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Synthesis of multi-observers for discrete-time nonlinear systems with delayed output

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Abstract. This paper deals with the state estimation of discrete-time nonlinear systems subject to delayed output measurements. These systems are represented in each operating zone by a specific model with the appropriate dimension that describes accurately the system behavior. The simulation results show clearly that the proposed multi-observer leads to satisfactory results of the estimation error convergence.

Keyword: Discrete-time nonlinear systems, Decoupled multimodel, Time-delay Systems, State estimation, Multi-observer

1 Introduction

Most physical systems have nonlinear behavior. Generally, they involve time-delays that can not be neglected since the non-consideration of their effects can lead to degradation of system performance and can even lead to instability. The time-delay occurs naturally in the modeling of systems encountered in physics, engineering, economics, chemistry, biology [21]. Furthermore, even if the process does not contain time-delay, the closed loop system can generate significant time-delays (through sensors, actuators, computational system ...).
In a variety of engineering applications, the output measurements are available after a non-negligible time-delay. In fact, this type of time-delay is often encountered when the measured output data are transmitted through a low speed communication system. Also, it can arise when systems to be controlled or monitored are located far from the computing unit. Moreover, sensor technology can be the source of time-delays associated with output measurements.
For instance, the measured output data of systems controlled by a remote control are transmitted to the control system through a communication system that inevitably introduces a significant time-delay between the process and the control or the supervision system. On the other hand, we can cite the example of bioreactors where most measurements are obtained after an important time interval since these measured data result from analyzes that take time.

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Another typical example of systems, where the transmission of output data is performed by means of a communication system with low rate, is network-connected systems. This usually introduces significant time-delays which must be taken into account to ensure the viability of the control and monitoring system. In order to simplify the control and observer synthesis for systems with time-delays, several studies attempt to consider the constancy of the delay. However, this assumption is rarely verified in reality; indeed, the invariance of the time-delay is an unrealistic assumption.

Nonlinear systems (with time-delay or delay-free systems) are frequently discussed in the literature [7],[9],[6],[13],[17],[24]. In practice, such systems have often a complex structure and they are difficult to be handled. Thus, in order to exploit the techniques used in the handling of linear systems, researchers tend to assume the linearity of these systems. However, nonlinear systems have a linear behavior only in specific operating zones. Outside these areas, the linear model of the system is no longer valid which leads to the degradation of the model performance. The manipulation of nonlinear systems requires the obtaining of a model that can be usable in practice and describes accurately the system behavior. Researchers have attempted to ensure this compromise by introducing a new approach called multimodel approach. This approach consists in representing the nonlinear system by local models (or partial models). Each partial model describes the behavior of the system in a restricted operating zone. Ltaief proposes in [11] an optimal systematic determination of models'base for multimodel representation.

Multimodel structures are classified, according to the kind of coupling between the partial models, into two main types; coupled and decoupled structure. The first one (known as Takagi-Sugeno model) is obtained by interpolating the partial linear models. The partial models of this structure are of the same dimension, the major drawback of Takagi-Sugeno (T-S) multimodel is the complexity induced by the consideration of an invariable structure of partial models and therefore an over-parameterized multimodel.

Unlike the T-S multimodel; the partial models of the decoupled structure are described independently of each other, which provides flexibility in modeling since each operating zone is characterized by a local model that accurately reflects the degree of complexity. The significance of this structure appears clearly in the handling of complex systems with high nonlinearities.

Different techniques are proposed in the literature to obtain a multimodel, namely the linearization method, convex polytopic transformation or identification. In the first method, the model is linearized around several operating points [23], [8], then the obtained partial models are linked by nonlinear functions. The main drawback of this technique is the loss of information. Similarly, the choice of the number and position of the various operating points remains difficult to achieve.

