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Switched Complex System Analysis for Modeling and Control

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Résumé *This paper introduces a methodology of modeling for a class of Non Linear Complex Switched Systems in interaction with their environment. The models have to be adequate either for identification or for diagnosis and control. The effectiveness of this modeling technique is illustrated by experimental results obtained on a greenhouse.*

Keywords. *Variable Structure Systems, Switched Systems, Interaction with environment, Identification, Diagnosis and Control, Non Stationarity, Hybrid Systems, multiple models*

1 Introduction

Nowadays, technology progress increase the complexity of industrial systems, operating in different environments in changing conditions and characteristics (rapidly or discontinuously).

If in identification of a such system we use a single model, then it will have to quickly adapt itself to the new environment after each change. An appropriate control can be computed using several models combined adequately in function of the operating region.

In several applications involving this kind of complex systems, learning approaches are applied such as neural networks, fuzzy logic based methods or neuro fuzzy techniques. These approaches uses black box representations or combined black box models with learning and data processing methods, like eg Principal Component Analysis to extract information which will be used for the system supervision. In several cases these models are local representations which operating points dependant and do not allow physical interpretations

Another way may be to combine local grey box representations based on partial knowledge on the process and the involved phenomena. Several possible subsystems, local models are combined in a hybrid representation. The interconnected subsystem and structure are driven by some discrete events which are present and depend on involved phenomena.

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Some knowledge on this kind of events is necessary to built up detection procedures in order to be able to follow the structure commutations and then estimate the models combination and their parameters.

In linear systems, where different environments are described by different functions, a single model may not be adequate to identify the changes in the system (i.e., a model may not exist in the assumed framework to match the environment). Hence, multiple models are required both to identify the different environments as well as to control them rapidly. In literature different approach have proposed use of switches and supervision framework, for complex system. Approaches are based either on a classic linear modeling and control using statistical method [1] or on a hierarchical fuzzy logic used in the same time for models identification and commutation supervision [2], or non linear extension of PCA [3] or driven by a prediction error generated [4], or using a bank of Kalman filters [5] or Markov chains [6]. We suggest here a modeling approach based on behavioral models. Different models are supervised by discrete events which affect system operating zone, and switched via different selectors associated to events.

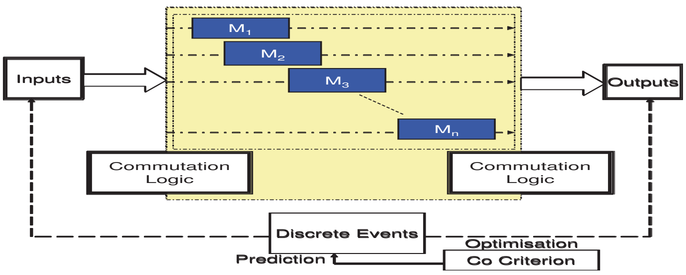

Fig.1. Varying Structure Systems with non linearities and commutations.

We can propose some structures for the sub models and nonlinear functions combining commutations and switching between structures. Then we have to define some methods for supervision and control of the main partial models. In general, the global process needs first to be stabilized in its global behaviour around some operational point. This corresponds to some operating conditions for the main components or subsystems in coordination of the discrete events. Switching and commutations have also to be managed.

The method consists to determine the particular zones where the system is supervised by (internal or external) discrete events characterizing its behavior. The models have to be adequate either for identification and behavioral analysis or for diagnosis and trajectory planning and following (in a non stationary environment or time varying characteristics). Thus, one can consider the behavior driven by multiple models orchestrated by events governing the system operation, and controlling various model commutations.

Also, in this paper, we want to study the problems of stability of some class of hybrid systems and how to deal with observability and controllability of such systems. Despite an amount of theory and proposed methods to test observability, it seems to be rather difficult to find an optimal way and efficient rules which cope with some class of hybrid non-linear systems. This is why we have chosen some simple mechanical systems with different phases when operating, or commutation of structures. Thus, we present in thes section 2 the formulation of the problem, in section 3, the stability tools useful for this approach are presented and we illustrate the effectiveness of this technique in section 4 by experimental results.

2 Problem formulation

2.1 System description

Definition of a class of systems having variable structures, commutations in their dynamic behavior, non linearities (hard or smooth), varying parameters and other non standard features, is difficult to be done in general. So we can restrict our case to some simple situations with known involved physical phenomena. Figure 1 can be obviously used to depict the features we are interested by.

