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Design of recursive parametric estimation algorithm for large-scale nonlinear systems described by Wiener mathematical models

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Abstract. *This paper proposes a recursive parametric estimation algorithm for large-scale nonlinear systems, which can be composed of several nonlinear interconnected systems. Each interconnected system, which can operate in a deterministic or a stochastic environment, is described by a discrete-time Wiener mathematical model with unknown, but constant or slowly time-varying parameters. We propose to estimate the parameters intervening in the Wiener mathematical model by using the prediction error method and the recursive least square techniques. A numerical simulation example of the parametric estimation of two stochastic nonlinear interconnected systems was treated to test the performance and the efficiency of the developed recursive parametric estimation algorithm.*

Key words: *Large-scale nonlinear systems; Interconnected nonlinear systems; Wiener mathematical models; Recursive parametric estimation algorithm.*

1. Introduction

Several studies of the dynamic large-scale systems, consisting of several interconnected systems, have been developed and published in the literature (see, e.g., Kamoun and Titli, 1988, Lunze, 1992; Jamshidi, 1996, Kamoun, 1994, 1995, Groumpos, 1994, Kamoun, 2008, Šiljak, 2013). Most of the published works were concerned the large-scale systems, which could be described by linear mathematical models with constant or slowly time-varying parameters. In addition, several types of the nonlinear mathematical models were used in the description and the parametric estimation of the nonlinear systems (see, e.g., Haber and Keviczky, 1999; Favier and Kibangou, 2009a, 2009b; Billings, 2013). In addition, several Wiener mathematical models which permits to describe nonlinear systems have been developed, by considering different configurations of the static nonlinear part (see, Wigren, 1993; Boutayeb and Darouach, 1995; Haber and Keviczky, 1999; Kamoun, 2003; Chaari *et al*., 2008).

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The remaining of this paper was organized as follows. The Section 2 dealt with the description of the nonlinear large-scale systems by discrete-time Wiener mathematical models. The Section 3 presents the formulation of the parametric estimation problem of large-scale nonlinear systems consisting of several interconnected monovariable nonlinear systems and which can be described by the class of Wiener stochastic mathematical models. An illustrative numerical simulation example is treated in the Section 4. Finally, some conclusions will come in Section 5.

2. Description of nonlinear large-scale systems

This section is reserved to the development of discrete-time mathematical models that can describe the dynamics of large-scale nonlinear systems. We particularly focused to the dynamic large-scale systems consisting of several interconnected nonlinear systems and that can be described by the class of Wiener mathematical models with known structure (order, delay), and unknown and slowly time-varying parameters.

In this case, we can spot two types of Wiener mathematical models belonging to this class to formulate the parametric estimation problem of nonlinear large-scale systems. In the first type of the Wiener mathematical model, the dynamic linear part is described by an Interconnected Deterministic Auto-Regressive Moving Average (IDARMA) mathematical model. In the second type of mathematical models, we assume that there is noise acting on the system output. Thereby, the dynamic linear portion is described by an Interconnected Auto-Regressive Moving Average with eXogenous (IARMAX) mathematical model in the second type.

In the following sub-sections, we treat the description of the nonlinear large-scale systems consisting of several interconnected nonlinear systems and operating in a deterministic or a stochastic environment. We suppose that the considered systems can be described by the class of the Wiener mathematical models.

2.1. Wiener deterministic mathematical model

In this context, we consider a nonlinear large-scale system $\mathcal{S}$ that can be made up of N interconnected nonlinear systems $\mathcal{S}_1,\ldots,\mathcal{S}_N$. We suppose that these nonlinear interconnected systems operated in a deterministic environment with unknown and slowly time-varying parameters. Thus, this system can be described by a Wiener mathematical model which structure can be illustrated by the following Figure 1:

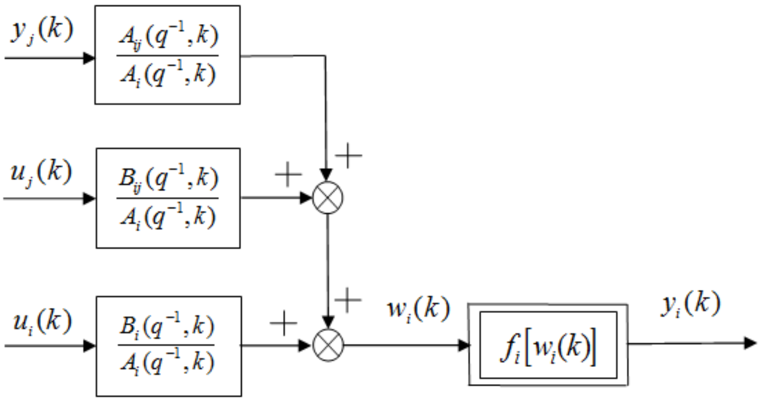

Figure 1. Structure of the Wiener deterministic mathematical model.

