International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 8, N°1, April 2014, pp. 1998–2011.

Robust H_∞ Filtering for 2-D Discrete Fornasini-Marchesini Systems

B. BOUKILI A. HMAMED F. TADEO

Abstract

The robust H_∞ filtering problem for two-dimensional (2-D) systems described by uncertain Fornasini-Marchesini models is studied. Attention focuses on the design of H_∞ filters such that the filter error system is asymptotically stable and preserves a guaranteed H_∞ performance. By using the homogeneous polynomially parameter-dependent approach and adding slack matrix variables, the coupling between the Lyapunov matrix and the system matrices is eliminated. Then, a linear matrix inequality (LMI)-based approach is developed for designing the H_∞ filter. An illustrative example shows the effectiveness of this approach.

Key words – Robustness, 2-D systems, H_∞ filtering, Linear Matrix Inequalities (LMI), Uncertain systems, Fornasini-Marchesini model.

1 Introduction

In recent years, the control and filtering problems for 2-D systems have drawn considerable attention, as 2-D systems have important applications in the areas of multidimensional digital filtering, image data processing and transmission, thermal process modeling, etc. A number of important results have been obtained so far. To mention a few, the stability analysis and stabilization for 2-D systems has been investigated in [5, 7, 8, 13, 14, 15, 17], the robust H_∞ filtering for 2-D systems in [3, 4, 6, 9, 12, 19], the H_∞ filtering for 2-D systems with time delays in [10, 11, 20] and the H_∞ filtering for Markovian jump parameter systems in [16].
These previous results on robust H_∞ filtering problem for 2-D systems are

[*]BOUKILI Bensalem and HMAMED Abdelaziz are with Department of Physics, Faculty of Sciences Dhar El Mehraz, B.P. 1796 Fes-Atlas, Morocco. (e-mail:{b_boukili, hmamed_abdelaziz}@yahoo.fr).

[†]TADEO Fernando is with the Department of Systems Engineering and Automatic Control, University of Valladolid, 47005 Valladolid, Spain (e-mail: fernando@autom.uva.es).

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

mostly based on the method of quadratic stability conditions, and are hence inevitably conservative, since the same Lyapunov functions are used for the entire uncertainty domain. To overcome this, one possible way that has been well-recognized is to consider a parameter-dependent Lyapunov function whose aim is to reduce the over design in the quadratic framework. The basic idea is to decouple the product terms between the Lyapunov matrix and system matrices by introducing slack matrix variables to the well-established linear matrix inequality (LMI) performance conditions.
Thus, we study in this paper, the problem of robust H_∞ filtering for 2-D discrete system in Fornasini-Marchesini model with parameter uncertainties, that belong to polytopes. The key in our approach is to utilize slack variables and the polynomially parameter-dependent idea. The reported results are based on homogenous polynomially parameter-dependent matrices of on arbitrary degree. It is proved that as the degree grows, increasing precision is obtained, providing less conservative filter designs. The proposed condition include results in the quadratic framework (that entail fixed matrices for the entire uncertainty domain), and the linearly parameter-dependent framework (that use linear convex combinations of matrices) as special cases.
The theoretical results are given as LMIs conditions, which can be solved by standard numerical software, as illustrated in the example at the end of the paper.
Notations : The notation used throughout the paper is standard. The superscript T stands for matrix transposition. The notation $P > 0$ means that P is real symmetric and positive definite. I is the identity matrix with appropriate dimension. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry, and diag{...} stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2 Problem Description

Consider the 2-D discrete systems described by the Fornasini-Marchesini model:

$$\begin{aligned} x(i+1,j+1) &= A_{1\alpha}x(i,j+1) + A_{2\alpha}x(i+1,j) + B_{1\alpha}w(i,j+1) + B_{2\alpha}w(i+1,j) \\ y(i,j) &= C_\alpha x(i,j) + D_\alpha w(i,j) \\ z(i,j) &= L_\alpha x(i,j) \end{aligned} \tag{1}$$

where $x(i,j) \in R^n$ is the state vector, $y(i,j) \in R^m$ is the measured output vector, and $z(i,j) \in R^p$ is the signal to be estimated, $w(i,j) \in R^q$ is the disturbance input vector which belongs to $L_2\{[0,\infty),[0,\infty)\}$. The system matrices are supposed to be unknown but belong to a given convex bounded polyhedral domain, namely

$$\begin{aligned} \Omega_\alpha &= (A_{1\alpha}, A_{2\alpha}, B_{1\alpha}, B_{2\alpha}, C_\alpha, D_\alpha, L_\alpha) \in \mathcal{R} \\ \mathcal{R} &= \{\Omega_\alpha | \Omega_\alpha = \sum_{i=1}^{s} \alpha_i \Omega_i; \alpha \in \Gamma\} \end{aligned} \tag{2}$$

with $\Omega_i = (A_{1i},\ A_{2i},\ B_{1i},\ B_{2i},\ C_i,\ D_i,\ L_i)$ denoting the vertices of the polytope, and Γ the unit simplex:

$$\Gamma = \{(\alpha_1, \alpha_2, ..., \alpha_s) : \sum_{i=1}^{s} \alpha_i = 1, \alpha_i > 0\}. \tag{3}$$

The boundary condition of the state vector is supposed to be

$$lim_{n \mapsto \infty} \sum_{k=1}^{n} (|x(0,k)|^2 + |x(k,0)|^2) < \infty. \tag{4}$$