In figure 1 we can remark that several models (M_p)are involved. We can consider multi-model approaches [7]. Use of these models and even their combinations are orchestrated by some occurring events noted discrete event in figure 1. Switching between models or ODEs (Ordinary Differential Equations) or combinations can appear. It is driven by a Commutation Logic. The same holds for the pertinent variables selection at each time interval or period of operating. It is obvious that importance of output variables and input commands may change in function of the selected model(s). Some models combinations may also be used in function of :

- time periods leading to selection of some kind or category of model equations,
- some running logic in the energetic behavior or command of the system.
- some connection and or interaction with other dynamics or environment

Discrete event systems can be considered for supervision of the switchings and commutations which drive the model selection and the the multi-model representation.

The discrete events taken into account and the choice of their effect may be driven by some higher level or simply selected according of optimization of some criterion or performance index [8] [9] [18].

The switchings and commutations often appear abruptly but changes from one representation to another one may be very smooth or not. In another hand we must note also that such systems representation is not unique and differences can appear between the behavioral representation and physical system description or modeling for diagnosis and control.

This can be referred to as a switched multi-model representation driven by some Discrete Event Systems.

The system equation can then be written in following form :

$$\begin{cases} \dot{x} = f_{m_i}(x, u, t) \\ m_i = < S, I, O, \delta_{int}, \delta_{ext}, \lambda, t_{\alpha i} > \end{cases} \quad (1)$$

where :

- x is the set of continuous states of system
- u is the set of input controls
- S is the set of sequential states
- I is the set of external events
- O is the set of internal events
- $\delta_{int} : S \rightarrow S$ is the internal state transition function
- $\delta_{ext} : S_- \times I \rightarrow S_+$ is the external state transition function
- $\lambda : S \rightarrow O$ is the output function
- $t_{\alpha i}$ is the time advance function

2.2 Piecewise continuous systems

The system is defined by the following contents :

- A set of sub-models, representing process in different regions $D_p \subset \mathbb{R}^n$ of dynamic state space vector $x(t) \in \mathbb{R}^n$

- Switching rules or conditions for each operating zone which drives the switchings between models.

Suppose that we have several simple sub- models

$$\dot{x} = f_p(x, u, t), \ t \in \mathbb{R} \quad (2)$$

for $p = 1, 2, ...q$ and that each one of these models is valid in some state subspace $D_p \subset \mathbb{R}^n$. u is the set of input of the system. To describe the global system's dynamic behavior, we have to gather all the locally valid model equations and then :

$$\begin{cases} \dot{x} = f_p(x, u, t) \\ p = < S, I, O, \delta_{int}, \delta_{ext}, \lambda, t_{\alpha i} > \end{cases} \quad (3)$$

S is the discrete state of the complex system (ie depending on the region of operation and valid sub-model). The discrete system is in S_p when operation in the region D_p. The events may be considered as defined by state x of the system and any internal x-dependant variable, and then, events depend on input values of the S_{ed} system noted $I(t)$ [10]. As consequence the validity domain $D_p(I(t))$ depends also on this input.

This example is simple regard to the sub-model transition and combination. The model changes are driven in function of the position of the dynamic state vector in the space. This kind of representation is commonly used in fuzzy logic modeling and identification for complex systems.

2.3 Some examples

Several examples may be considered in this case :
- vehicles and mobile robots dynamics when rolling in its environment (contact between road and wheels can be lost and retrieved, braking and accelerating, ...),
- legged robots,
- greenhouses,
- helicopter and flying robots

Example 1. **:** Let us consider the mass-spring system shown in the figure (2), without frictions [11][12]. The spring (with stiffness k) attached to the center of mass M.

