Design of fault tolerant control strategy based on fault isolation filter: Application to the IFATIS benchmark problem

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Abstract This paper deals with a reconfiguration method against major actuator failures. These failures, as a blocking or a complete loss of an actuator, may reduce drastically the desired performances of a closed-loop system. The proposed approach is composed of two stages. The first stage is the detection and isolation of the failed component using a directionnal filter designed under a particular eigenstructure assignment. The second stage is represented by the reconfiguration mechanism which make possible to compensate the fault effects or in some case to reduce the objectives. This approach was applied to a benchmark proposed in the framework of the IFATIS "Intelligent Fault Tolerant Control in Integrated Systems" project (IFATIS-IST 32122) and gave good results for actuator fault accommodation.

1 Introduction

In highly automated plants where maintenance or repairing cannot be always achieved immediately, fault tolerant control systems have become a high priority. A fault-tolerant controller engages suitably fixed or varied structure to guarantee stability and satisfactory performance when all components are in normal condition, as well as the case when some components are of malfunction. A fault-tolerant control system (FTCS) design must provide levels of performance and reliability that are beyond what control systems lacking fault tolerance can provide.

The main task to be tackled in achieving fault-tolerance is the design of a controller with suitable structure to guarantee stability and satisfactory performance, not only when all control components are operational, but also in the case when sensors, actuators (or other components e.g. the control computer hardware or software) malfunction. Therefore, it is important to implement FTC strategies in order to minimize degradation of product quality and economic loss. Various approaches for fault-tolerant control have been suggested in the literature [18]). Actually, fault-tolerant control concepts can be separated into “passive” and “active” approaches. The key difference between them consists in that
the active FTC system includes an fault detection and isolation (FDI) system and the fault handling is carried out based on information on faults delivered by the FDI system, while in a passive FTC system the system components and controllers are so designed that they are robust to possible faults to a certain degree [11], [6], [16]. In practice, using only passive approach is risqued. Generally, the passive approaches have the following characteristics:

- robustness to a certain known faults
- using a hardware redundancy (multiple actuator and sensor...)

In this respect, several advanced control strategies have been proposed in the literature [24], [25].

Adaptive control seems to be the most natural approach to accommodate faults: the faults effects appear as parameter changes and are identified on line, and the control law is reconfigured automatically based on new parameters [4], [20], [22]. Other approaches have been proposed, based on pseudo inverse [26], eigenstructure [27], multiple model ([29], [28]).

A classical way to achieve fault-tolerant control relies on supervised control where an FDI unit provides information about the location and time occurrence of any fault. Faults are compensated via an appropriate control law triggered according to diagnosis of the system [15]. Nevertheless, it is to be noticed that only few methods have been applied to real plants [1], [2], [15], [19].

The active fault tolerant control, consists of the following steps (Fig. 1),

* Fault detection and isolation:

* Performance evaluation : to express the severity of the failure and its consequences on the performances of the system

* Fault compensation via control reconfiguration : Appropriate control law is triggered accordingly to the diagnosis of the system.

This paper is organised as follow: first, in section 2 fault diagnosis and estimation is studied. A method based on the design of directional residual is proposed. Section 3 adresses control reconfiguration, accommodation to soft failures is first considered, before dealing with critical faults. The application to the IFATIS Benchmark is developped in section 4. Finally, section 4 gives some conclusions.
2 Fault diagnosis and estimation

Fault diagnosis implies to design residuals that are close to zero in fault-free situations and clearly deviate from zero in the presence of faults. Residual must possess the ability to discriminate between all possible modes of faults, which explains the use of the term isolation. Generation of residuals having directional properties in response to a particular faults is an attractive way for enhancing fault isolability. The fault detection filter which is proposed in this paper is a special full-order state observer which generates output residuals having directional properties in response to each fault. First developed by [3], the fault detection filter has been revisited by [13] from the geometric state-space control theory and by [21] in the context of eigenstructure assignment. To apply the fault detection filter in stochastic systems, a new interpretation of the fault detection filter have been suggested by [17], however, the treatment of multiple faults was not studied, convergence and stability conditions of the filter as well. Further improvements were suggested by [8]. Recently, ([5]; [8]) have proposed a new robust multiple fault detection filter which is derived by solving an optimization problem in the context where we can not achieve a perfect decoupling. The fault isolation filter presented here is very similar to the predictor structure of the standard Kalman filter allowing the establishment of its convergence and stability conditions.