On the other hand, the convex polytopic method consists in transforming the scalar fonctions which represent the sources of nonlinearities. This method is not always obvious since the knowledge of bounds of the decision variables is

not always possible.
Concerning the third method, it provides a partial models' base if input/output data around different operating points are available [10],[18],[1].
The state estimation of linear and nonlinear systems has been widely discussed in recent decades seeing that the control of systems requires generally the knowledge of all state variables which is not always feasible in practice. Indeed, the measure of all state variables of the system is often difficult and even impossible.
The state estimation of nonlinear systems with time-delay described by T-S multimodel has been extensively investigated in the last recent years. Cao [3],[2] proposed an observer-based controller for the stabilisation of nonlinear systems described by T-S fuzzy model. Gassara [5] designs an observer to estimate simultaneously the states and the actuator faults.
On the other hand, the state estimation of the decoupled multimodel is not widely treated in the literature, indeed, few works were interested in this topic [19], [14], [15]. Orjuela [20] proposed a new method for designing an observer for nonlinear systems described by a decoupled multimodel with delayed measurements and provided sufficient conditions for the exponential convergence of the continuous-time observer. The convergence of the estimation error requires a rigorous constraint on the derivative of the time-delay.
Otherwise, most of these works are carried out in continuous-time, in fact, the state estimation of discrete-time nonlinear systems with time-delays described by T-S or decoupled multimodel has not attracted as much attention as that of continuous-time systems. Furthermore, several results have focused on the observation of systems with time-delays on the state or on the input.
Hereafter, the state estimation of discrete-time nonlinear systems with variable time-delay on output measurements and modeled by a decoupled multimodel will be investigated.
Convergence conditions of the estimation error will be formulated under Linear Matrix Inequalities based on the Lyapunov-Krasovskii approach.
The present paper is organized as follows. The section 2 gives the problem formulation. In the third section, the sufficient conditions of the asymptotic convergence of the estimation error for the decoupled multimodel with delayed measurements are provided. Simulation examples are proposed in Section 4 to illustrate the significance of the proposed multi-observer. Finally, a conclusion finishes this paper.

2 Problem statement and preliminaries

2.1 Decoupled multimodel

The multimodel approach is considered as an efficient alternative that leads to an accurate representation of nonlinear systems. In the literature, there exist two common structures of multimodel; Takagi-Sugeno and decoupled structures. The T-S multimodel is firstly investigated by Takagi and Sugeno [22], this structure is an aggregation of partial models sharing a unique state vector. On the other hand, the decoupled multimodel proposed by Filev [4] involves decoupled partial models, each local model has its own state vector. Indeed, this introduces a degree of flexibility in the modeling stage. Also, this structure shows up partial models whose state vectors are with different sizes and therefore a structure that is different from conventional forms. In identification context, the decoupled multimodel provides additional degrees of freedom insofar as it enables the modeling of complex systems whose structures may vary in terms of the operating mode.

Consider a discrete-time nonlinear system described by the following decoupled multimodel:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u(k) \\ y_i(k) = C_i x_i(k) \\ y(k) = \sum\limits_{i=1}^{Nm} \mu_i(z(k)) C_i x_i(k) \end{cases} \tag{1}$$

where:

$x_i \in \Re^{n_i}$ et $y_i \in \Re^p$ are the state and the output vectors of the i^{th} partial model, respectively.

$u \in \Re^m$ et $y \in \Re^p$ are the input and the output vectors, respectively.

N_m is the number of partial models.

$$\mu_i(z(k)) = \frac{e^{\left(\frac{-(z(k)-c_i)^2}{\sigma_d^2}\right)}}{\sum\limits_{i=1}^{N_m} e^{\left(\frac{-(z(k)-c_i)^2}{\sigma_d^2}\right)}}, \quad i = 1, 2, ..., N_m \tag{2}$$

c_i are the centers and σ_d is the dispersion.

The $\mu_i(z(k))$ are the weighting functions that ensure the transition between the partial models. The properties of the weighting functions are given by the following expressions:

$\sum\limits_{i=1}^{N_m} \mu_i(z(k)) = 1, \quad 0 \leq \mu_i(z(k)) \leq 1, \quad \forall i = 1, \ldots, N_m, \quad \forall k$

$z(k)$ is the decision variable. Several choices of this premise variable is possible, it depends on measurable variables such as the input, the output or the measurable states of the system.