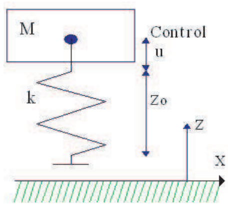

Fig.2. Vertical Hopper (1 DOF)

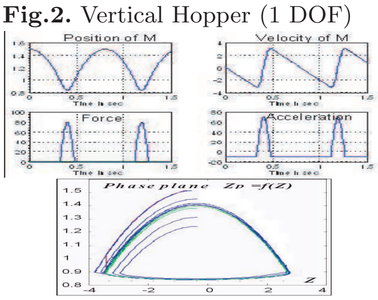

Fig.3. Closed orbit

The system equation can be written as follow :

$$M\ddot{z} + k(z - z_o)\xi(z) = -Mg \quad (4)$$

with a commutation function $\xi(z) = \frac{1}{2}(1 - sign(z - z_o))$.

$$\dot{x} = f_p(x, u, t) \quad (5)$$

with $S = 1$ if $\xi(z) = 1$
and $S = 0$ if otherwise
The commutation variable $\xi(z) = \frac{1}{2}(1 - sign(z - z_o))$ is equal to unity when the spring is in contact with ground (we have the ODE 1 : $\dot{x} = f_1(x, u, t)$) and zero otherwise (we then have the ODE 0 : $\dot{x} = f_0(x, u, t)$). This simple model allows to analyze energetic interactions between the robot, the control and ground. This shows existence of periodic cycles, corresponding to system oscillations.
The system equation can then be written in state space form :

$$\begin{cases} \dot{x}_1 = x_2 = \dot{z} \\ \dot{x}_2 = \ddot{z} = -\frac{k}{M}\xi(x_1)(x_1 - z_o) - g \\ S_+ = \{S_-, I = \xi(z), t\} \end{cases} \quad (6)$$

The system has obviously an equilibrium point at 0 and its solution describes a closed orbit $\Omega_o(x_o, t)$ around 0.
The non perturbed oscillations correspond to a closed orbit (see bottom of figure (3))[12]. The periodic orbit, obtained for the free system, depends on initial state x_o(e.g. $(z_m, \dot{z} = 0)$ see top of figure (3)) and is defined by the following equation [11] :

$$V_f(z, \dot{z}) = \frac{1}{2}M\dot{z}^2 + Mgz + \xi(z)\frac{k}{2}(z - z_o)^2 = V_o \quad (7)$$

$$V_o = V(z_m, 0) = V(0, \dot{z}_d) \quad (8)$$

So the figure (3) show us the velocity, the acceleration of the mass M and the forces applied in the spring when this last down touch the ground.

Example 2. **:** Another example can be considerated for the same system, [21]. Two springs (with stiffness k) attached on both dimensions of the mass M. This is illustrated in the figure(4).

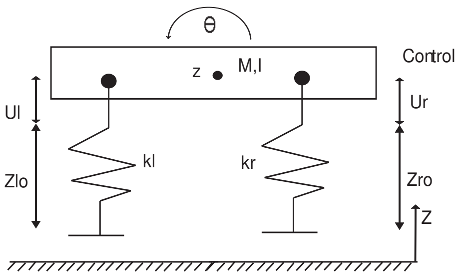

Fig.4. Mass Spring Model

The dynamic interaction with the ground is composed by two phases : flying and stance phases [12][13]. This system is composed by interconnection of three subsystems (mass, spring and ground) and energy evolutions. In this case, we can considerate the following phases :

– Contact Phases. In these phases the controls are active when the springs are in contact.
- The two springs are in contact : $\dot{x} = f_1(x, u, t)\ \ t \in \mathbb{R}_+\ \ and\ \ x(t) \in D_1 \subset \mathbb{R}^4$

$$\begin{cases} \dot{x}_3 = \ddot{z} = -\frac{k_l}{M}(z_l - z_{l0} - u_l) - \frac{k_r}{M}(z_r - z_{r0} - u_r) - g \\ \dot{x}_4 = \ddot{\theta} = \frac{lk_l}{I}(z_l - z_{l0} - u_l) - \frac{lk_r}{I}(z_r - z_{r0} - u_r) \end{cases} \quad (9)$$

- The right spring is in contact : $\dot{x} = f_2(x, u, t)\ \ t \in \mathbb{R}_+\ \ and\ \ x(t) \in D_2 \subset \mathbb{R}^4$

$$\begin{cases} \dot{x}_3 = \ddot{z} = -\frac{k_r}{M}(z_r - z_{r0} - u_r) - g \\ \dot{x}_4 = \ddot{\theta} = -\frac{lk_r}{I}(z_r - z_{r0} - u_r) \end{cases} \quad (10)$$

- The left spring is in contact : $\dot{x} = f_3(x, u, t)\ \ t \in \mathbb{R}_+\ \ and\ \ x(t) \in D_3 \subset \mathbb{R}^4$

$$\begin{cases} \dot{x}_3 = \ddot{z} = -\frac{k_l}{M}(z_l - z_{l0} - u_l) - g \\ \dot{x}_4 = \ddot{\theta} = \frac{lk_l}{I}(z_l - z_{l0} - u_l) \end{cases} \quad (11)$$

– Flying Phase : $\dot{x} = f_4(x, u, t)\ \ t \in \mathbb{R}_+\ \ and\ \ x(t) \in D_4 \subset \mathbb{R}^4$

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = \ddot{z} = -g \\ \dot{x}_4 = \ddot{\theta} = 0 \end{cases} \quad (12)$$

The relationship of these different phases is illustrated in the figure (5).