The dynamic linear part of the first Wiener mathematical model, as shown in Figure 1, is described by the following equation:

$$A_i(q^{-1},k)\, w_i(k) = B_i(q^{-1},k)\, u_i(k) + \sum_{j=1,\, j\neq i}^{N} B_{ij}(q^{-1},k)\, u_j(k) + \sum_{j=1,\, j\neq i}^{N} A_{ij}(q^{-1},k)\, y_j(k) \qquad (1)$$

where $u_i(k)$ and $w_i(k)$ represent the input and the output of the block of the dynamic linear part the discrete-time k, respectively, $u_j(k)$ and $y_j(k)$ denote the inputs of the other interconnected system S_j, $i, j = 1,\ldots,N;\, j \neq i$, $y_i(k)$ represents the output of the interconnected system S_i, and $A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $A_{ij}(q^{-1},k)$ and $B_{ij}(q^{-1},k)$ are polynomials with unknown and time-varying parameters, which are defined by:

$$A_i(q^{-1},k) = 1 + a_{i,1}(k)q^{-1} + \ldots + a_{i,n_{A_i}}(k)q^{-n_{A_i}} \qquad (2)$$

$$B_i(q^{-1},k) = b_{i,1}(k)q^{-1} + \ldots + b_{i,n_{B_i}}(k)q^{-n_{B_i}} \qquad (3)$$

$$A_{ij}(q^{-1},k) = 1 + a_{ij,1}(k)q^{-1} + \ldots + a_{ij,n_{A_{ij}}}(k)q^{-n_{A_{ij}}} \qquad (4)$$

and

$$B_{ij}(q^{-1},k) = b_{ij,1}(k)q^{-1} + \ldots + b_{ij,n_{B_{ij}}}(k)q^{-n_{B_{ij}}} \qquad (5)$$

where $i, j = 1,\ldots,N;\, j \neq i$, and, n_{A_i}, n_{B_i}, $n_{A_{ij}}$ and $n_{B_{ij}}$ are the orders of the polynomials $A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $A_{ij}(q^{-1},k)$ and $B_{ij}(q^{-1},k)$, respectively.

The nonlinear static part of the first Wiener mathematical model is given by:

$$y_i(k) = f_i[w_i(k)] \qquad (6)$$

where $f_i[.]$ represents the nonlinear function.

The equation (6) can be approximated by the following polynomial:

$$y_i(k) = \sum_{r=1}^{p_i} \beta_{i,r} w_i^r(k) + \Delta y_i[w_i(k)] \tag{7}$$

where $\Delta y_i[w_i(k)]$ represents an approximated error of the nonlinear function that can be assimilated to a noise acting on the system output.

We assume that the choice of the non-linearity degree p_i is made in such a way that the approximated error $\Delta y_i[w_i(k)]$ can be neglected. Then, from the mathematical model of the dynamic linear part (1), we can rewrite the output of the large-scale system in the following form:

$$\begin{aligned} y_i(k) = \sum_{r=1}^{p} \beta_{i,r} [-\sum_{h=0}^{n_{A_i}} a_{i,h}(k) w_i(k-h) + \sum_{h=1}^{n_{B_i}} b_{i,h}(k) u_i(k-h) \\ + \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_{B_{ij}}} b_{ij,h}(k)\, u_j(k-h) + \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_{A_{ij}}} a_{ij,h}(k)\, y_j(k-h)]^r + \Delta y_i[w_i(k)] \end{aligned} \tag{8}$$

with: $i, j = 1, \ldots, N;\ j \neq i$.

It can be remarked that the developed mathematical model becomes more complex with the increase of the non-linearity degree.

2.2. Wiener stochastic mathematical model

In this second type of the Wiener mathematical model, we assume that there is a noise sequence which is composed of independent random variables $e_i(k)$. In this case, the dynamic linear part of the Wiener mathematical model is of type IARMAX. The structure of the Wiener stochastic mathematical model with time-varying parameters can be represented by the following Figure 2:

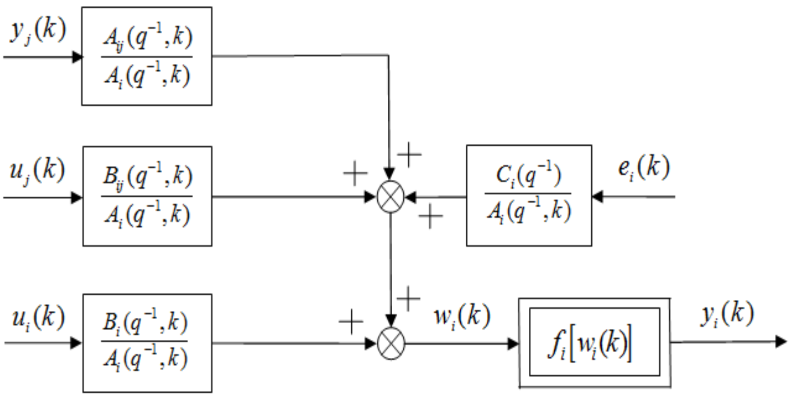

Figure 2. Structure of the Wiener stochastic mathematical model.

The dynamic linear part of the Wiener stochastic mathematical model is described by the following equation:

$$A_i(q^{-1},k)\, w_i(k) = B_i(q^{-1},k)\, u_i(k) + \sum_{j=1, j\neq i}^{N} B_{ij}(q^{-1},k)\, u_j(k) + \sum_{j=1, j\neq i}^{N} A_{ij}(q^{-1},k)\, y_j(k) + C_i(q^{-1})\, e_i(k) \quad (9)$$

where $\{e_i(k)\}$ is a sequence of independent random variables with zero mean and constant variance σ_i^2 and $C_i(q^{-1})$ is a polynomial with unknown but constant parameters, such that:

$$C_i(q^{-1}) = 1 + c_{i,1} q^{-1} + \ldots + c_{i,n_{C_i}} q^{-n_{C_i}} \quad (10)$$

where n_{C_i} represents the order of the polynomial $C_i(q^{-1})$.