Consider the equation (1). In order to rewrite this equation in a compact form, the vector $x(k)$ can be defined as follows:

$$x(k) = \left[x_1^T(k) \cdots x_i^T(k) \cdots x_{N_m}^T(k)\right]^T \in \Re^n, \qquad n = \sum_{i=1}^{N_m} n_i$$

Thus, the system (1) can be written in the following compact form:

$$\begin{cases} x(k+1) = \mathbb{A}x(k) + \mathbb{B}u(k) \\ y(k) = \mathbb{C}(k)x(k) \end{cases} \tag{3}$$

where matrices $\mathbb{A} \in \Re^{n\times n}$, $\mathbb{B} \in \Re^{n\times m}$ and $\mathbb{C} \in \Re^{p\times n}$ are defined as follows:

$$\mathbb{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_{N_m} \end{bmatrix},$$

$$\mathbb{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_{N_m} \end{bmatrix} \quad \text{and} \quad \mathbb{C}(k) = \begin{bmatrix} \mu_1(k)\, C_1^T \\ \vdots \\ \mu_i(k)\, C_i^T \\ \vdots \\ \mu_{N_m}(k)\, C_{N_m}^T \end{bmatrix}$$

The matrix $\mathbb{C}(k)$ can be written as follows:

$$\mathbb{C}(k) = \sum_{i=1}^{N_m} \mu_i(z(k))\mathbb{C}_i$$

where $\mathbb{C}_i$ is a bloc matrix of the form:

$$\mathbb{C}_i = [0 \ldots C_i \ldots 0]$$

2.2 Decoupled multimodel with delayed output measurements

Consider a discrete-time nonlinear system with time-varying delay on the measurements described by the following decoupled multimodel:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u(k) \\ y(k) = \sum_{i=1}^{N_m} \mu_i(z(k-d(k)))C_i x_i(k-d(k)) \end{cases} \tag{4}$$

where $d(k)$ is a time-varying delay on output measurements and assumed known, it satisfies the following condition:

$$d_{min} \leq d(k) \leq d_{max} \tag{5}$$

with d_{min} and d_{max} are constant positive scalars denoting the lower and the upper time-delays, respectively.
The state estimation of the system (4) is ensured by the following multi-observer:

$$\begin{cases} \hat{x}_i(k+1) = A_i \hat{x}_i(k) + B_i u(k) + L_i(y(k) - \hat{y}(k)) \\ \hat{y}(k) = \sum_{i=1}^{N_m} \mu_i(z(k-d(k)))C_i \hat{x}_i(k-d(k)) \end{cases} \tag{6}$$

The system (4) can be written in a compact form:

$$\begin{cases} x(k+1) = \mathbb{A}x(k) + \mathbb{B}u(k) \\ y(k) = \mathbb{C}(k-d(k))x(k-d(k)) \end{cases} \tag{7}$$

where

$$\mathbb{C}(k-d(k)) = \sum_{i=1}^{N_m} \mu_i\left(z\left(k-d(k)\right)\right)\mathbb{C}_i$$

The compact form of the multi-observer (6) is given by the following expression:

$$\begin{cases} \hat{x}\left(k+1\right) = \mathbb{A}\hat{x}(k) + \mathbb{B}u\left(k\right) + \mathbb{L}(y(k) - \hat{y}(k)) \\ \hat{y}\left(k\right) = \mathbb{C}(k-d(k))\hat{x}\left(k-d(k)\right) \end{cases} \tag{8}$$

The estimation error is defined as follows:

$$\tilde{x}(k+1) = x(k+1) - \hat{x}(k+1) \tag{9}$$

The purpose of this paper is to investigate the state estimation of discrete-time nonlinear systems with delayed measurements described by a decoupled multi-model. The convergence conditions of the multi-observer are formulated using Lyapunov-Krasovskii functionals.

3 A multi-observer synthesis for decoupled multimodel with time-varying delay

In physical systems, the state variables are often unmeasurable in practice and no information on them are available. So, it is necessary to estimate their values using the available information on the input and output.
The convergence conditions are formulated in terms of linear matrix inequalities given in the following theorem:
Theorem 1: Consider the system (7) and the multi-observer (8). The estimation error is asymptotically stable if there exist symmetric matrices ($P>0$) and ($Q>0$) and a matrix X such that the following linear matrix inequalities hold:

$$\begin{bmatrix} \mathbb{A}^T P\mathbb{A} + (d_{max} - d_{min} + 1)Q - P & -\mathbb{A}^T X\mathbb{C}_i & 0 \\ \mathbb{C}_i^T X^T\mathbb{A} & -Q & \mathbb{C}_i^T X^T \\ 0 & X\mathbb{C}_i & -P \end{bmatrix} < 0 \tag{10}$$

$$i = 1...N_m$$

The observer gain is $\mathbb{L} = P^{-1}X$.

