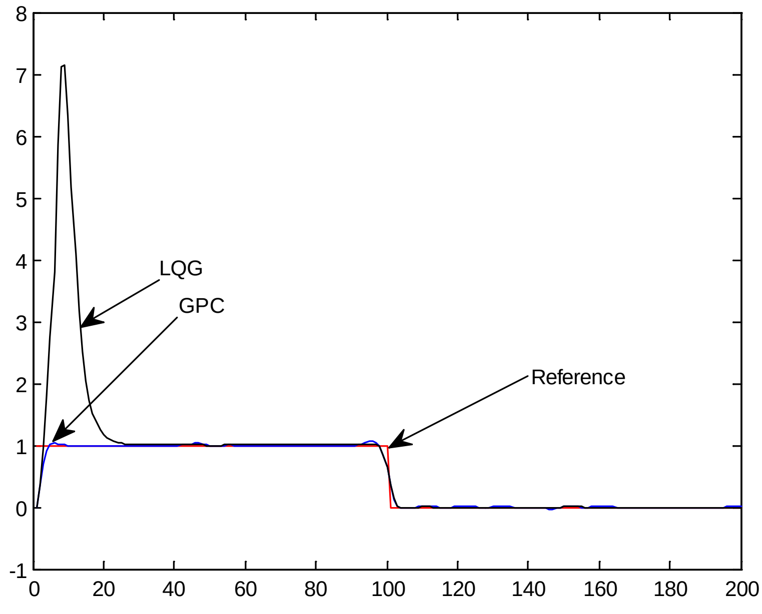

Fig. 4. Terminal voltage V_t

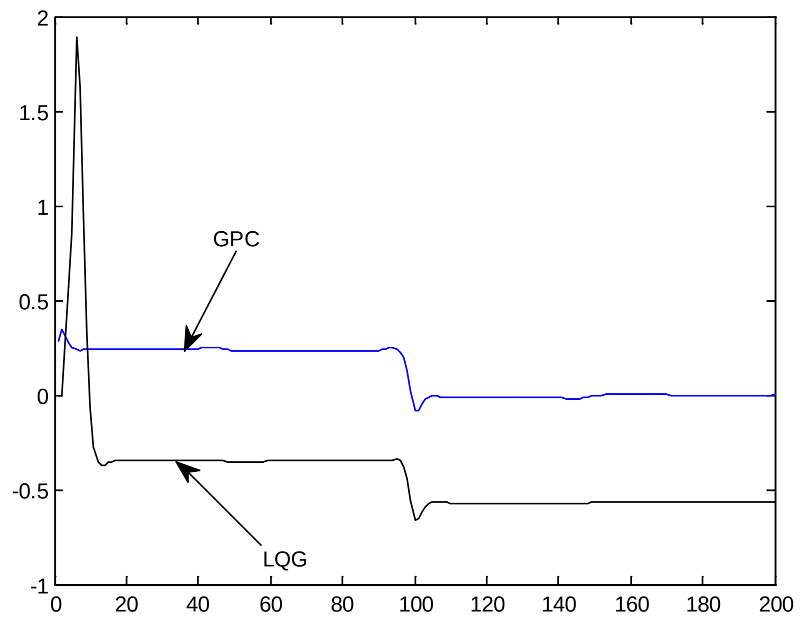

Fig. 5. Governor Input U_g

6. Conclusion

In this paper, a Multivariable Generalized Predictive Controller and Multivariable Linear Quadratic Gaussian controller were designed for a sample power system comprising a water turbine driving a synchronous generator. The model of hydropower plant was constructed based on mathematical equations that summarize the behavior of the hydropower plant. From the simulation results, it is clear that, the GPC control exhibits better performance for P_t (real power output at the generator terminals) and V_t (terminal voltage) responses than the LQG control.