Taking into account the equations (9) and (7), we can write the output of the considered system by the following expression:

$$\begin{aligned} y_i(k) = \sum_{r=1}^{p} \beta_{i,r} [& -\sum_{h=0}^{n_{A_i}} a_{i,h}(k) w_i(k-h) + \sum_{h=1}^{n_{B_i}} b_{i,h}(k) u_i(k-h) + \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_{B_{ij}}} b_{ij,h}(k)\, u_j(k-h) \\ & + \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_{A_{ij}}} a_{ij,h}(k)\, y_j(k-h) + \sum_{h=1}^{n_{C_i}} c_{i,h} e_i(k-h) + e_i(k)]^r + \Delta y_i[w_i(k)] \end{aligned} \quad (11)$$

We can obtain several types of the Wiener mathematical models for the description of the interconnected nonlinear systems by considering various configurations of the static nonlinear part.

For simplicity, we assume, in what follows, that the polynomials $A_i(q^{-1},k)$, $B_i(q^{-1},k)$, $A_{ij}(q^{-1},k)$, $B_{ij}(q^{-1},k)$ and $C_i(q^{-1})$ of the Wiener mathematical model (11) have the same order n_i (i.e., $n_i = n_{A_i} = n_{B_i} = n_{A_{ij}} = n_{B_{ij}} = n_{C_i}$).

3. Parametric estimation of the nonlinear interconnected systems described by Wiener stochastic mathematical models

This section is devoted to the parametric estimation of the nonlinear interconnected systems, which can be described by the Wiener stochastic mathematical models (11). The parametric estimation was performed based on the prediction error method and the recursive least square techniques, while using several measured values of the input and the output of the considered system.

Taking into account the selected assumptions that mentioned in the second part, the stochastic nonlinear large-scale system output $y_i(k)$, which is described by the Wiener mathematical model (11), can be rewrite as follows:

$$
\begin{aligned}
y_i(k) = \sum_{r=1}^{p} \beta_{i,r}[-\sum_{h=0}^{n_i} a_{i,h}(k) w_i(k-h) + \sum_{h=1}^{n_i} b_{i,h}(k) u_i(k-h) + \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_i} b_{ij,h}(k)\, u_j(k-h) \\
+ \sum_{j=1, j\neq i}^{N} \sum_{h=1}^{n_i} a_{ij,h}(k)\, y_j(k-h) + \sum_{h=1}^{n_i} c_{i,h} e_i(k-h) + e_i(k)]^r
\end{aligned} \tag{12}
$$

with: $i, j = 1, \ldots, N;\ j \neq i$.

By examining the Wiener mathematical model (12), we can identify two points. The first point concerns the non-linearity of the mathematical model relative to the unknown parameters $\beta_{i,r} a_{i,h}(k)$, $h = 1,\ldots,n_i$. The parametric estimation can be conducted using an appropriate method of identification, based on the knowledge of the values of several variables $u_i(k)$, $w_i(k)$ and $y_i(k)$. As for the second point, it relates to the sequence $\{w_i(k-h),\ h = 1,\ldots,n_i\}$ of the output of the linear dynamic part of the Wiener mathematical model, which is not observable.

It should be noted that the steps of the estimate parameters that involved in the Wiener mathematical model (12) and the variable estimation $w_i(k)$ must take place sequentially according to a criterion of validation of the developed mathematical model. Therefore, these two steps might present some implementation difficulties.

To overcome the posed parametric estimation problem, we assume that the nonlinear function of the considered Wiener mathematical model is bijective and symmetric about the origin. This assumption implies that the non-linear function $y_i(k) = f_i[w_i(k)]$ can admit an inverse function $w_i(k) = f_i^{-1}[y_i(k)]$, which is defined on the same domain as the output $y_i(k)$ and is described by the following function:

$$
w_i(k) = f_i^{-1}[y_i(k)] \tag{13}
$$

where $f_i^{-1}[y_i(k)]$ represents the inverse function of $f_i[w_i(k)]$.

Thus, one can opt for a Wiener inverse mathematical model, as shown Figure 3.

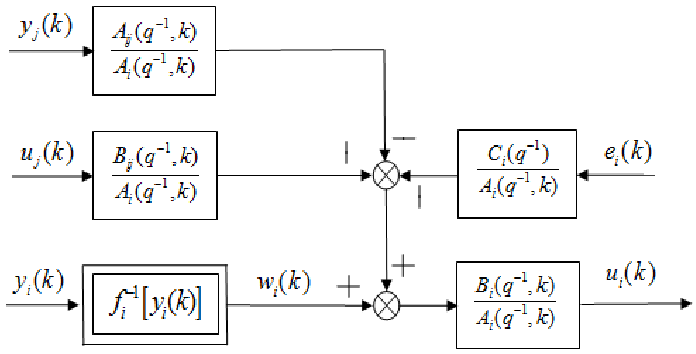

Figure 3. Structure of the Wiener inverse mathematical model.

The inverse function (13) of the static non-linear part of the considered Wiener mathematical model can be described by the following polynomial:

$$w_i(k)=\sum_{r=1}^{p_i}\beta_{i,r}\,y_i^r(k)+\delta_i(k) \tag{14}$$

where $\delta_i(k)$, $i=1,\ldots,N$, represents the approximated error of the inverse function, which is assumed negligible.