Proof:The estimation error dynamic is defined as :

$$\begin{aligned} \tilde{x}(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= \mathbb{A}\tilde{x}(k) - \mathbb{L}\mathbb{C}(k-d(k))\tilde{x}(k-d(k)) \end{aligned} \tag{11}$$

Consider the following Lyapunov-Krasovskii functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{12}$$

with

$$V_1(k) = \tilde{x}^T(k)P\tilde{x}(k) \tag{13}$$

$$V_2(k) = \sum_{i=k-d(k)}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) \tag{14}$$

$$V_3(k) = \sum_{j=-d_{\max}+2}^{-d_{\min}+1} \sum_{i=k+j-1}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) \tag{15}$$

The forward difference $\varDelta V_1(k)$ is obtained by the following expression:

$$\begin{aligned} \varDelta V_1(k) =& V_1(k+1) - V_1(k) \\ =& \tilde{x}^T(k+1)P\tilde{x}(k+1) - \tilde{x}^T(k)P\tilde{x}(k) \\ =& \tilde{x}_a^T(k)\varGamma(k)\tilde{x}_a(k) \end{aligned} \tag{16}$$

where $\tilde{x}_a\left(k\right) = \begin{bmatrix} \tilde{x}(k) \\ \tilde{x}(k-d(k)) \end{bmatrix}$

$\varGamma\left(k\right) =$
$\begin{bmatrix} \mathbb{A}^T P\mathbb{A} - P & -\mathbb{A}^T P\mathbb{L}\mathbb{C}\left(k-d(k)\right) \\ -\mathbb{C}^T\left(k-d(k)\right)\mathbb{L}^T P\mathbb{A} & \mathbb{C}^T\left(k-d(k)\right)\mathbb{L}^T P\mathbb{L}\mathbb{C}\left(k-d(k)\right) \end{bmatrix}$

The forward difference $\varDelta V_2(k)$ is:

$$\begin{aligned} \varDelta V_2(k) =& V_2(k+1) - V_2(k) \\ =& \tilde{x}^T(k)Q\tilde{x}(k) - \tilde{x}^T(k-d(k))Q\tilde{x}(k-d(k)) \\ &+ \sum_{i=k+1-d(k+1)}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) - \sum_{i=k+1-d(k)}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) \end{aligned} \tag{17}$$

It is obvious that:

$$\begin{aligned} \sum_{i=k+1-d(k+1)}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) =& \sum_{i=k+1-d(k+1)}^{k-d_{\min}} \tilde{x}^T(i)Q\tilde{x}(i) \\ &+ \sum_{i=k+1-d_{\min}}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) \\ &\le \sum_{i=k+1-d_{\max}}^{k-d_{\min}} \tilde{x}^T(i)Q\tilde{x}(i) \\ &+ \sum_{i=k+1-d(k)}^{k-1} \tilde{x}^T(i)Q\tilde{x}(i) \end{aligned} \tag{18}$$

then

$$\begin{aligned} \varDelta V_2(k) \le& \tilde{x}^T(k)Q\tilde{x}(k) - \tilde{x}^T(k-d(k))Q\tilde{x}(k-d(k)) \\ &+ \sum_{i=k+1-d_{\max}}^{k-d_{\min}} \tilde{x}^T(i)Q\tilde{x}(i) \end{aligned} \tag{19}$$

The forward difference $\Delta V_3(k)$ is:

$$\begin{aligned}\Delta V_3(k) =& V_3(k+1) - V_3(k) \\ =& \tilde{x}^T(k)(d_{\max} - d_{\min})Q\tilde{x}(k) - \sum_{i=k+1-d_{\max}}^{k-d_{\min}} \tilde{x}^T(i)Q\tilde{x}(i)\end{aligned} \tag{20}$$