Here, we are interested in estimating the signal $z(i,j)$ by a robust filter of the form

$$\begin{aligned} \tilde{x}(i+1,j+1) &= A_{f1}\tilde{x}(i,j+1) + A_{f2}\tilde{x}(i+1,j) + B_{f1}y(i,j+1) + B_{f2}y(i+1,j) \\ \tilde{z}(i,j) &= C_f \tilde{x}(i,j) \end{aligned} \tag{5}$$

where $\tilde{x}(i,j) \in R^{n_f}$ is the state vector of the filter, and $\tilde{z}(i,j) \in R^p$ is the estimation of $z(i,j)$.
If the augmented state vector is $\xi(i,j) = [x^T(i,j)\ \ \tilde{x}^T(i,j)]^T$ and the estimation error is $e(i,j) = z(i,j) - \tilde{z}(i,j)$, then the filtering error system can be written as follows:

$$\begin{aligned} \xi(i+1,j+1) &= \bar{A}_\alpha \bar{\xi}(i,j) + \bar{B}_\alpha \bar{w}(i,j) \\ \bar{e}(i,j) &= \bar{C}_\alpha \bar{\xi}(i,j) \end{aligned} \tag{6}$$

where

$$\bar{A}_\alpha = \left[\ \bar{A}_{1\alpha} \mid \bar{A}_{2\alpha}\ \right] = \left[\begin{array}{cc|cc} A_{1\alpha} & 0 & A_{2\alpha} & 0 \\ B_{f1}C_\alpha & A_{f1} & B_{f2}C_\alpha & A_{f2} \end{array}\right],\ \bar{C}_\alpha = \left[\begin{array}{c|c} L_\alpha - C_f & 0 \\ \hline 0 & L_\alpha - C_f \end{array}\right],$$

$$\bar{B}_\alpha = \left[\ \bar{B}_{1\alpha} \mid \bar{B}_{2\alpha}\ \right] = \left[\begin{array}{c|c} B_{1\alpha} & B_{2\alpha} \\ B_{f1}D_\alpha & B_{f2}D_\alpha \end{array}\right].$$

and

$$\bar{\xi}(i,j) = \begin{bmatrix} \xi(i,j+1) \\ \xi(i+1,j) \end{bmatrix},\ \bar{w}(i,j) = \begin{bmatrix} w(i,j+1) \\ w(i+1,j) \end{bmatrix},\ \bar{e}(i,j) = \begin{bmatrix} e(i,j+1) \\ e(i+1,j) \end{bmatrix}. \tag{7}$$

The transfer function of the filtering error system is then

$$\|T_{ew}(z_1, z_2, \alpha) = \bar{C}_\alpha[z_1 z_2 I_{2n} - z_2\bar{A}_{1\alpha} - z_1\bar{A}_{2\alpha}]^{-1}[z_2\bar{B}_{1\alpha} + z_1\bar{B}_{2\alpha}] \tag{8}$$

Thus, the robust H_∞ filtering error problem can be given as follows.
Problem description. Given the system in (1) subject to parameter uncertain in (2), determine the filter of the form (5), such that the filter error system (6) is robustly asymptotically stable for all $\alpha \in \Gamma$ and satisfies

$$\|T_{ew}(z_1, z_2, \alpha)\|_\infty < \gamma \quad (\forall \alpha \in \Gamma) \tag{9}$$

where γ is a given positive scalar and $\|T_{ew}(z_1, z_2, \lambda)\|_\infty < \gamma$ is defined as the H_∞ performance of the filtering error system in (6).
Before deriving our main results, we give the following Lemmas.

Lemma 2.1. *[1] Given the 2-D FM system in (1) and the filter in (5), for any fixed $\alpha \in \Gamma$, the filtering error system in (6) is asymptotically stable and satisfies (9) if there exist matrices $P_\alpha \in \mathbb{R}^{r\times r} > 0$ and $S_\alpha \in \mathbb{R}^{r\times r} > 0$ satisfying*

$$\begin{bmatrix} -R_\alpha & \bar{A}_\alpha^T P_\alpha & 0_{2r\times 2m} & \bar{C}_\alpha^T \\ * & -P_\alpha & P_\alpha \bar{B}_\alpha & 0_{r\times 2p} \\ * & * & -\gamma^2 I_{2m} & 0_{2m\times 2p} \\ * & * & * & -I_{2p} \end{bmatrix} < 0 \tag{10}$$

where $R_\alpha = diag\{P_\alpha - S_\alpha\ ,\ S_\alpha\}$ and $r = n + n_f$

Lemma 2.2. *[18] Let $\xi \in \mathbb{R}^n$, $\mathcal{Q} \in \mathbb{R}^{n\times n}$ and $\mathcal{B} \in \mathbb{R}^{m\times n}$ with rank $(\mathcal{B}) < n$ and $(\mathcal{B})^\perp$ such that $\mathcal{B}\mathcal{B}^\perp = 0$. Then, the following conditions are equivalent:*

1. $\xi^T \mathcal{Q}\xi < 0, \forall \xi \neq 0 : \mathcal{B}\xi = 0$
2. $\mathcal{B}^{\perp T}\mathcal{Q}\mathcal{B}^\perp < 0$
3. $\exists \mu \in \mathbb{R} : \mathcal{Q} - \mu\mathcal{B}^T\mathcal{B} < 0$
4. $\exists \mathcal{X} \in \mathbb{R}^{n\times m} : \mathcal{Q} + \mathcal{X}\mathcal{B} + \mathcal{B}^T\mathcal{X}^T < 0$

3 H_∞ Filtering Analysis

In this section, we assume that the filter matrices in (5) are known and we will study the condition under which the filter error system (6) is asymptotically stable with H_∞-norm bounded γ. Based on Lemma 2.1, we devote ourselves to the design of robust H_∞ filters. We first give the following proposition.