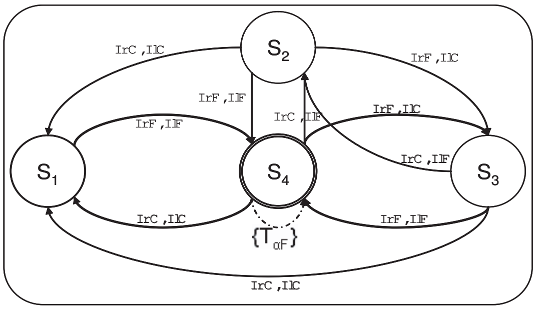

Fig.5. State Behavioural Model

Each discrete state represents one phase and the arc of connection repe- resents the commutation possible for every discrete state.

The figure (6-8) shows us respectively, the trajectory given for the mass spring system, its closed orbit, the swing angle stabilized and the commu- tation sequence possible for damping any rotational motion and maintain hopping along z axis. In another terms, the system stability is studied as a stability of one periodic motion.

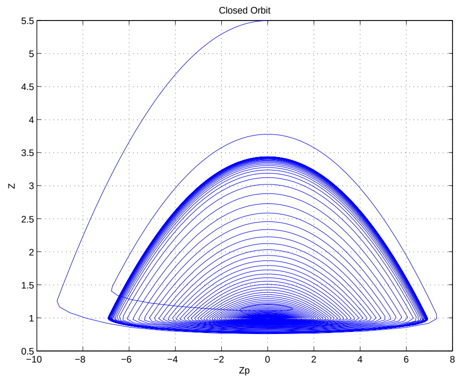

Fig.6. Closed Orbit

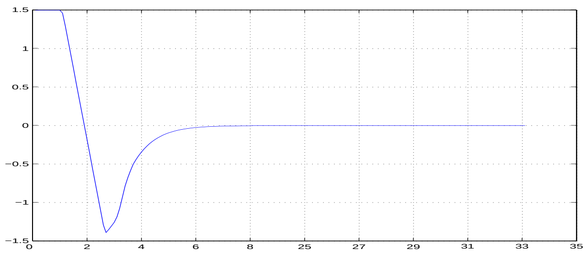

Fig.7. Sweet Angle

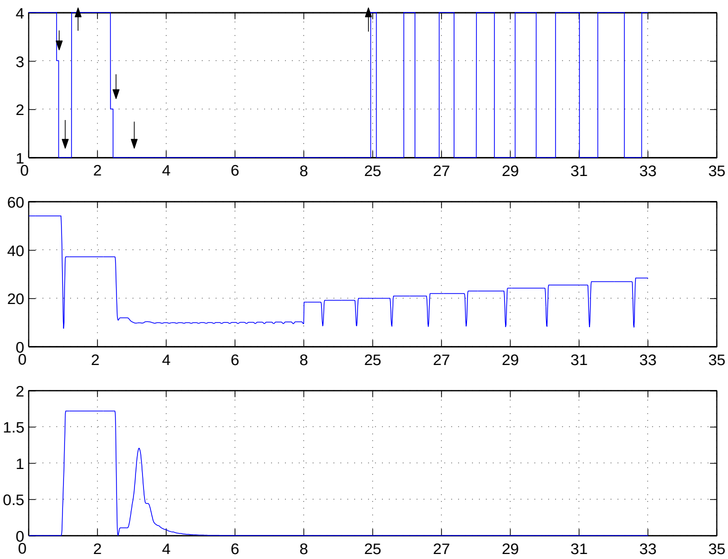

Fig.8. Switching and Energies of the System

A bouncing ball has the same model as the previous example, with additional damping, during contact phase, which introduces a lost of energy. Then the system trajectory goes asymptotically to zero.