Consider the following discrete time linear system

\[ x_{k+1} = Ax_k + Bu_k + Fn_k + w_k \]  
\[ y_k = Cx_k + v_k \]

where \( x_k \in \mathbb{R}^n \) is state vector, \( y_k \in \mathbb{R}^m \) the output vector, \( u_k \in \mathbb{R}^p \) the input vector. \( F = [f_1 \ldots f_i \ldots f_q] \) is faults distribution matrix and \( n_k \in \mathbb{R}^q \) is the
fault vector. We assume \( \text{rank}(C) = m \) and \( \text{rank}(F) = q \). The noises \( w_k \) and \( v_k \) are zero mean uncorrelated random sequences with

\[
E \left( \begin{bmatrix} w_k \\ v_k \end{bmatrix} \begin{bmatrix} w_j^T \\ v_j^T \end{bmatrix} \right) = \begin{bmatrix} W & 0 \\ 0 & I \end{bmatrix} \delta_{kj}
\]  

(3)

where \( W \succeq 0 \). The initial state \( x_0 \), uncorrelated with \( w_k \) and \( v_k \), is a gaussian random variable with \( E \{ x_0 \} = \bar{x}_0 \) and \( E \{ (x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T \} = \bar{P}_0 \).

Consider the following residual generator

\[
\hat{x}_{k+1} = A\hat{x}_k + Bu_k + K(y_k - C\hat{x}_k)
\]

(4)

\[
r_k = L(y_k - C\hat{x}_k)
\]

(5)

where \( \hat{x}_k \) is the state of the filter, \( r_k \) the output of the filter and where \( L \in \mathbb{R}^{q,m} \) and \( K \in \mathbb{R}^{n,m} \) are unknown matrices that we will be designed in order to fulfill fault detection and isolation requirements. The remaining degrees of freedom will be used to minimize the noises effects on the output residual.

From (1) and (4), the estimation error \( e_k = x_k - \hat{x}_k \) and the output of the filter \( r_k \) propagate as

\[
e_{k+1} = (A - KC)e_k + Fn_k + w_k - Kv_k
\]

(6)

\[
r_k = L(Ce_k + v_k)
\]

(7)

Let us define the detectability indexes introduced by (Keller 1999). The linear time invariant system (1;2) has the following fault detectability indexes

\[
\rho_i = \min\{ \nu : CA^{\nu-1}f_i \neq 0, \nu = 1, 2, \ldots \}
\]

(8)

Define the fault detectability matrix \( D = C\Psi \) with

\[
\Psi = \begin{bmatrix} A^{\rho_1-1}f_1 & \cdots & A^{\rho_1-1}f_q \\
A^{\rho_2-1}f_1 & \cdots & A^{\rho_2-1}f_q \\
\vdots & \ddots & \vdots \\
A^{\rho_q-1}f_1 & \cdots & A^{\rho_q-1}f_q \end{bmatrix}
\]

(9)

The generated residuals will be sensitive to the faults by means of a diagonal transfer function between residuals and faults.

Under the following assumptions \( \text{rank}(D) = q \), the goal is to compute \( K \) and \( L \) such that

\[
W(z) = LC(zI - (A - KC))^{-1}F
\]

(10)

\[
= \text{diag}(z^{-\rho_1}, z^{-\rho_2}, \ldots, z^{-\rho_q})
\]

(11)

where (11) ensures the diagonal structure of transfer from faults to residuals allowing the multiple faults isolation.

This is achieved by assigning \( A^{\rho_i-1}f_i \) as eigenvector of \( (A - KC) \) such that

\[
(A - KC)\Psi = 0
\]

(12)

\[
LC\Psi = I
\]

(13)
After having parameterized the eigenstructure assignment (12) and algebraic constraint (13), the remaining design of freedom is used to minimize the noise effects on the fault estimation error. This is equivalent to minimize the trace of the fault estimation error covariance matrix \( P^m_k \) given by the following expression

\[
P^m_k = E \left( (r_k - E(r_k))(r_k - E(r_k))^T \right)
\]

where \( E(r_k) = \begin{bmatrix} n_{k-\rho_1} \hdots n_{k-\rho_i} \hdots n_{k-\rho_q} \end{bmatrix}^T \).