Proposition 3.1. *Given the 2-D FM system in (1) and the filter (5), for any fixed* $\alpha \in \Gamma$*, the filtering error system in (6) is asymptotically stable and satisfies (9) if there exist matrices* $P_\alpha \in \mathbb{R}^{r\times r} > 0$*,* $S_\alpha \in \mathbb{R}^{r\times r} > 0$*,* $K_\alpha \in \mathbb{R}^{2r\times r}$*,* $E_\alpha \in \mathbb{R}^{r\times r}$*,* $Q_\alpha \in \mathbb{R}^{2m\times r}$*,* $F_\alpha \in \mathbb{R}^{2p\times r}$*,* $G_\alpha \in \mathbb{R}^{2r\times 2p}$*,* $H_\alpha \in \mathbb{R}^{r\times 2p}$*,* $M_\alpha \in \mathbb{R}^{2m\times 2p}$*, and* $N_\alpha \in \mathbb{R}^{2p\times 2p}$*, satisfying*

$$\begin{bmatrix} \Gamma_{11} & -K_\alpha + \bar{A}_\alpha^T E_\alpha^T + \bar{C}_\alpha^T H_\alpha^T & K_\alpha \bar{B}_\alpha + \bar{A}_\alpha^T Q_\alpha^T + \bar{C}_\alpha^T M_\alpha^T & -G_\alpha + \bar{A}_\alpha^T F_\alpha^T + \bar{C}_\alpha^T N_\alpha^T + \bar{C}_\alpha^T \\ * & P_\alpha - E_\alpha - E_\alpha^T & E_\alpha \bar{B}_\alpha - Q_\alpha^T & -H_\alpha - F_\alpha^T \\ * & * & Q_\alpha \bar{B}_\alpha + \bar{B}_\alpha^T Q_\alpha^T - \gamma^2 I_{2m} & \bar{B}_\alpha^T F_\alpha^T - M_\alpha \\ * & * & * & -I_{2p} - N_\alpha - N_\alpha^T \end{bmatrix} < 0 \quad (11)$$

where $\Gamma_{11} = K_\alpha \bar{A}_\alpha + \bar{A}_\alpha^T K_\alpha^T + G_\alpha \bar{C}_\alpha + \bar{C}_\alpha^T G_\alpha^T - R_\alpha$.

Proof. The equivalence is obtained by considering

$$\mathcal{Q} = \begin{bmatrix} -R_\alpha & 0_{2r\times r} & 0_{2r\times 2m} & \bar{C}^T \\ 0_{r\times 2r} & P_\alpha & 0_{r\times 2m} & 0_{r\times 2p} \\ 0_{2m\times 2r} & 0_{2m\times r} & -\gamma^2 I_{2m} & 0_{2m\times 2p} \\ \bar{C} & 0_{2p\times r} & 0_{2p\times 2m} & -I_{2p} \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} \bar{A}_\alpha & -I_r & \bar{B}_\alpha & 0_{r\times 2p} \\ \bar{C}_\alpha & 0_{2p\times r} & 0_{2p\times 2m} & -I_{2p} \end{bmatrix},$$

$$\mathcal{X} = \begin{bmatrix} X_\alpha & Y_\alpha \end{bmatrix}, \text{ where } X_\alpha = \begin{bmatrix} K_\alpha \\ E_\alpha \\ Q_\alpha \\ F_\alpha \end{bmatrix}, \in \mathbb{R}^{(3r+2m+2p)\times r},$$

$$Y_\alpha = \begin{bmatrix} G_\alpha \\ H_\alpha \\ M_\alpha \\ N_\alpha \end{bmatrix} \in \mathbb{R}^{(3r+2m+2p)\times(2p)},$$

in condition (4) of Lemma 2.2, with

$$\mathcal{B}^\perp = \begin{bmatrix} I_{2r} & 0_{2r\times 2m} \\ \bar{A}_\alpha & \bar{B}_\alpha \\ 0_{2m\times 2r} & I_{2m} \\ \bar{C}_\alpha & 0_{2p\times 2m} \end{bmatrix}.$$

Using condition (2) of Lemma 2.2, this gives condition (10), which completes the proof. □

Remark 3.2. *In Proposition 3.1, the slack variables* X_α *and* Y_α *are introduced. By setting* $Y_\alpha = 0$*, Proposition 3.1 coincides with the previous results for 1-D discrete systems [2]. Thus, Proposition 3.1 would generally render a* *less conservative evaluation the upper bound of the H_∞ norm in 2-D systems FM model case, which can be seen from the numerical example later in the paper.*