The previous examples are the simplest cases the possible legged robots. We can consider also the case of biped or four legged robots [12][13] [14]. These cases involved several models (each one corresponding to some legs in contact with ground and others in flying motion. The commutations between models depend on events like heating ground and impacts on obstacles.

3 Stability tools for switching system

The need of tools for stability analysis and control design, when switched systems are involved, is important. In this section we simply recall useful recent result presented for Hybrid systems and using multiple Lyapunov functions as a generalization of the Lyapunov's second method.

Stability proof depends on the existence and/or construction of an appropriate Lyapunov candidate function V and is rather not obvious for hybrid systems. The inherent discontinuous nature of hybrid system suggests use of multiple Lyapunov functions concatenated together in function of sub models commutations and transitions.

This may produce a non-traditional multiple Lyapunov functions useful to prove stability [15], [16][17] of the hybrid system. Let us recall a useful theorem based on the second Lyapunov method for stability analysis.

A Lyapunov function for the system (1), at an equilibrium point x_{ep} in the domain D_p is real valued function $V_p(x)$ defined in the domain D_p satisfying the conditions :

(C1) : Positive definiteness : $V_p(0) = 0$ and $V_p(x) > 0$ for $x \neq 0$

(C2) : Negative derivative : for any $x \in D_p$: $\dot{V}_p(x) = \frac{\partial V_p(x)}{\partial x} f_p(x, u, t) \leq 0$

Theorem 1. *Given an P-switched non-linear system, suppose that each vector field $f_p(x, u, t)$ has an associated Lyapunov function $V_p(x)$ in the domain D_p, each one defined for the equilibrium point $x_e = 0$. Let S_{k+1} be a switching sequence of the discrete state such that S_{k+1} can take values p only if $x_{k+1} \in D_p$, and in addition :*

(C3) : $V_p(x(t_p, k + 1)) \leq V_p(x(t_p, k))$ for all t_{pk} the switching times

Beginning with different assumptions, this more general result assumes a so-called weak Lyapunov function for V_p, in which condition (C3) is replaced by :

(C4) : $V_p(x(t)) \leq h.V_p(x(t_p)$ with $t \in (t_p, t_{p+1})$

thus, the set V contains a number of candidate Lyapunov functions that are used as a measure of the hybrid system energy, $V = \{V_1, \ldots, V_p\}$. Since the energy changes according to :

$\dot{V}_q(x) = \frac{\partial V_q(x)}{\partial x} f_p(x, u, t) \leq 0$

for an arbitrary $V_q \in V$, this means that the change of energy depends on the vector field $f_p(x, u, t)$ and thus on the discrete state S_p. To express

where in the continuous state space the energy decreases when there is switching from the Lyapunov function V_q to V_r, the following sets are defined [20] :
$D_p^q = \{x \in \mathbb{R}^n | \frac{\partial V_q(x)}{\partial x} f_p(x,u,t) \leq 0\}$
$D_R^q = \{x \in \mathbb{R}^n | \ V_q(x) \geq V_r(x)\}$
So, assume that the candidate Lyapunov functions V_q and V_r are used as a measure of the energy for different discrete behavioral states S_i and S_j , and consider once more the case where $i \neq j$ and $q \neq r$. If the discrete state is S_i and the threshold point R_{ij} is reached, then the discrete state becomes S_j , implying that the vector is changed. Two possible situations may occur, on the one hand we have a same directions of the vectors field $f_i(x,u,t)$ and $f_j(x,u,t)$, and on the other hand opposite directions of the vector fields $f_i(x,u,t)$ and $f_j(x,u,t)$.

4 Application to a greenhouse modeling and control

4.1 Greenhouse description

The main goal of the greenhouse is to improve the weather conditions. This system is sensitive to the external disturbances as for example radiation, temperature etc. and can filter the disadvantages like wind, rain. By controlling the internal temperature, the internal hygrometry and the carbon dioxide we can normally create optimal conditions for the plants. This is illustrated in the figure (9). The problem of modeling and control

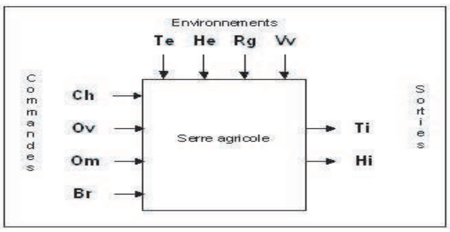

Fig.9. Greenhouse Model

design for our system is then complex and intricate because there are, at least, eight inputs and two outputs involved in a non linear and switched way :
- 4 actuators (heating Ch (boolean), opening Ov (%), shade Rd (%), misting system Br (boolean)) ;
- 4 meteorology disturbance (external temperature Te ($°C$), external hygrometry He (%), solar radiation Rg (W/m2), wind speed Vv (km/h)) ;
- 2 controlled outputs (internal temperature Ti ($°C$), internal hygrometry Hi (%)).