Solution to this problem makes use of the following theorem:

**Theorem 1** Filter Parametrization

under \( \text{rank}(D) = q \), the residual generator \((4;5)\) can be parametrized according to

\[
K = \omega \Pi + \bar{K}_k \Sigma
\]

\[
L = \Pi + \bar{L}_k \Sigma
\]

with \( \Sigma = \beta (I - D \Pi) \), \( \Pi = (D)^+ \) and \( \omega = A \Psi \), where \( o \beta \in \mathbb{R}^{m-q \times m} \) is an arbitrary matrix determined so that \( \Sigma \) is of full rows rank and \( \bar{K}_k \in \mathbb{R}^{m-q \times q} \) and \( \bar{L}_k \in \mathbb{R}^{m-q \times q} \) are the reduced gain describing the remaining degree of freedom.

Next \( \bar{K}_k \) and \( \bar{L}_k \) are computed so that the trace of the fault estimation error covariance matrix is minimized.

The problem of minimizing the trace of the estimation error covariance matrix under algebraic constraints has been studied by [12] for the design of reduced-order Kalman filter and [14] for the design of a Kalman filter with unknown inputs. In this paper, the minimisation concern the state error estimation and the fault error estimation, it will be made with respect to the free parameter \( \bar{K}_k \) and \( \bar{L}_k \).

Under the stability and convergence conditions given by

\[
\text{rank} \begin{bmatrix} zI - A \Psi \\ C & 0 \end{bmatrix} = n + q, \ \forall z \in \mathbb{C}, \ |z| \geq 1
\]

and

\[
\text{rank} \begin{bmatrix} -e^{jw}I + A \Psi W^{1/2} \end{bmatrix} = n, \ \forall w \in [0,2\pi]
\]

The proposed fault isolation filter described by the following relations:

\[
\hat{x}_{k+1} = A \hat{x}_k + Bu_k + (\omega \Pi + \bar{K}_k \Sigma)(y_k - C \hat{x}_k)
\]

\[
\bar{P}_{k+1} = (\bar{A} - \bar{K}_k \bar{C})\bar{P}_k (\bar{A} - \bar{K}_k \bar{C})^T + \bar{W} + \bar{K}_k \bar{V} \bar{K}_k^T
\]

\[
r_k = (\Pi + \bar{L}_k \Sigma)(y_k - C \hat{x}_k)
\]

\[
P^m_k = (\Pi + \bar{L}_k \Sigma)H_k(\Pi + \bar{L}_k \Sigma)^T
\]
with

\[ K_k = AP_k C^T (CP_k C + V)^{-1} \] (23)
\[ L_k = -H_k \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \] (24)
\[ H_k = CP_k C^T + I \] (25)

where \( \bar{A} = A - \omega \Pi C \), \( \bar{C} = \Sigma C \), \( \bar{V} = \Sigma \Sigma^T \), \( W = W + \omega \Pi \Pi^T \omega^T \).

**Demonstration 1** The faults estimation can be expressed as

\[ r_k = L(C\bar{e}_k + v_k) + \left[ n_{k-\rho_1}^1 \ldots n_{k-\rho_i}^i \ldots n_{k-\rho_q}^q \right]^T \] (26)

from the state estimation errors without faults which propagates as

\[ \bar{e}_{k+1} = (A - K C)\bar{e}_k + w_k - K v_k \] (27)

where the fault \( n_{k-\rho_i}^i \) of detectability index \( \rho_i \) affects directly the reduced output residual \( r_k \) with a time delay equals to its detectability index. \( r_k \) can also be viewed as a stochastic deadbeat observer of the fault magnitudes.