4 H_∞ Filter Design

In this section, a procedure will be established for designing a H_∞ filter in (5), that is, to determine the filter matrices in (5) such that the filter error system (6) is asymptotically stable with H_∞-norm bounded γ.
Based on Proposition 3.1, we select for variables K_α, E_α, Q_α and F_α the following structures [2, 18]:

$$\begin{aligned} I(i,j) &= \begin{bmatrix} \lambda_i I & 0 \\ 0 & \lambda_j I \end{bmatrix}, \bar{\bar{K}} = \begin{bmatrix} \hat{K} \\ \hat{K} \end{bmatrix}, \bar{F}_\alpha = \begin{bmatrix} F_{1\alpha} \\ F_{2\alpha} \end{bmatrix}, K_{11\alpha} = \begin{bmatrix} K_{1\alpha} \\ K_{2\alpha} \end{bmatrix}, K_{21\alpha} = \begin{bmatrix} K_{3\alpha} \\ K_{4\alpha} \end{bmatrix}, \\ \bar{Q}_\alpha &= \begin{bmatrix} Q_{1\alpha} \\ Q_{2\alpha} \end{bmatrix}, E_\alpha = \begin{bmatrix} E_{1\alpha} & \hat{K} \\ E_{2\alpha} & \lambda_1 \hat{K} \end{bmatrix}, K_\alpha = \begin{bmatrix} K_{11\alpha} & I(2,3)\bar{\bar{K}} \\ K_{21\alpha} & I(4,5)\bar{\bar{K}} \end{bmatrix}, \\ Q_\alpha &= \begin{bmatrix} \bar{Q}_\alpha & 0 \end{bmatrix}, F_\alpha = \begin{bmatrix} \bar{F}_\alpha & 0 \end{bmatrix}, \; N_\alpha = 0, \; G_\alpha = 0, \; H_\alpha = 0, \; M_\alpha = 0. \end{aligned} \tag{12}$$

The matrices P_α and S_α are also partitioned in $n \times n$ blocks as follows

$$P_\alpha = \begin{bmatrix} P_{1\alpha} & P_{2\alpha} \\ P_{2\alpha}^T & P_{3\alpha} \end{bmatrix} \quad , \quad S_\alpha = \begin{bmatrix} S_{1\alpha} & S_{2\alpha} \\ S_{2\alpha}^T & S_{3\alpha} \end{bmatrix}, \tag{13}$$

and the following change of variables is adopted:

$$\begin{bmatrix} \hat{A}_{f1} & \hat{B}_{f1} \\ \hat{A}_{f2} & \hat{B}_{f2} \\ \hat{C}_f & 0 \end{bmatrix} = \begin{bmatrix} \hat{K} & 0 & I \\ 0 & \hat{K} & 0 \end{bmatrix} \begin{bmatrix} A_{f1} & B_{f1} & 0 \\ A_{f2} & B_{f2} & 0 \\ 0 & 0 & C_f \end{bmatrix},$$

where $E_{1\alpha}$, $E_{2\alpha}$, $K_{11\alpha}$, $K_{21\alpha}$, Q_α, and F_α are supposed to depend only on the parameter α, while $\hat{K}$ is supposed to be fixed for the entire uncertainty domain and, without loss of generality, invertible. The scalar parameters λ_i, $i = 1...5$, will be searched through the entire uncertainty domain as part of optimization problems.

Theorem 4.1. *If there exist symmetric parameter-dependent positive definite matrices P_α , S_α as in (13) and parameter-dependent matrices K_α, E_α, Q_α, and F_α as in (12), $\bar{A}_{f1}$, $\bar{A}_{f2}$, $\bar{B}_{f1}$, $\bar{B}_{f2}$, $\bar{C}_f$, $\gamma > 0$ and scalars λ_i,*

$i = 1, ..., 5,$ *such that*

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & A_{1\alpha}^T F_{1\alpha}^T \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \hat{A}_{f2}^T - K_{2\alpha} & \Psi_{26} & \Psi_{27} & \Psi_{28} & -\hat{C}_f^T & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & A_{2\alpha}^T F_{1\alpha}^T & \Psi_{310} \\ * & * & * & \Psi_{44} & \hat{A}_{f2}^T - K_{4\alpha} & \Psi_{46} & \Psi_{47} & \Psi_{48} & 0 & -\hat{C}_f^T \\ * & * & * & * & \Psi_{55} & \Psi_{56} & \Psi_{57} & \Psi_{58} & -F_{1\alpha}^T & -F_{2\alpha}^T \\ * & * & * & * & * & \Psi_{66} & \Psi_{67} & \Psi_{68} & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & B_{1\alpha}^T F_{1\alpha}^T & B_{1\alpha}^T F_{2\alpha}^T \\ * & * & * & * & * & * & * & \Psi_{88} & B_{2\alpha}^T F_{1\alpha}^T & B_{2\alpha}^T F_{2\alpha}^T \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 (14)$$