Thus, the forward difference of the Lyapunov functional $V(k)$ is given by the following equation:

$$\begin{aligned}\Delta V(k) =& \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \\ & \leq \tilde{x}_a^T(k)\Gamma(k)\tilde{x}_a(k) + \bar{d}\tilde{x}^T(k)Q\tilde{x}(k) \\ & - \tilde{x}^T(k-d(k))Q\tilde{x}(k-d(k))\end{aligned} \tag{21}$$

then

$$\Delta V(k) \leq \tilde{x}_a^T(k)\Omega(k)\tilde{x}_a(k) \tag{22}$$

where $\Omega(k) =$
$\begin{bmatrix} \mathbb{A}^T P\mathbb{A} - P + \bar{d}Q & -\mathbb{A}^T P\mathbb{L}\mathbb{C}\,(k-d(k)) \\ -\mathbb{C}^T\,(k-d(k))\,\tilde{L}^T P\tilde{A}\,\mathbb{C}^T\,(k-d(k))\,\mathbb{L}^T P\mathbb{L}\mathbb{C}\,(k-d(k)) - Q \end{bmatrix}$

and $\bar{d} = (d_{\max} - d_{\min} + 1)$

Applying Schur Complement and exploiting weighting functions properties, the following linear matrix inequalities hold.

$$\begin{bmatrix} \mathbb{A}^T P\mathbb{A} + \bar{d}Q - P & -\mathbb{A}^T X\mathbb{C}_i & 0 \\ \mathbb{C}_i^T X^T\mathbb{A} & -Q & \mathbb{C}_i^T X^T \\ 0 & X\mathbb{C}_i & -P \end{bmatrix} < 0 \tag{23}$$

$$i = 1...N_m$$

4 Simulation results

Consider a manipulator arm with revolute joints actuated by a DC motor. This system is discretized using the Euler discretization with a sampling period $T_s = 0.1$ [16].

$$\begin{cases} x_1(k+1) = x_1(k) + T_s x_3(k) \\ x_2(k+1) = x_2(k) + T_s(-48.6x_1(k) - 1.25x_2(k) \\ \qquad\qquad +48.6x_3(k) + 21.6u(k)) \\ x_3(k+1) = x_3(k) + T_s(x_4(k) - 3.33\sin(x_3(k)) \\ x_4(k+1) = x_4(k) + T_s(1.95x_1(k) - 1.95x_3(k)) \\ y(k) = x_2(k) \end{cases} \tag{24}$$

where $x_1(k)$ and $x_2(k)$ are the angular rotation and the angular velocity of the motor, respectively. $x_3(k)$ and $x_4(k)$ stand for the angular position and the angular velocity of the link, respectively.

Generally, the representation of a nonlinear system in local models is ensured by several methods. In this work, an identification method is adopted. This method necessitates the collection of input/output measurements for different operating points, the identification of the multimodel structure (partition of the operating space, determination of models' number, choice of decision variable), the determination of unknown parameters and the validation of the obtained multimodel on an other data set. The optimisation of the parameters is ensured by the Levenberg-Marquardt algorithm [12] which combines the gradient and Gauss-Newton algorithms in order to guarantee both the stability and the rapidity of the convergence.
The identification of the system (24) can be modeled by a decoupled multimodel including three partial models given as follows:

Partial model 1:

$$A_1 = \begin{bmatrix} 0.6932 \; 0.4883 \\ 0.2570 \; 0.1994 \end{bmatrix}, B_1 = \begin{bmatrix} 2.2278 \\ 0.7960 \end{bmatrix}, C_1 = \begin{bmatrix} 1 \; 0 \end{bmatrix}.$$

Partial model 2:

$$A_2 = \begin{bmatrix} 0.6430 \; 0.4569 \\ 0.3850 \; 0.1755 \end{bmatrix}, B_2 = \begin{bmatrix} 1.5408 \\ 0.5050 \end{bmatrix}, C_2 = \begin{bmatrix} 1 \; 0 \end{bmatrix}.$$

Partial model 3:

$$A_3 = \begin{bmatrix} 0.5078 \; 0.6999 \\ 0.2000 \; 0.4244 \end{bmatrix}, B_3 = \begin{bmatrix} 2.9129 \\ 1.5525 \end{bmatrix}, C_3 = \begin{bmatrix} 1 \; 0 \end{bmatrix}.$$

The decision variable is assumed to be the input signal $u(k)$.
Centers and dispersion have the following values:
$c_1 = -0.7, \quad c_2 = 0, \quad c_3 = 0.7, \quad \text{and} \quad \sigma_d = 0.4$
Hereinafter, the state estimation of the above system (24) that is described by the three partial models will be investigated.
The evolution of the time-delay is expressed in multiple integer of the sampling period and has the following expression:

$$d(k) = 6 + 5sin(0.015k)$$

The evolution of the input signal is given by the following expression:

$$u(k) = 0.9 \sin (2k\pi/250)$$

Applying Theorem 1, the following observer gain is obtained:

$$\mathbb{L} = \begin{bmatrix} 0.0077 \\ 0.0189 \\ 0.0082 \\ 0.0174 \\ 0.0154 \\ 0.0124 \end{bmatrix}$$