4.2 Greenhouse nominal models

The greenhouse system can be viewed as one multi-model linear system writing in the following form :

$$\begin{cases} x(k+1) = A_{m_i}x(k) + B_{m_i}u(k) + F_{m_i}w(k) \\ m_i =< S, I, O, \delta_{int}, \delta_{ext}, \lambda, t_{\alpha i} > \end{cases} \qquad (13)$$

where $A_{m_i} \in \mathbb{R}^{n\times n}$ is the state matrix, $B_{m_i} \in \mathbb{R}^{n\times m}$ is the control matrix and $F_{m_i} \in \mathbb{R}^{n\times p}$ is the disturbance matrix.

In linear systems, where different environments are described by different functions f_{mi}, a single model may not be adequate to identify the changes in the system (i.e., a model may not exist in the assumed framework to match the behavior in all the regions). Hence, multiple models are required both to identify the system in the different regions, as shown in the Figure(10), as well as to control the system.

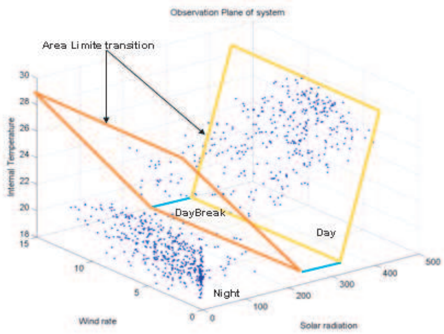

Fig.10. Representation of state equation in each domain validity

When the region changes, the input output characteristics of the system will change. If a single identification model is used, it will have to adapt itself quickly to the new region of behavior before that an appropriate control action can be taken.

In the sequel, let us consider the following assumptions to simplify the modeling and analysis.

Assumptions - The system behavior can be approximated by three pertinante operating modes : Night , Daybreak and Day.

These three models have been obtained by the previous studies based on use of a non linear extension of PCA [3] of neural network approach [2].

- Input disturbances variables set are solar radiation Rg and wind rate Vv. They can take the following values Lower, Middle, Upper and Lower, Upper respectively.

Let us try to represent the greenhouse by a model which captures the nominal dynamics in its behavior. A nominal identification Model can be written, among several simplification, as follows :

$$\begin{cases} \begin{bmatrix} Ti(k+1) \\ Hi(k+1) \end{bmatrix} = A_{m_i} \begin{bmatrix} Ti(k) \\ Hi(k) \end{bmatrix} + B_{m_i} \begin{bmatrix} Ch(k) \\ Br(k) \\ Ov(k) \\ Rd(k) \end{bmatrix} + F_{m_i} \begin{bmatrix} Te(k) \\ He(k) \\ Vv(k) \\ Rg(k) \end{bmatrix} \\ m_i =< S, I, O, \delta_{int}, \delta_{ext}, \lambda, t_{\alpha i} > \end{cases} \quad (14)$$

4.3 Identification of the operating zone

Let us associate the observations via external environment with events notion as it is shown in the table 1. The observations will be described [19] to specify a both the events trace and occurrence date where they appear. These events are defined by input value of the system. The output value is calculated by comparaison with the state where the system is from at each instant [10]. As we have said previously, each sub model is defined for specific operating point (or region). So, we can consider that this operating point as the center of a specific domain, or sub model validity domain, for description of greenhouse behavior. Let $D_q(I(k))$ be the validity domain (Figure 10) when we are in the state S_q. S_q can switch to S_r, if the threshold $R_{q,r}(I(k))$ is true and the input value I(k) defines exactly this validity domain.