where

$$\begin{array}{ll} \Psi_{11} = K_{1\alpha}A_{1\alpha} + A_{1\alpha}^T K_{1\alpha}^T + \lambda_2(\kappa \hat{B}_{f1} C_\alpha & \Psi_{12} = \lambda_2 \kappa \hat{A}_{f1} + A_{1\alpha}^T K_{2\alpha}^T + \lambda_3 C_\alpha^T \hat{B}_{f1}^T \\ \quad +C_\alpha^T \hat{B}_{f1}^T \kappa^T) + S_{1\alpha} - P_{1\alpha} & \quad +S_{2\alpha} - P_{2\alpha} \\ \Psi_{15} = A_{1\alpha}^T E_{1\alpha}^T + C_\alpha^T \hat{B}_{f1}^T - K_{1\alpha} & \Psi_{16} = A_{1\alpha}^T E_{2\alpha}^T + \lambda_1 C_\alpha^T \hat{B}_{f1}^T - \lambda_2 \hat{K} \\ \Psi_{17} = K_{1\alpha}B_{1\alpha} + \lambda_2 \hat{B}_{f1} D_\alpha + A_{1\alpha}^T Q_{1\alpha}^T & \Psi_{18} = K_{1\alpha}B_{2\alpha} + \lambda_2 \hat{B}_{f2} D_\alpha + A_{1\alpha}^T Q_{2\alpha}^T \end{array} \quad (15)$$

$$\begin{array}{ll} \Psi_{19} = A_{1\alpha}^T F_{1\alpha}^T + L_\alpha^T & \Psi_{22} = \lambda_3(\hat{A}_{f1} + \hat{A}_{f1}^T) + S_{3\alpha} - P_{3\alpha} \\ \Psi_{23} = K_{2\alpha}A_{2\alpha} + \lambda_3 \hat{B}_{f2} C_\alpha + \lambda_4 \hat{A}_{f1}^T & \Psi_{24} = \lambda_3 \hat{A}_{f2} + \lambda_5 \hat{A}_{f1}^T \\ \Psi_{26} = \lambda_1 \hat{A}_{f1} - \lambda_3 \hat{K} & \Psi_{27} = K_{2\alpha}B_{1\alpha} + \lambda_3 \hat{B}_{f1} D_\alpha \\ \Psi_{34} = \lambda_4 \hat{A}_{f2} + A_{2\alpha}^T K_{4\alpha}^T + \lambda_5 C_\alpha^T \hat{B}_{f2}^T - S_{2\alpha} & \Psi_{35} = A_{2\alpha}^T E_{1\alpha}^T + C_\alpha^T \hat{B}_{f2}^T - K_{3\alpha} \\ \Psi_{36} = A_{2\alpha}^T E_{2\alpha}^T + \lambda_1 C_\alpha^T \hat{B}_{f2}^T - \lambda_4 \hat{K} & \Psi_{37} = K_{3\alpha}B_{1\alpha} + \lambda_4 \hat{B}_{f1} D_\alpha + A_{2\alpha}^T Q_{1\alpha}^T \\ \Psi_{38} = K_{3\alpha}B_{2\alpha} + \lambda_4 \hat{B}_{f2} D_\alpha + A_{2\alpha}^T Q_{2\alpha}^T & \Psi_{310} = A_{2\alpha}^T F_{2\alpha}^T + L_\alpha^T \\ \Psi_{44} = \lambda_5(\hat{A}_{f2} + \hat{A}_{f2}^T) - S_{3\alpha} & \Psi_{46} = \lambda_1 \hat{A}_{f2}^T - \lambda_5 \hat{K} \\ \Psi_{47} = K_{4\alpha}B_{1\alpha} + \lambda_5 \hat{B}_{f1} D_\alpha & \Psi_{48} = K_{4\alpha}B_{2\alpha} + \lambda_5 \hat{B}_{f2} D_\alpha \\ \Psi_{55} = P_{1\alpha} - E_{1\alpha} - E_{1\alpha}^T & \Psi_{56} = P_{2\alpha} - \hat{K} - E_{2\alpha}^T \\ \Psi_{57} = E_{1\alpha}B_{1\alpha} + \hat{B}_{f1} D_\alpha - Q_{1\alpha}^T & \Psi_{58} = E_{1\alpha}B_{2\alpha} + \hat{B}_{f2} D_\alpha - Q_{2\alpha}^T \\ \Psi_{66} = P_{3\alpha} - \lambda_1(\hat{K} + \hat{K}^T) & \Psi_{67} = E_{2\alpha}B_{1\alpha} + \lambda_1 \hat{B}_{f1} D_\alpha \\ \Psi_{68} = E_{2\alpha}B_{2\alpha} + \lambda_1 \hat{B}_{f2} D_\alpha & \Psi_{77} = Q_{1\alpha}B_{1\alpha} + B_{1\alpha}^T Q_{1\alpha}^T - \gamma^2 I \\ \Psi_{28} = K_{2\alpha}B_{2\alpha} + \lambda_3 \hat{B}_{f2} D_\alpha & \Psi_{13} = K_{1\alpha}A_{2\alpha} + \lambda_2 \hat{B}_{f2} C_\alpha + A_{1\alpha}^T K_{3\alpha}^T + \lambda_4 C_\alpha^T \hat{B}_{f1}^T \\ \Psi_{78} = Q_{1\alpha}B_{2\alpha} + B_{1\alpha}^T Q_{2\alpha}^T & \Psi_{88} = Q_{2\alpha}B_{2\alpha} + B_{2\alpha}^T Q_{2\alpha}^T - \gamma^2 I \\ \Psi_{33} = K_{3\alpha}A_{2\alpha} + A_{2\alpha}^T K_{3\alpha}^T + \lambda_4(\hat{B}_{f2} C_\alpha & \Psi_{14} = \lambda_2 \hat{A}_{f2} + A_{1\alpha}^T K_{4\alpha}^T + \lambda_5 C_\alpha^T \hat{B}_{f1}^T \\ \quad +C_\alpha^T \hat{B}_{f2}^T) - S_{1\alpha} & \end{array} \quad (16)$$