The dynamic linear part of the Wiener inverse mathematical model, as given Figure 3, can be expressed as follows:

$$\begin{aligned}B_i(q^{-1},k)u_i(k)&=A_i(q^{-1},k)w_i(k)-\sum_{j=1,\,j\neq i}^{N}B_{ij}(q^{-1},k)\;u_j(k)\\&-\sum_{j=1,\,j\neq i}^{N}A_{ij}(q^{-1},k)\;y_j(k)-C_i(q^{-1})e_i(k)\end{aligned} \tag{15}$$

which can be written as follows:

$$\begin{aligned}b_{i,1}(k)u_i(k-1)&=-\sum_{h=2}^{n_i}b_{i,h}(k)u_i(k-h)+\sum_{h=0}^{n_i}a_{i,h}(k)w_i(k-h)-\sum_{j=1,\,j\neq i}^{N}\sum_{h=1}^{n_i}b_{ij,h}(k)\;u_j(k-h)\\&-\sum_{j=1,\,j\neq i}^{N}\sum_{h=1}^{n_i}a_{ij,h}(k)\;y_j(k-h)-\sum_{h=1}^{n_i}c_{i,h}e_i(k-h)-e_i(k)\end{aligned} \tag{16}$$

with: $a_{i,0}(k)=1,\ \forall\ k.$

Taking into account the equation (14), we can rewrite the expression (16) as follows:

$$\begin{aligned}b_{i,1}(k)u_i(k-1)&=-\sum_{h=2}^{n_i}b_{i,h}(k)u_i(k-h)+\sum_{h=0}^{n_i}\sum_{r=1}^{p}a_{i,h}(k)\beta_{i,r}y_i^r(k-h)-\sum_{j=1,\,j\neq i}^{N}\sum_{h=1}^{n_i}b_{ij,h}(k)\;u_j(k-h)\\&-\sum_{j=1,\,j\neq i}^{N}\sum_{h=1}^{n_i}a_{ij,h}(k)\;y_j(k-h)-\sum_{h=1}^{n_i}c_{i,h}e_i(k-h)-e_i(k)\end{aligned} \tag{17}$$

In the following, we propose to formulate the posed parametric estimation problem, in order to estimate the parameters intervening in the expression of the Wiener inverse mathematical model (17), by using the prediction error method and the recursive least square techniques. This problem can be solved on the basis of the knowledge of several values of the inputs $u_i(k)$ and the outputs $y_i(k)$ from the considered nonlinear system.

We assume that the parameter $b_{i,1}(k)$, $i=1,\ldots,N$, that involved in the Wiener inverse mathematical model (17), is well-known, and that the approximated error of the nonlinear function is negligible. Taking into account these assumptions, we can express the Wiener inverse mathematical model (17) in the following developed form:

$$\begin{aligned}b_{i,1}(k)u_i(k-1) = &-b_{i,2}(k)u_i(k-2) - \ldots - b_{i,n_i}(k)u_i(k-n_i) + \beta_{i,1}y_i(k) + a_{i,1}(k)\beta_{i,1}y_i(k-1)\\ &+ \ldots + a_{i,n_i}(k)\beta_{i,1}y_i(k-n_i) + \ldots \ \ldots + \beta_{i,p}y_i^p(k) + a_{i,1}(k)\beta_{i,p}y_i^p(k-1)\\ &+ \ldots + a_{i,n_i}(k)\beta_{i,p}y_i^p(k-n_i) - b_{ij,1}(k)u_j(k-1) - \ldots - b_{ij,n_i}(k)u_j(k-n_i)\\ &- a_{ij,1}(k)y_j(k-1) - \ldots - a_{ij,n_i}(k)y_j(k-n_i) - c_{i,1}e_i(k-1)\\ &- \ldots - c_{i,n_i}e_i(k-n_i) - e_i(k)\end{aligned} \quad (18)$$

or equivalently, in the following matrix form:

$$b_{i,1}(k)u_i(k-1) = \theta_i^T(k)\psi_i(k) - e_i(k) \quad (19)$$

where the parameters vector $\theta_i(k)$ and the observations vector $\psi_i(k)$ are defined as follows:

$$\begin{aligned}\theta_i^T(k) = \Big[&b_{i,2}(k)\ \ldots\ b_{i,n_i}(k)\ \beta_{i,1}\ f_{i,11}(k)\ \ldots\ f_{i,n_i1}(k)\ \ldots\ \ldots\ \beta_{i,p_i}\ f_{i,1p_i}(k)\ \ldots\ f_{i,n_ip_i}(k)\\ &b_{ij,1}(k)\ b_{ij,2}(k)\ \ldots\ b_{ij,n_i}(k)\ a_{ij,1}(k)\ a_{ij,2}(k)\ \ldots\ a_{ij,n_i}(k)\ c_{i,1}\ \ldots\ c_{i,n_i}\Big]\end{aligned} \quad (20)$$

and

$$\begin{aligned}\psi_i^T(k) = \big[&-u_i(k-2)\ \ldots\ -u_i(k-n_i)\ \ y_i(k)\ \ y_i(k-1)\ \ldots\ y_i(k-n_i)\ \ldots\ \ldots\ y_i^p(k)\\ &y_i^p(k-1)\ \ldots\ y_i^p(k-n_i)\ -u_j(k-1)\ -u_j(k-2)\ \ldots\ -u_j(k-n_i)\ -y_j(k)\\ &-y_j(k-1)\ \ldots\ -y_j(k-n_i)\ -e_i(k-1)\ -e_i(k-2)\ \ldots\ -e_i(k-n_i)\big]\end{aligned} \quad (21)$$

with: $f_{i,11}(k),\ldots,f_{i,n_i1}(k) = a_{i,1}(k)\beta_{i,1},\ldots,a_{i,n_i}(k)\beta_{i,1}$; $f_{i,1p_i}(k),\ldots,f_{i,n_ip_i}(k) = a_{i,1}(k)\beta_{i,p_i},\ldots,a_{i,n_i}(k)\beta_{i,p_i}$.