Figure 1 illustrates the evolution of weighting functions:

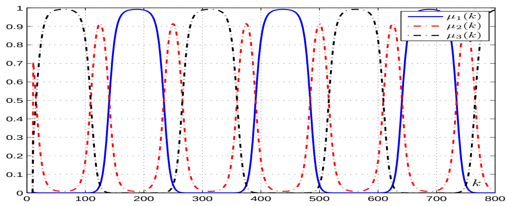

Fig. 1. Evolution of weighting functions.

The evolutions of the real and estimate outputs and the estimation error are shown in Figure 2:

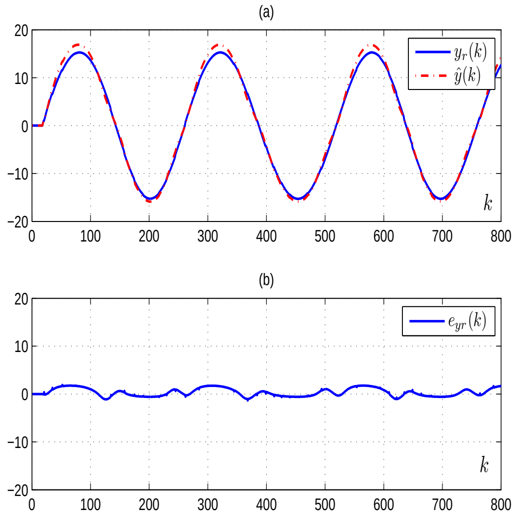

Fig. 2. (a) Evolutions of the real and estimate outputs (b) Evolution of the estimation error.

Figures 3, 4, 5, 6, 7 and 8 show the evolutions of the states 1, 2, 3, 4, 5 and 6 and their estimates.

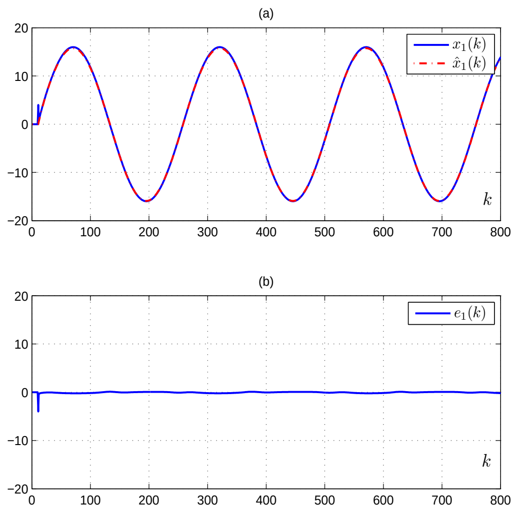

Fig. 3. (a) Evolutions of the state 1 and its estimate (b) Evolution of the estimation error.

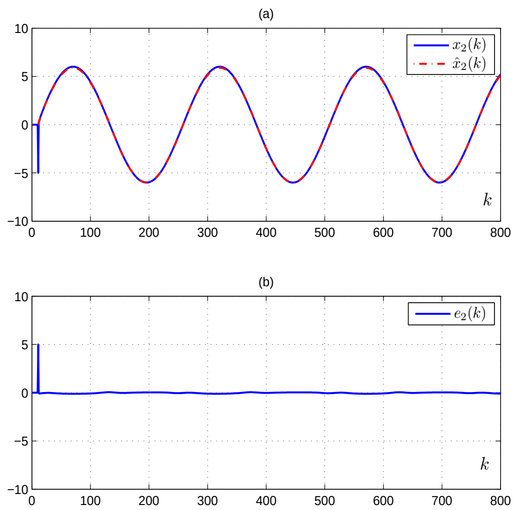

Fig. 4. (a) Evolutions of the state 2 and its estimate (b) Evolution of the estimation error.