Let \( e_n^k = \hat{n}_k - E(\hat{n}_k) \) the fault estimation error. By substituting (15) and (16) in (26) and (27), we obtain

\[ \bar{e}_{k+1} = (A - (\omega \Pi + \bar{K}_k \Sigma)C)\bar{e}_k + w_k - (\omega \Pi + \bar{K}_k \Sigma)v_k \] (28)

\[ e_n^k = (\Pi + L_k \Sigma)(C\bar{e}_k + v_k) \] (29)

The estimation errors covariance matrices \( \bar{P}_k = E(\bar{e}_k \bar{e}_k^T) \) and \( P_k^a = E(e_n^k e_n^k^T) \) satisfy

\[ \bar{P}_{k+1} = (A - (\omega \Pi + \bar{K}_k \Sigma)C)\bar{P}_k (A - (\omega \Pi + \bar{K}_k \Sigma)C)^T + W + (\omega \Pi + \bar{K}_k \Sigma)(\omega \Pi + \bar{K}_k \Sigma)^T \] (30)

\[ P_k^a = (\Pi + L_k \Sigma)(CP_k C^T + I)(\Pi + L_k \Sigma)^T \] (31)

The traces of \( \bar{P}_{k+1} \) and \( P_k^a \) are minimized with respect to \( \bar{K}_k \) and \( L_k \) if and only if

\[ \bar{K}_k = (A\bar{P}_k C^T - \omega \Pi H_k \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \] (32)
\[ L_k = -\Pi H_k \Sigma^T (\Sigma H_k \Sigma^T)^{-1} \] (33)

where \( \Pi \Sigma^T = 0 \), (32) gives (23), where \( CP_k C^T + \bar{V} \) is assured to be non singular if \( \beta \) is a full rows rank. From filter equation, considering theorem 1, we have then:

\[ \hat{x}_{k+1} = A\hat{x}_k + Bu_k + \omega r_k + \bar{K}_k \gamma_k \] (34)

Furthermore, it easy to show that:

\[ \gamma_k = \Sigma (y_k - C\hat{x}_k) \] (35)
is decoupled from the faults, while

$$r_k = (I + L)\Sigma(y_k - C\hat{x}_k)$$

(36)

is sensitive to the faults since $\Pi D = I$ and $\Sigma D = 0$. The stability and convergence conditions can be deduced from the results obtained by [10]. This approach can also be applied to detect and isolate the sensor faults. Based on the augmented system by the evolution model of the sensor faults, it will be converted to the actuator faults. Consequently, the filter outputs gives the sensor faults estimation.

3 Control reconfiguration

Fault-tolerant control systems are characterized by their capabilities, after fault occurrence, to recover performance close to the nominal desired performance. In addition, their ability to react successfully during a transient period between the fault occurrence and the performance recovery is an important feature. Accommodation capability of a control system depends on many factors but in particular the severity of the failure. In the next we consider first non critical fault which can be compensated via an appropriate additive control input and we propose then a method for control reconfiguration when degraded performances are required due to the severity of the fault.

3.1 Accommodation to soft failures

Once the FDI module indicates which sensor or actuator is faulty, the fault magnitude is estimated and a new control law is added to the nominal one to thwart the fault effect on the system. As sensor and actuator faults do not act in the same way on the system, the additive control law is not the same for both cases, but in the sequel only actuator faults are considered. The fault magnitude is estimated and a new control law is added to the nominal one to thwart the fault effect on the system. From equation (26) it is clear that $r_k$ represents an estimation of the fault. Thus, a fault free estimate of the actual state vector can be computed from (35) and (36)

$$x_{rec}^k = \left[ \begin{array}{c} \Sigma C \\ (I + L)\Sigma C \end{array} \right]^{-1} \left[ \begin{array}{c} \Sigma C\hat{x}_k \\ r_k + (I + L)\Sigma\hat{x}_k \end{array} \right]$$

(37)

and used for control reconfiguration. Reconfiguration strategy works as follows.

The fault free state estimate is used to feed back the controller while an additive control signal $u_{ad}$ is used to compensate for the fault effect on the system. Therefore, the total control law applied to the system is given by

$$u_k = -\left[ K_1 K_2 \right] \left[ \begin{array}{c} x_{rec}^k \\ z_{rec}^k \end{array} \right] + u_{ad}$$

(38)

with

$$z_{k+1} = z_k + T_s(y_k^* - Cx_{rec}^k).$$

(39)
where $K_1,K_2$ is computed by the LQI technique ([7]). The additional control law $u_{ad}$ must be computed such that the faulty system is as close as possible to the nominal one. In other terms, $u_{ad}$ must satisfy

$$Bu_{ad} + Fn_k = 0 \quad (40)$$

Using the estimation of the fault magnitude described in the previous section, the solution of (40) can be obtained by the following relation if matrix $B$ is of full row rank:

$$u_{ad} = -B^+F r_k \quad (41)$$

where $B^+$ is the pseudo-inverse of matrix $B$.