The parametric estimation problem can be solved using the prediction error method and the recursive least square techniques. In this context, we can define the predicted value $b_{i,1}(k)\hat{u}_i(k-1)$ of the adjusted inverse model as follows:

$$b_{i,1}(k)\,\hat{u}_i(k-1) = \hat{\theta}_i^T(k-1)\psi_i(k) \quad (22)$$

where $\hat{\theta}_i^T(k-1)$ represents the vector of the estimated parameters at the discrete-time $k-1$. This vector is defined at the discrete-time k by:

$$\begin{aligned}\hat{\theta}_i^T(k) = \Big[&\hat{b}_{i,2}(k)\ \ldots\ \hat{b}_{i,n_i}(k)\ \hat{\beta}_{i,1}(k)\ \hat{a}_{i,1}(k)\hat{\beta}_{i,1}(k)\ \ldots\ \hat{a}_{i,n_i}(k)\hat{\beta}_{i,1}(k)\\ &\ldots\ \ldots\ \hat{\beta}_{i,p}(k)\ \hat{a}_{i,1}(k)\hat{\beta}_{i,p}(k)\ \ldots\ \hat{a}_{i,n_i}(k)\hat{\beta}_{i,p}(k)\ \hat{b}_{ij,1}(k)\ \hat{b}_{ij,2}(k)\\ &\ldots\ \hat{b}_{ij,n_i}(k)\ \hat{a}_{ij,1}(k)\ \hat{a}_{ij,2}(k)\ \ldots\ \hat{a}_{ij,n_i}(k)\ \hat{c}_{i,1}(k)\ \ldots\ \hat{c}_{i,n_i}(k)\Big]\end{aligned} \quad (23)$$

The prediction error $\varepsilon_i(k)$ can be described by:

$$\varepsilon_i(k) = b_{i,1}(k)u_i(k-1) - \hat{\theta}_i^T(k-1)\psi_i(k) \quad (24)$$

The parameters intervening in the Wiener inverse mathematical model (18) can be estimated by using the following Recursive Extended Least Square RELS algorithm:

$$\begin{aligned}
&\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + P_i(k)\hat{\psi}_i(k)\varepsilon_i(k) \\
&P_i(k) = \frac{1}{\lambda_i(k)}\left[P_i(k-1) - \frac{P_i(k-1)\hat{\psi}_i(k)\hat{\psi}_i^T(k)P_i(k-1)}{\lambda_i(k) + \hat{\psi}_i^T(k)P_i(k-1)\hat{\psi}_i(k)}\right] \\
&\varepsilon_i(k) = b_{i,1}(k)u_i(k-1) - \hat{\theta}_i^T(k-1)\hat{\psi}_i(k)
\end{aligned} \tag{25}$$

where $\lambda_i(k)$ is the forgetting factor which permits to improve the capacity of the adaptation gain matrix $P_i(k)$, while providing a better monitoring of the parametric variations ($0 \prec \lambda_i(k) \prec 1$) and $\hat{\psi}_i(k)$ is the vector of the approximated observations $\psi_i(k)$, which is defined by:

$$\begin{aligned}
\hat{\psi}_i^T(k) = [&-u_i(k-2) \ \dots \ -u_i(k-n_i) \ \ y_i(k) \ \ y_i(k-1) \ \dots \ y_i(k-n_i) \ \dots \ \dots \ y_i^p(k) \\
&y_i^p(k-1) \ \dots \ y_i^p(k-n_i) \ -u_j(k-1) \ -u_j(k-2) \ \dots \ -u_j(k-n_i) \ -y_j(k) \\
&-y_j(k-1) \ \dots \ -y_j(k-n_i) \ -\varepsilon_i(k-1) \ -\varepsilon_i(k-2) \ \dots \ -\varepsilon_i(k-n_i)]
\end{aligned} \tag{26}$$

It should be noted that the Wiener mathematical model (18) contains a certain redundancy of the estimated parameters $\hat{a}_{i,h}(k)$ and $\hat{\beta}_{i,r}(k)$, $h = 1,\dots,n_i$, $r = 1,\dots,p_i$. In this case, we propose to determine the estimated parameters $\hat{a}_{i,h}(k)$ from the knowledge of the estimated parameters $\hat{\beta}_{i,r}(k)$ and $\hat{a}_{i,h}(k)\hat{\beta}_{i,r}(k)$. The h^{th} estimated parameter of $a_{i,h}(k)$, which denotes $\tilde{a}_{i,hr}(k)$, can be determined by the following equation:

$$\tilde{a}_{i,hr}(k) = \frac{\hat{\theta}_i\left[k; n_i + (r-1)(n_i+1) + h\right]}{\hat{\theta}_i\left[k; n_i + (r-1)(n_i+1)\right]} \tag{27}$$

where $\hat{\theta}_i[k;h]$ represents the h^{th} component of the vector of the estimated parameters $\hat{\theta}_i(k)$, defined by (23), with: $1 \le r \le p_i$; $1 \le h \le n_i$; $1 \le i \le N$.