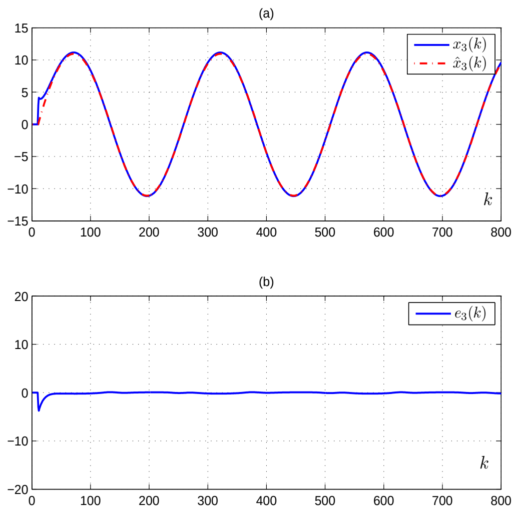

Fig. 5. (a) Evolutions of the state 3 and its estimate (b) Evolution of the estimation error.

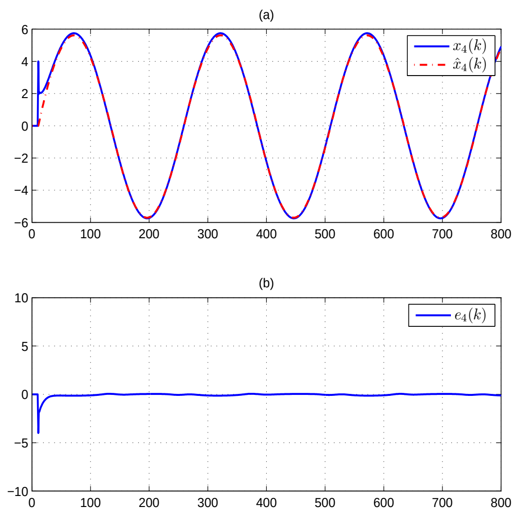

Fig. 6. (a) Evolutions of the state 4 and its estimate (b) Evolution of the estimation error.

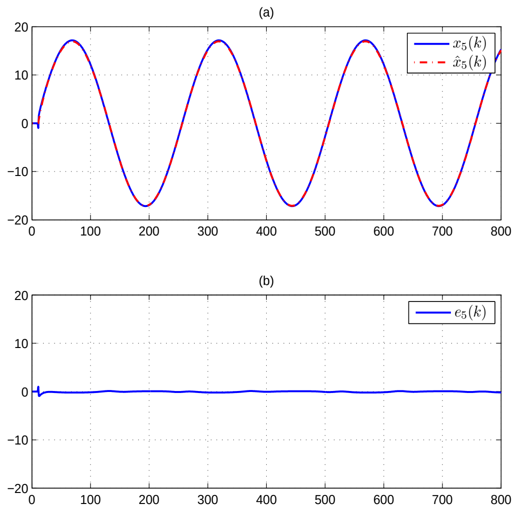

Fig. 7. (a) Evolutions of the state 5 and its estimate (b) Evolution of the estimation error.

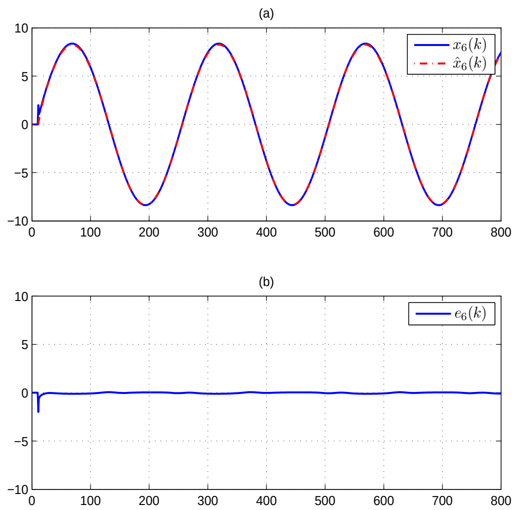

Fig. 8. (a) Evolutions of the state 6 and its estimate (b) Evolution of the estimation error.

It can be easily seen that the state estimates converge rapidly with a good accuracy to the real states due to the proposed multi-observer.

5 Conclusion

In this paper, we have designed an multi-observer for state estimation of discrete-time nonlinear systems subject to delayed output measurements. The convergence conditions of the estimation error have been formulated in terms of linear matrix inequalities. A numerical simulation on a manipulator arm has provided satisfactory results in terms of estimation error convergence which shows the effectiveness of the proposed delay-dependant conditions.

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