Appendix 1

List of Symbols

V_d, V_q	Stator voltage in d-axis and q-axis circuit
V_t	Terminal voltage
E_q'	Transient EMF in the quadratic axis of the generator
x_{ad}	Stator – rotor mutual reactance
E_{fd}	Field voltage
r_{fd}	Field resistance

X_{fd} Self reactance of field winding
U_e Exciter input
δ Rotor angle
P_m Mechanical power
P_w Water power
H Inertia constant
$\omega(t)$ Rotor speed of the generator
ω_0 Angular frequency of the infinite bus bar
K_d Mechanical damping torque coefficient
T_d Damping torque coefficient due to damper windings
P_t Real power output at the generator terminals
τ_e Exciter time constant
τ_g Governor valve time constant
τ_b Turbine time constant
U_g Governor input
G_v Governor valve position
K_v Valve constant
x_d Total d-axis synchronous reactance between the generator and the infinite busbar
x_q Total q-axis synchronous reactance between the generator and the infinite busbar
$x_d^{'}$ Total d-axis transient reactance including the generator and the infinite bus-bar
$T_{do}^{'}$ d-axis transient open-circuit time constant
x_T Reactance of the transformer
x_L Reactance of the transmission line
x_s Reactance of the system

Appendix 2

Expressions for parameters K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13} and K_{14} in the system model are:

$$K_1 = \frac{V_s x_{ds}}{(x_{ds}^{'})^2}, \qquad K_2 = -\frac{(x_d - x_q)V_s^2}{(x_{ds}^{'})^2}, \qquad K_3 = -\frac{x_{ds}}{x_{ds}^{'} T_{do}^{'}},$$

$$K_4 = -\frac{(x_d - x_q)V_s}{x_{ds}^{'} T_{do}^{'}}, \quad K_5 = \frac{x_q V_s}{x_{ds}^{'}} \quad K_6 = \frac{x_{t+} x_l}{x_{ds}^{'}}$$

$$K_7 = \frac{x_d^{'} V_s}{x_{ds}^{'}}, \; K_8 = -K_1 x_{3_0}\cos(x_{1_0}) - K_2\cos(2x_{1_0})$$

$$K_9 = -K_1\sin(x_{1_0})$$

$$K_{10} = -K_4\sin(x_{1_0})$$

$$K_{11} = -K_1 x_{3_0}\cos(x_{1_0}) + K_2\cos(2x_{1_0})$$

$$K_{12} = K_1\sin(x_{1_0})$$

$$K_{13} = ((K_5 - K_7^2)\sin(x_{1_0})\cos(x_{1_0}) - K_6 K_7 x_{3_0}\sin(x_{1_0}))((K_5\sin(x_{1_0}))^2 + (K_6 x_{3_0} + K_7\cos(x_{1_0}))^2)^{-1/2}$$

$$K_{14} = 2K_6(K_6 x_{3_0} + K_7\cos(x_{1_0}))\left((K_5\sin(x_{1_0}))^2 + (K_6 x_{3_0} + K_7\cos(x_{1_0}))^2\right)^{-1/2}$$