We can choose an average value $\tilde{a}_{i,hm}(k)$ of the estimated parameter $\hat{a}_{i,h}(k)$, which can be calculated using the following expression:

$$\tilde{a}_{i,hm}(k) = \frac{1}{p_i}\sum_{r=1}^{p}\frac{\hat{\theta}_i\left[k; n_i + (r-1)(n_i+1) + h\right]}{\hat{\theta}_i\left[k; n_i + (r-1)(n_i+1)\right]} \tag{28}$$

Similarly, the estimated parameter $\hat{\beta}_{i,r}(k)$, which is related to the estimated parameters $\hat{a}_{i,h}(k)$, $h = 1,\dots,n_i$, is found h times in the vector of the estimated parameters $\hat{\theta}_i(k)$. The calculation of the estimated parameter $\hat{\beta}_{i,r}(k)$ based on the knowledge of the average value $\tilde{a}_{i,hm}(k)$ of the estimated parameter $\hat{a}_{i,h}(k)$, such as:

$$\tilde{\beta}_{i,r}(k) = \frac{\hat{\theta}_i\left[k; n_i + (r-1)(n_i+1) + h\right]}{\tilde{a}_{i,hm}(k)} \tag{29}$$

It should be noted that in a general case of the mathematical model, where the parameters $\hat{a}_{i,h_1}(k)$ and $\hat{b}_{i,h_2}(k)$ are given, such as: $h_1 = 1,\dots,n_{A_i}$ and $h_2 = 2,\dots,n_{B_i}$, with: $n_{B_i} \le n_{A_i}$, then, the value of h_1^{th} estimated parameter of $a_{i,h_1}(k)$ can be determined using the following equation:

$$\tilde{a}_{i,h_1 r}(k) = \frac{\hat{\theta}_i\left[k; n_{B_i} + (r-1)(n_{A_i}+1) + h_1\right]}{\hat{\theta}_i\left[k; n_{B_i} + (r-1)(n_{A_i}+1)\right]} \quad (30)$$

with: $1 \le r \le p_i$; $1 \le i \le N$.

In addition, the average value $\tilde{a}_{i,h_1 m}(k)$ of the estimated parameter $\hat{a}_{i,h_1}(k)$, can be calculated through the following expansion:

$$\tilde{a}_{i,h_1 m}(k) = \frac{1}{p_i}\sum_{r=1}^{p} \frac{\hat{\theta}_i\left[k; n_{B_i} + (r-1)(n_{A_i}+1) + h_1\right]}{\hat{\theta}_i\left[k; n_{B_i} + (r-1)(n_{A_i}+1)\right]} \quad (31)$$

The global quality of the parametric estimation obtained by the recursive parametric estimation algorithm can be made starting the calculation of the following parametric distance $d_i(k)$:

$$d_i(k) = \left[\sum_{h=1}^{n_i}\left[\frac{a_{i,h}(k) - \tilde{a}_{i,hm}(k)}{a_{i,h}(k)}\right]^2 + \sum_{h=2}^{n_i}\left[\frac{b_{i,h}(k) - \hat{b}_{i,h}(k)}{b_{i,h}(k)}\right]^2 + \sum_{j=1, j\neq i}^{N}\sum_{h=1}^{n_i}\left[\frac{b_{ij,h}(k) - \hat{b}_{ij,h}(k)}{b_{ij,h}(k)}\right]^2 \right.$$
$$\left. + \sum_{j=1, j\neq i}^{N}\sum_{h=1}^{n_i}\left[\frac{a_{ij,h}(k) - \hat{a}_{ij,h}(k)}{a_{ij,h}(k)}\right]^2 + \sum_{h=1}^{n_i}\left[\frac{c_{i,h} - \hat{c}_{i,h}(k)}{c_{i,h}}\right]^2 \right]^{0.5} \quad (32)$$

We must indicate here that the calculation of the parametric distance $d_i(k)$ can only be done in the context of an example of numerical simulation, where the parameters intervening of the considered mathematical model are well known.

4. Numerical simulation

In this section, we treat a numerical simulation example. The objective of this numerical simulation was to test the performance and the efficiency of the developed recursive parametric estimation algorithm RELS with forgetting factor, which is defined by (25).

Let us consider a stochastic large-scale system that is constituted by two interconnected nonlinear monovariable systems S_1 and S_2, which are described by the following Wiener mathematical models, respectively:

$$\begin{aligned} y_2(k) &= \text{th}\left[w_2(k)\right] \\ w_1(k) &= -a_{1,1}(k)w_1(k-1) + b_{1,1}(k)u_1(k-1) + b_{12,1}(k)u_2(k-1) \\ &\quad + a_{12,1}(k)y_2(k-1) + e_1(k) + c_{1,1}e_1(k-1) \end{aligned} \quad (33)$$

$$\begin{aligned} w_2(k) &= -a_{2,1}(k)w_2(k-1) + b_{2,1}(k)u_2(k-1) + b_{21,1}(k)u_1(k-1) \\ &\quad + a_{21,1}(k)y_1(k-1) + e_2(k) + c_{2,1}e_1(k-1) \end{aligned} \quad (34)$$

with

$$y_1(k) = \text{th}\left[w_1(k)\right] \tag{35}$$

and

(36)

We assume that the considered nonlinear statics parts are bijective and symmetric about the origin. The nonlinear statics parts admit two inverse functions, which can be described as follows:

$$w_1(k) = \text{arcth}\left[y_1(k)\right] \tag{37}$$

and

$$w_2(k) = \text{arcth}\left[y_2(k)\right] \tag{38}$$

The inverse function of the nonlinear static part of the considered Wiener mathematical model can be approximated by the following polynomial:

$$w_i(k) = \sum_{r=1}^{p_i} \beta_{i,r}\ y_i^{2r-1}(k) \tag{39}$$

with: $i = 1, 2$.