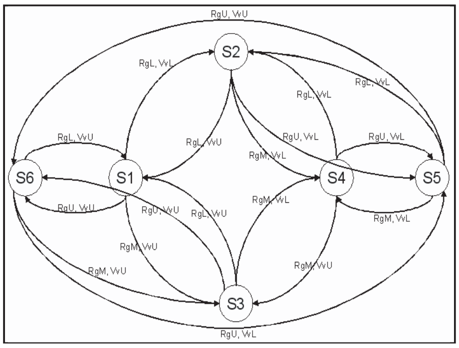

Fig.11. Discrete Event Model of the Greenhouse

Thus, we have the state commutation in the table 2. Where all conditions of transition are respected to check the operating particular modes of the system.

In the experimentation, we have considerated three days of March. Its the 10, 11 and 12 March. These choices were made owing to the fact that these days comprise in them even behaviors which all are not similar.

Values / Input	Lower	Middle	Upper
Vv	$< R'_1$	$-$	$\geq R'_1$
Rg	$< R'_1$	$> R'_1$ and $\leq R'_2$	$> R'_2$

Tab.1. Input values associated with Environment Specification

State	Name	Input Variable
S_1	$ColdNight$	$I = \{RgL, VvU\}$
S_2	$FreshNight$	$I = \{RgL, VvL\}$
S_3	$ColdDaybreak$	$I = \{RgM, VvU\}$
S_4	$FreshDaybreak$	$I = \{RgM, VvL\}$
S_5	$DryDay$	$I = \{RgU, VvL\}$
S_6	$ModerateDay$	$I = \{RgU, VvU\}$

Tab.2. Discrete State designation of the supervision device

The first level of figure (12) gives us the result of simulation between the interior temperature of the estimated greenhouse and reality. For the same, we can show the effectiveness of this approach, in modelling case because we have done a best result for estimation of our continuous state with the methodolgy proposed. In fact, this system can be modelling with this approach which consider the phenomena defined by the external environment, as its shown for the 12 March day. Here, it appears just one model selection even we know that one day is always composed of the three following sequence : "'*Night* ↔ *Daybreak* ↔ *Day*"'. Its proove interresting to use this approach in this case.

The second level, gives us the dynamics of the error in estimation.

The third level corresponds to the value of the output of the model with discrete event which manages the selection of the models. The index numbers 1, 2, and 3 are associated respectively to the Night model, the Daybreak one and Day model.

Lastly, the fourth level gives us the estimate of the discrete state of the system.

4.4 Concluding remarks Perspective of the modeling approach

We propose of this work is to define an approach to identify and then control and supervise such class of complex systems represented by switched models. The system is composed by different sub-models. Each model switches to another instantaneously when the thresholds that define some operating points or zone, is reached. In the goal to build the best prediction of system outputs, we have to get the best switching

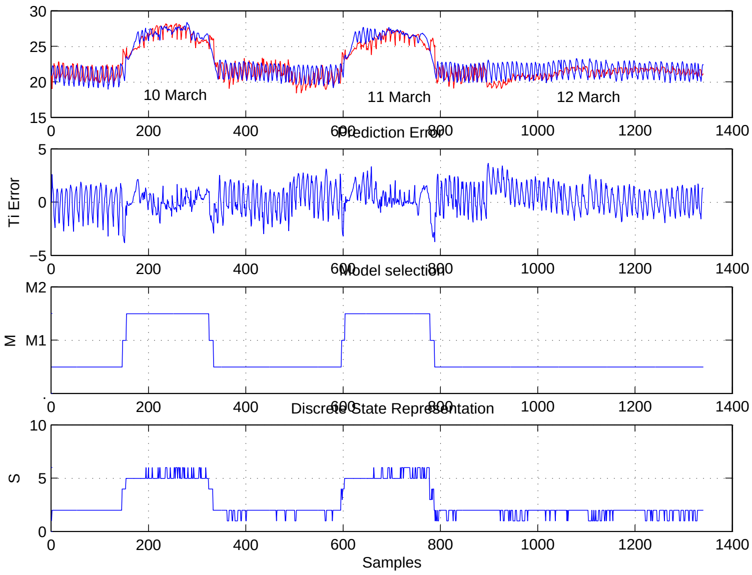

Fig.12. Greenhouse Simulation with Multiple Model

and supervision device depending on operating point, the behavior and environment.

The presented experimental results emphasize efficiency of this approach for modeling, behavior analysis and prediction for such class of complex systems.

In a future work, this approach will be used for diagnosis, fault detection and monitoring. A diagnostic framework will be considered to detect defaults and control the system.

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