holds for all $\alpha \in \Gamma$, then

$$\begin{bmatrix} A_{f1} & B_{f1} \\ A_{f2} & B_{f2} \\ C_f & 0 \end{bmatrix} = \begin{bmatrix} \hat{K}^{-1} & 0 & I \\ 0 & \hat{K}^{-1} & 0 \end{bmatrix} \begin{bmatrix} \hat{A}_{f1} & \hat{B}_{f1} & 0 \\ \hat{A}_{f2} & \hat{B}_{f2} & 0 \\ 0 & 0 & \hat{C}_f \end{bmatrix} \tag{17}$$

are the matrices of the robust stable filter that ensure a guaranteed cost H_∞ given by γ.

Remark 4.2. *When the scalars λ_1, λ_2, λ_3, λ_4 and λ_5 of Theorem* 4.1 *are fixed to be constants, then (*14*) is an LMI (it is linear in the variables). To select values for these scalars, optimization can be used (for example fminsearch in MATLAB) to improve some performance measures, such as the disturbance attenuation level γ.*

Remark 4.3. *In order to solve the parameter-dependent LMI conditions of Theorem 4.1, the technique proposed in [2] to handle parameter-dependent LMIs with parameters in the unit simplex can be applied. For this, the decision variables $P_{1\alpha}$, $P_{2\alpha}$, $P_{3\alpha}$, $S_{1\alpha}$, $S_{2\alpha}$, $S_{3\alpha}$, $E_{1\alpha}$, $E_{2\alpha}$, $K_{1\alpha}$, $K_{2\alpha}$, $K_{3\alpha}$, $K_{4\alpha}$, $Q_{1\alpha}$, $Q_{2\alpha}$, $F_{1\alpha}$ and $F_{2\alpha}$) are treated as homogenous polynomials of arbitrary degree g, so the corresponding LMI conditions, that although sufficient are increasingly precise when increasing g, are expressed just in terms of the vertices of the polytope.*

5 Numerical Example

Consider a 2-D static field model described by the differential equation:

$$\eta_{i+1,j+1} = \alpha_1 \eta_{i,j+1} + \alpha_2 \eta_{i+1,j} - \alpha_1 \alpha_2 \eta_{i,j} + \omega_{1(i,j)} \tag{18}$$

where $\eta_{i,j}$ is the state of coordinates (i, j), and α_1,α_2 are the vertical and horizontal correlative coefficients respectively, satisfying $\alpha_1^2 < 1$ and $\alpha_2^2 < 1$. Defining the augmented state vector $x_{i,j} = [\eta_{i,j+1}^T - \alpha_2 \eta_{i,j}^T \eta_{i,j}^T]^T$, and supposing that the measured equation and the signal to be estimated are

$$\begin{aligned} y_{i,j} &= \alpha_1 \eta_{i,j+1} + (1 - \alpha_1 \alpha_2) \eta_{i+1,j} + \omega_2 \\ z_{i,j} &= \eta_{i,j} \end{aligned} \tag{19}$$

it is not difficult to transform the above equations into a 2-D FM model in the form of (1), with the corresponding system matrices given by

$$\begin{aligned} A_{1\alpha} &= \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \end{bmatrix}, A_{2\alpha} = \begin{bmatrix} 0 & 0 \\ 1 & \alpha_2 \end{bmatrix}, B_{1\alpha} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B_{2\alpha} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ C_\alpha &= \begin{bmatrix} \alpha_1 & 1 \end{bmatrix}, D_\alpha = \begin{bmatrix} 0 & 1 \end{bmatrix}, H_\alpha = \begin{bmatrix} 0 & 1 \end{bmatrix}. \end{aligned}$$

where we assume that $0.15 \leq \alpha_1 \leq 0.45$, and $0.35 \leq \alpha_2 \leq 0.85$: therefore the above system can be represented by a four-vertex polytopic system.
By solving LMI (14), the results obtained are given in table 1. In this Example, we show that less conservative designs are achieved as the degree of the polynomial grows, when applying the HPPD approach.

Degree g	*Theorem* 4.1	*Theorem* 3[1]
0	3.8691	3.8709
1	2.4883	2.5450
2	2.4880	2.5028

Table 1: Comparisons of disturbance levels obtained for the Example with [1] and the proposed approach.

For degree $g = 0$, the H_∞ disturbance attenuation level is $\gamma = 3.8691$ (with $\lambda_1 = 1.0074$, $\lambda_2 = 0.0000$, $\lambda_3 = 0.0001$, $\lambda_4 = 0.0000$, $\lambda_5 = 0.0000$, obtained by optimization following Remark 4.2), and the filter matrices are

$$A_{f1} = \begin{bmatrix} 0.2772 & -0.0631 \\ 0.0647 & -0.0147 \end{bmatrix}, A_{f2} = \begin{bmatrix} 0.0992 & -0.0082 \\ 0.8269 & -0.0683 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0626 \\ -0.0146 \end{bmatrix}$$
$$B_{f2} = \begin{bmatrix} -0.0752 \\ -0.7663 \end{bmatrix}, C_f = \begin{bmatrix} 0.0001 & -1.0075 \end{bmatrix}.$$