The non-linearity degree p_i is equal to 2. Then, the expansions of the Wiener inverse mathematical models are defined as follows:

$$\begin{aligned} b_{1,1}(k)u_1(k-1) = \beta_{1,1}y_1(k) + a_{1,1}(k)\beta_{1,1}y_1(k-1) + \beta_{1,2}y_1^3(k) + a_{1,1}(k)\beta_{1,2}y_1^3(k-1) \\ - b_{12,1}(k)u_2(k-1) - a_{12,1}(k)y_2(k-1) - e_1(k) - c_{1,1}e_1(k-1) \end{aligned} \tag{40}$$

and

$$\begin{aligned} b_{2,1}(k)u_2(k-1) = \beta_{2,1}y_2(k) + a_{2,1}(k)\beta_{2,1}y_2(k-1) + \beta_{2,2}y_2^3(k) + a_{2,1}(k)\beta_{2,2}y_2^3(k-1) \\ - b_{21,1}(k)u_1(k-1) - a_{21,1}(k)y_1(k-1) - e_2(k) - c_{2,1}e_2(k-1) \end{aligned} \tag{41}$$

The output $y_1(k)$ of the nonlinear interconnected system S_1 can be rewritten as follows:

$$b_{1,1}(k)u_1(k-1) = \theta_1^T(k)\psi_1(k) - e_1(k) \tag{42}$$

where the vectors of the parameters $\theta_1(k)$ and the observations $\psi_1(k)$ are defined as follows:

$$\theta_1^T(k) = \left[\beta_{1,1}\ \ f_{1,11}(k)\ \beta_{1,2}\ \ f_{1,12}(k)\ b_{12,1}(k)\ a_{12,1}(k)\ c_{1,1}\right] \tag{43}$$

and

$$\psi_1^T(k) = \left[y_1(k)\ \ y_1(k-1)\ \ y_1^3(k)\ \ y_1^3(k-1)\ -u_2(k-1)\ -y_2(k-2)\ -e_1(k-1)\right] \tag{44}$$

with: $f_{1,11}(k) = a_{1,1}(k)\beta_{1,1}$; $f_{1,12}(k) = a_{1,1}(k)\beta_{1,2}$.

The output $y_2(k)$ of the nonlinear interconnected system S_2 can be rewritten by:

$$b_{2,1}(k)u_2(k-1) = \theta_2^T(k)\psi_2(k) - e_2(k) \tag{45}$$

where the vectors of the parameters $\theta_2(k)$ and the observations $\psi_2(k)$ are defined as follows:

$$\theta_2^T(k) = \left[\beta_{2,1}\ \ f_{2,11}(k)\ \beta_{2,2}\ \ f_{2,12}(k)\ b_{21,1}(k)\ a_{21,1}(k)\ c_{2,1}\right] \tag{46}$$

and

$$\psi_2^T(k) = \left[y_2(k)\ \ y_2(k-1)\ \ y_2^3(k)\ \ y_2^3(k-1)\ \ -u_1(k-1)\ \ -y_1(k-2)\ \ -e_2(k-1)\right] \tag{47}$$

with: $f_{2,11}(k) = a_{2,1}(k)\beta_{2,1}$; $f_{2,12}(k) = a_{2,1}(k)\beta_{2,2}$.

In this example of numerical simulation, the objective is to estimate the parameters intervening in the vectors $\theta_1(k)$ and $\theta_2(k)$ using the recursive parametric estimation algorithm RELS (25).

For the purpose of the simulation example, we assume the following numerical values for the parameters intervening in the Wiener mathematical models (40) and (41):

1. the values of parameters intervening in the mathematical model (40) are chosen, as such: $a_{1,1}(k) = -0.75 + 0.04\sin(0.3k)$, $b_{1,1}(k) = 0.66 + 0.03\cos(0.3k)$, $c_{1,1} = 0.1$, $b_{12,1}(k) = -0.76 + 0.03\cos(0.3k)$, $a_{12,1}(k) = -0.1 + 0.05\sin(0.2k)$;
2. the values of parameters intervening in the mathematical model (41) are chosen, as such: $a_{2,1}(k) = 0.58 + 0.04\sin(0.3k)$, $b_{2,1}(k) = 0.55 + 0.03\cos(0.3k)$, $c_{2,1} = -0.1$, $b_{21,1}(k) = -0.5 + 0.03\cos(0.3k)$, $a_{21,1}(k) = 0.1 + 0.05\sin(0.2k)$;

Let us add that the noise sequences $\{e_i(k), i = 1,2\}$ are assumed to be independent and correspond to a Gaussian distribution with zero mean and variances $\sigma_1^2 = 0.0773$ and $\sigma_2^2 = 0.0591$, respectively.

The relative data to this numerical simulation example for the practical implementation of the parametric estimation algorithm RELS (25), are given hereafter:

1. the input $u_i(k)$, $i = 1,2$, that applied to the interconnected nonlinear system S_i, is a high level pseudo-random binary sequence [–2, +2];
2. the initial conditions of the two vectors of the estimated parameters $\hat{\theta}_i(0)$ and the two adaptation gain matrices $P_i(0)$ are chosen, in such a way that: $\hat{\theta}_i(0) = 0$ and $P_i(0) = 1000I$, where I is a matrix unit, $i = 1,2$;
3. the forgetting factors $\lambda_1(k)$ and $\lambda_2(k)$ are chosen, in such a way that: $\lambda_1(k) = 0.995$, $\lambda_2(k) = 0.99$;
4. the number of measurements is selected as: $M_i = 1,\ldots 1000$.