References

1. Henderson, DS.: An advanced electronic load governor for control of Micro hydroelectric power generation, IEEE Transactions Energy Conversion, Vol. 13, No. 3, September (1998)
2. Henderson, DS.: Recent Developments of an Electronic Load Governor for Micro Hydroelectric Generation, International Conference on Renewable Energy – Clean Power 2001, (1993), 84-88
3. Working Group on Prime Mover and Energy Supply Models for System Dynamic Performance Studies, Hydraulic turbine and turbine control models for system dynamic studies, Transactions on Power Systems, Vol. 7, NO. 1, February (1992), 167-179
4. Vournas, C.D., Papaionnou, G.: Modeling and stability of a hydro plant with two surge tanks, IEEE Trans. Energy Conversion, vol. 10, no. 2, June (1995), 368-375
5. Goyal H., Hanmandlu M., Kothari D.P.: An Artificial Intelligence based ,Approach for Control of Small Hydro Power Plants, Centre for Energy Studies, Indian Institute of Technology, New Delhi-110016 (India)
6. Munoz-Hernandez, G.A.M.: MIMO Generalized Predictive Control for a Hydroelectric Power Station, IEEE Transactions Energy Conversion, Vol. 21, Issue: 4, (2006), 921-929
7. Goyal, H., Bhatti, T. S., Kothari, D. P.: A novel technique proposed for automatic control of small hydro power plants, International Journal of Global Energy Issues, 24 (1/2) , (2005) , 29-46.
8. Hydro-thermal System, *Proceedings of IEE*, Vol. 135, 1988, 268-74,
9. Hanmandlu, M., Goyal, H.: Proposing a new advanced control technique for micro hydro power plants, Electrical power and Energy Systems, (2008) 272-282
10. Hanmandlu, M., Goyal, H., Kothari, D.P. : An Advanced Control Scheme for Micro Hydro Power Plants, International Conference on Power Electronics, Drives and Energy Systems, (2006), 1-7
11. Clarke, D. W., Mohtadi, C., Tuffs, P. S.: Generalized Predictive Control-part I. The Basic Algorithm, Vol. 23, No. 2. Automatica. (1987), 137-148
12. Clarke, D. W., Mohtadi, C., Tuffs, P. S.: Generalized Predictive Control-part II. Extentions and Interpretations, Vol. 23, No. 2. Automatica. (1987), 149-160
13. Pivonka, Nepevny, P.,: Generalized Predictive Control with Adaptive Model Based on Neural Networks, Proceedings of the 6th Wseas Int. Conf. on Neural Networks, Lisbon, Portugal, June 16-18, (2005), 1-4
14. C. Bordons, Camacho, E. F.: Adaptive Generalized Predictive Controller for a wide class of industrial Processes, Vol 6, No. 2, (1998), 372-387
15. Sepehri, N., Wu, G.: Experimental evaluation of Generalized Predictive Control Applied to a Hydraulic Actuator, Robotica, Vol. 16, (1998), 463-474
16. Chidrawar, S., Patre, B.: Generalized Predictive Control and Neural Generalized Predictive Control, Leonardo Journal of Sciences, Issue 13, July-December, (2008), 133-152
17. Walker, P.A.W., Abdallah, O.H.,: Discrete Control of an A.C. Turbo generator by Output Feedback, Proceedings of the IEE, Control & Science, Vol. 125, No. 9, Oct. (1978), 1031-38
18. Recommended Practice for Excitation System Models for Power System Stability Studies, IEEE Standard, August, (1992)

19. Aggoune M.E., Boudjemaa F., Bensenouci A., et al.,: Design of Variable Structure Voltage Regulator Using Pole Assignment Technique, IEEE Transactions on Automatic Control, Vol. 39, No. 10, (1994), 2106-10
20. Bensenouci A., Variable Structure Control for Voltage/Speed Control in Power System, Proc. 2nd IASTED, Crete, Greece. Jun 25-28, (2002)
21. Demello F.P., and Concordia C., Concepts of Synchronous Machine Stability as affected by Excitation Control, IEEE Transactions on Power Apparatus and Systems, Vol. 88, No. 4, (1969), 316-328
22. Anderson P.M., and Fouad, A.A.,: Power System Control and Stability, IEEE Press, 1993
23. Bensenouci A.,: Design of a Robust Hi/H2/MOC LMI-based Iterative Multivariable PID for Speed and Voltage Control of a Sample Power System, Journal of Engineering and Computer Sciences, Qassim University, Vol. 3, No. 2, July (2010), 127-146
24. Daniel Quiroga, O.,: Modelling and nonlinear control of voltage frequency of hydroelectric power plants, doctoral thesis, Universidad Politécnica de Cataluna, Instituto de Organizacion y Control de Sistemas Industriales, July (2000)
25. Muñoz-Hernández, G. A. :Simulation studies of a GPC controller for a hydroelectric plant, Transactions of the Institute of Measurement and Control, vol. 29 no. 1, March (2007), 35-51
26. Munoz-Hernandez, G.A., Mansoor, S.P., Jones, D.I.: Modelling and Controlling Hydropower Plants, Advances in Industrial Control, Dinorwig Simulation Models (2013), pp 269-282, Springer
27. Ait Lafkih, M.,: State observance in adaptive multivariable control, Advances in Modeling & Analysis, C, AMSE Press, Vol. 39, N° 2, (1993), 43-49