For degree $g = 1$ (linearly parameter dependent approach), we get $\gamma = 2.4883$ (with $\lambda_1 = 1.8950$, $\lambda_2 = 0.0497$, $\lambda_3 = 0.0588$, $\lambda_4 = 0.1118$, $\lambda_5 = -0.1922$) and the filter matrices are

$$A_{f1} = \begin{bmatrix} 0.5711 & -0.1615 \\ 0.0403 & -0.0134 \end{bmatrix}, A_{f2} = \begin{bmatrix} -0.0889 & 0.0110 \\ 0.2529 & 0.2846 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.1155 \\ -0.0098 \end{bmatrix}$$
$$B_{f2} = \begin{bmatrix} 0.0268 \\ -0.3841 \end{bmatrix}, C_f = \begin{bmatrix} -0.0348 & -1.4971 \end{bmatrix}.$$

For degree $g = 2$, $\gamma = 2.4880$ (with $\lambda_1 = 2.2051$, $\lambda_2 = 0.0428$, $\lambda_3 = 0.0552$, $\lambda_4 = 0.1077$, $\lambda_5 = -0.2072$), and the filter matrices are

$$A_{f1} = \begin{bmatrix} 0.5973 & -0.1540 \\ 0.0412 & -0.0122 \end{bmatrix}, A_{f2} = \begin{bmatrix} -0.0868 & 0.0085 \\ 0.2542 & 0.2867 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0947 \\ -0.0077 \end{bmatrix}$$
$$B_{f2} = \begin{bmatrix} 0.0219 \\ -0.3237 \end{bmatrix}, C_f = \begin{bmatrix} -0.0586 & -1.7711 \end{bmatrix}.$$

The maximum singular values curve of the filtering error transfer function

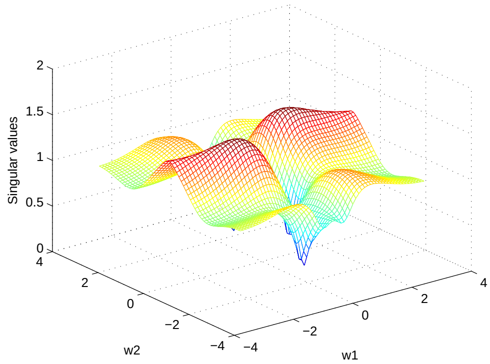

Figure 1: Singular value curve of the filtering error system in the Example, with the filter (5), for (α_1,α_2)=(0.15,0.35), from the results using a polynomial of degree $g = 2$.

with the filter in Eq.(5) in Example for different vertices are given in Table 2. The results obtained for this example using the proposed approach clearly

α_1	0.15	0.15	0.45	0.45
α_2	0.35	0.85	0.35	0.85
γ_{min}	1.5092	1.5267	1.7528	1.7373

Table 2: H_∞ norms at the vertices of Theorem 4.1 corresponding to degree $g = 2$

show the improvement with respect to previous results in the literature.

6 Conclusion

In this article, we have investigated the H_∞ filtering problem for 2-D discrete-time systems described by an uncertain Fornasini-Marchesini model. The Lyapunov function approach was used, so that by adding slack matrix variables, a new LMI-based condition for H_∞ performance analysis has been proposed. A numerical example is used to illustrate the effectiveness of the proposed method.

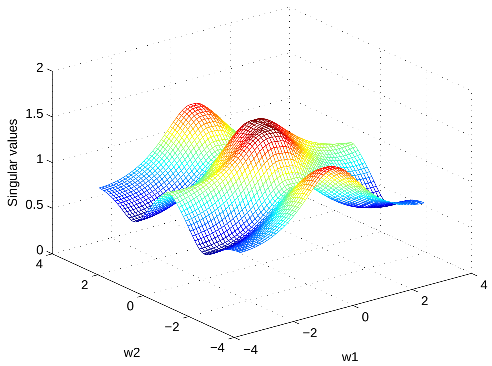

Figure 2: Singular value curve of the filtering error system in the Example, with the filter (5), for (α_1,α_2)=(0.15,0.85), from the results using a polynomial of degree $g = 2$

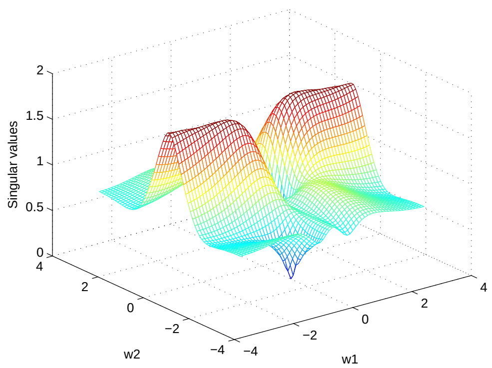

Figure 3: Singular value curve of the filtering error system in the Example, with the filter (5), when (α_1,α_2)=(0.45,0.35), from the results using a polynomial of degree $g = 2$

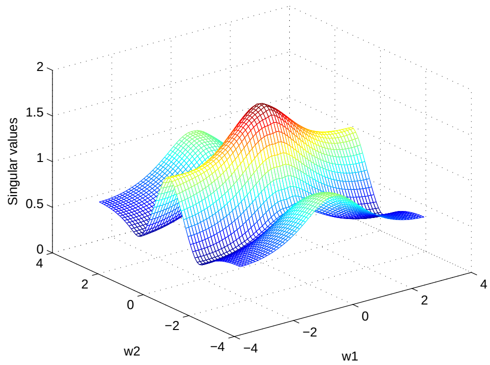

Figure 4: Singular value curve of the filtering error system in the Example, with the filter in Eq.(5), when (α_1,α_2)=(0.45,0.85), from the results using a polynomial of degree $g = 2$

References

[1] H. Gao, X. Meng, and T.Chen "New Design to Robust H_∞ Filters for 2-D Systems", IEEE Signal Processing Letters, 15: 217-220, 2008.