Some obtained numerical simulation results of the considered nonlinear system S_i are given in Figures 4 and 5. Thus, the evolution curves of the prediction error

$\varepsilon_1(k)$, the parametric distance $d_1(k)$ and their overall variances $\sigma^2_{\varepsilon_1}(k)$ and $\sigma^2_{d_1}(k)$ are given in Figure 4. Figure 5 represents the evolution curves of the prediction error $\varepsilon_2(k)$, the parametric distance $d_2(k)$ and their overall variances $\sigma^2_{\varepsilon_2}(k)$ and $\sigma^2_{d_2}(k)$.

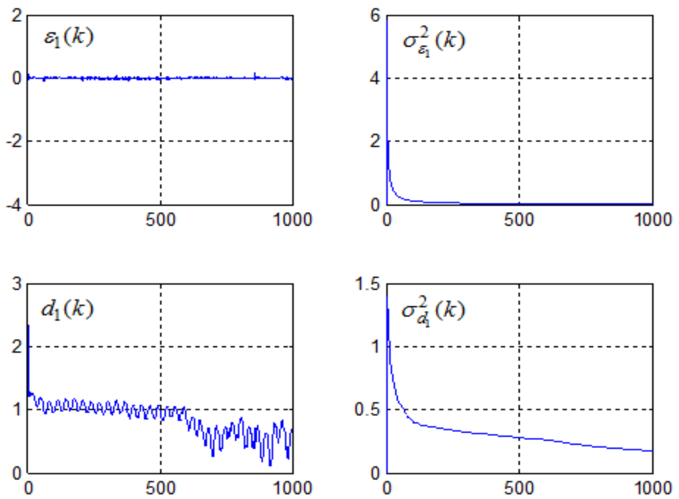

Figure 4. Evolution curves of the prediction error $\varepsilon_1(k)$, the parametric distance $d_1(k)$ and their overall variances $\sigma^2_{\varepsilon_1}(k)$ and $\sigma^2_{d_1}(k)$.

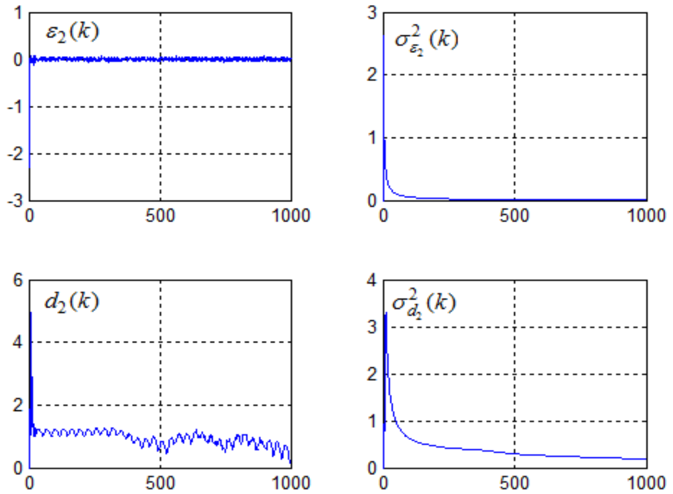

Figure 5. Evolution curves of the prediction error $\varepsilon_2(k)$, the parametric distance $d_2(k)$ and their overall variances $\sigma^2_{\varepsilon_2}(k)$ and $\sigma^2_{d_2}(k)$.

The interpretation of the evolution curves of the different elements shown in Figures 4 et 5 proves the quality of the estimation, which is obtained by the recursive parametric estimation algorithm RELS (25). It can be remarked that the parametric distances $d_1(k)$ and $d_2(k)$ (statistical average) have some quite low values.

The Table 1 represents the values of the statistical averages of the prediction errors $\varepsilon_1(k)$ and $\varepsilon_2(k)$, the parametric distances $d_1(k)$ and $d_2(k)$, and their overall variances $\sigma^2_{\varepsilon_1}(k)$, $\sigma^2_{d_1}(k)$, $\sigma^2_{\varepsilon_2}(k)$ and $\sigma^2_{d_2}(k)$ of the considered nonlinear system.

Table 1. The values of statistical averages of the interconnected system $\mathcal{S}_i$.

$\bar{m}_{\varepsilon_1(k)}$	$\sigma^2_{\varepsilon_1(k)}$	$\bar{m}_{\varepsilon_2(k)}$	$\sigma^2_{\varepsilon_2(k)}$
–0.0046	0.0821	–0.0045	0.0629
$\bar{m}_{d_1(k)}$	$\sigma^2_{d_1(k)}$	$\bar{m}_{d_2(k)}$	$\sigma^2_{d_2(k)}$
0.5122	0.2271	0.4316	0.2725

We remark that the shapes of the prediction errors have some relatively low values too. Thus, we can affirm that the obtained results in this numerical simulation example are satisfactory and clearly prove the good performance, that can be achieved by the recursive parametric estimation algorithm RELS (25) during the estimate of the parameters of the two mathematical models (40) and (41) describing the two nonlinear interconnected systems $\mathcal{S}_1$ and $\mathcal{S}_2$.

5. Conclusion

In this paper, we have developed Wiener mathematical models that can describe the dynamics of the large-scale nonlinear systems, which can operate in deterministic and stochastic environments. We have treated particularly the nonlinear large-scale dynamic systems, which can be composed of several nonlinear interconnected systems. We have supposed that each interconnected system can be described by a Wiener mathematical model, with unknown and time-varying parameters.

The parametric estimation problem of the considered class of the nonlinear interconnected systems was solved by using the prediction error method and the recursive least square techniques. We have developed a recursive parametric estimation algorithm RELS with forgetting factor, which can be estimated the parameters intervening in the considered interconnected systems.

We have treated a numerical simulation example in order to test the performance and the efficiency of the developed recursive parametric estimation algorithm RELS.

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