[2] M.J. Lacerda, R.C.L.F Oliveira, and P.L.D. Peres, "Robust H_2 and H_∞ Filter Design for uncertain linear systems via LMIs and polynomial matrices", Signal Processing, 91: 1115-1122, 2011.

[3] C. El-Kasri, A. Hmamed, T. Alvarez, and F. Tadeo, "Robust H_∞ Filtering of 2D Roesser Discrete Systems: A Polynomial Approach", Mathematical Problems in Engineering. Volume 2012, Article ID 521675, 15 pages.

[4] C. Du, L. Xie, and Y.C. Soh, "H_∞ filtering of 2-D discrete systems". IEEE Trans. Signal Processing, 48(6):1760-1768, 2000.

[5] C. Du, L. Xie, and C. Zhang, "H_∞ control and robust stabilization of two-dimensional systems in Roesser models", Automatica, 37:205-211, 2001.

[6] B. Boukili, A. Hmamed, A. Benzaouia, and A. El Hajjaji, "H_∞ Filtering of Two-Dimensional T-S Fuzzy Systems", Circuits, Systems and Signal Process., DOI 10.1007/s00034-013-9720-2, 2013.

[7] M. Benhayoun, F. Mesquine, and A. Benzaouia, “Delay-dependent Stabilizability of 2D Delayed Continuous Systems with Saturating Control”. Circuits, Systems and Signal Processing, DOI 10.1007/s00034-013-9585-4, 2013.

[8] H. Gao, J. Lam, S. Xu, and C. Wang, “Stabilization and H_∞ control of two-dimensional Markovian jump systems”, IMA Journal of Mathematical Control and Information, 21:377-392, 2004.

[9] S. Xu, J. Lam, Y. Zou, Z. Lin, and W. Paszke, “Robust H_∞ filtering for uncertain 2-D continuous systems,” IEEE Transactions on Signal Processing, 53(5): 1731-1738, 2005.

[10] S. F. Chen, and I. K. Fong, “Robust H_∞ filtering for 2-D state-delayed systems with NFT uncertainties,” IEEE Transactions on Signal Processing, 54(1): 274-285, 2006.

[11] C. El-Kasri, A. Hmamed, E. H. Tissir and F. Tadeo, “Robust H_∞ filtering for uncertain two-dimensional continuous systems with time-varying delays”, Multidimensional Systems and Signal Processing, 24(4): 685-706, 2013

[12] C. El-Kasri, A. Hmamed, and F. Tadeo, “Reduced-Order H_∞ Filters for Uncertain 2-D Continuous Systems, Via LMIs and Polynomial Matrices”, Circuits, Systems, and Signal Processing, DOI 10.1007/s00034-013-9689-x, 2013.

[13] A. Hmamed, F. Mesquine, F. Tadeo, M. Benhayoun, and A. Benzaouia, “Stabilization of 2D saturated systems by state feedback control”, Multidimensional Systems and Signal Processing, 21:277-292, DOI 10.1007/s11045-010-0107-2, 2010.

[14] A. Hmamed, M. Alfidi, A. Benzaouia, and F. Tadeo, “LMI conditions for robust stability of 2D linear discrete-time systems”, Mathematical Problems in Engineering, vol. 2008, Article ID 356124, 11 pages, 2008

[15] A. Hmamed, M. Alfidi, A. Benzaouia, and F. Tadeo, “Robust stabilisation under linear fractional parametric uncertaines of two-dimensional system with Rosser Models”, Int. J. of Sciences and Techniques of Automatic Control and Computer Engineering, 1(3): 336-348, 2007.

[16] L. Wu, P. Shi, H. Gao, and C. Wang, “H_∞ filtering for 2D Markovian jump systems,” Automatica, 44(7): 1849-1858, 2008.

[17] C. Du, L. Xie, and C. Zhang, "H_∞ Control and Robust Stabilization of Two-dimensional Systems in Roesser Models", Automatica, 37(2): 205-211, 2001.

[18] Z.S. Duan, J. X. Zhang, C.S. Zhang, and E. Mosca, "Robust H_2 and H_∞ filtering for uncertain linear systems", Automatica, 42(11): 1919-1926, 2006.

[19] C.E. Souza, L. Xie, and D. F. Coutinho, "Robust filtering for 2-D discrete-time linear systems with convex-bounded parameter uncertainty", Automatica, 46: 673-681, 2010.

[20] A. Hmamed, C. E. Kasri, E. H. Tissir, T. Alvarez, and F. Tadeo, "Robust H_∞ filtering for uncertain 2-D continuous systems with delays", International Journal of Innovative Computing, Information and control, 9(5): 2167:2183, 2013.