### 3.2 Reconfiguration in case of critical failures

In case of critical failures which cannot be compensated via an additive control design, the nominal performances cannot be preserved and degraded performances should be tolerated by the process operator. From a practical point of view, many industrial processes offer the possibility to deal with the output performances to minimize a reduction of objectives. The principle of active approaches, illustrated by Fig. 2, is very simple. After the fault occurrence, the system deviates from its nominal operating point defined by its input/output variables $M_0$ to a faulty one $M_f$. The goal of fault-tolerant control is to determine a new control law that takes the degraded system parameters into account and drives the system to a new operating point $M_r$ such that the main performances (stability, accuracy,...) are preserved (i.e., are as close as possible to the initial performances).

![Figure 2. The reconfiguration problem](image)

We show that under steady state operating conditions, a solution to the above mentioned can be easily proposed. Let us consider the dynamical system
(S) described by the discrete state equation (1) and (2). The different variables \(u_k, x_k, y_k\) designate variations around the nominal operating conditions \(U(0), X(0), Y(0)\) and under steady state conditions, the nominal operating point \(M_0 : (U_0, Y_0)\) satisfy to the constraints:

\[
S(U_0, Y_0) = 0
\]  

(42)

with the output controlled variables \(Y_0 = CX_0\). It is to be noted that what we call the operating point correspond to the set-points assigned to the controllers. But, if a critical failure occurs, and affects the system, then the solution is no longer valid and the output of the system move to a new operating point \(M_f : (U_f, Y_f)\) where subscript \(f\) designate the faulty conditions. Consequently, the initial performances are not reachable and the performance index must accommodate to new operating conditions closer to the initial ones.

The reconfiguration strategy which is proposed here relies on the acceptation of degraded performance for the reconfigured operating condition \(M_r : (U_r, Y_r)\).

Let us decompose the output vector into \(Y_r = [Y_p^r, Y_s^r]^T\) in order to exhibit primary and secondary output variables. The primary are considered of a prime importance for the system and should be kept constant at the nominal set-point values, thus, leading to the condition \(Y_p^r = Y_p^0\), while the secondary are free to evolve inside a region of the state space corresponding to acceptable degraded performances. This can be reflected via the optimization of the performance index \(\Psi([Y_s^r - Y_s^0], U)\) which is to be to defined according to operator requirements. Under the constraints given by Eq (42), and with respect to the criteria given in (43) which is to be optimized, the new operating point leading to the control reconfiguration, minimise the Lagrangian function:

\[
\ell = \Psi(Y_r^s - Y_0^s, U_r) + \mu^T(Y_r^s - Y_0^s) + \lambda^T S(U_r, Y_r) \tag{43}
\]

where components of vectors \(\lambda\) and \(\mu\) are the Lagrange multipliers.

4 Application to the IFATIS Benchmark

4.1 Benchmark Description

The approach which has been developed in the previous section is now applied to a plant benchmark which was used a benchmark in the framework of the EC project IFATIS (IFATIS, IST-2001-32122). IFATIS acronym stands for "Intelligent Fault Tolerant Control in Integrated Systems".

The plant illustrated by Fig.3 is composed of two cylindrical tanks which are used for mixing two liquids supplied by pumps driven by DC motors.

The system instrumentation includes 2 actuators and 6 sensors:

Actuators: the input-rates provided by the two pumps,

- Sensors: \(L_1\) and \(L_2\) are the level measurements while \(Q_1, Q_2\) and \(Q_{f1}, Q_{f2}\) the flow-rate at the output of the mixing tank are also measured.
4.2 Results

The linearized model of the tanks around the operating point \((L_{10}, L_{20}) = (0.5m, 0.6m), (Q_{10}, Q_{20}) = (6.6768e - 005, 7.4644e - 005)\) is given by the following discrete state-space

\[
A = \begin{bmatrix}
0.98866 & 0.0005794 \\
0.00056081 & 0.98628
\end{bmatrix}, \quad B = \begin{bmatrix}
-0.0024894 & 0 \\
0 & -0.0025626
\end{bmatrix}
\]

(44)

\[
C = \begin{bmatrix}
16.073 & 0 \\
0 & 16.088
\end{bmatrix}
\]

(45)

The effectiveness of the theory developed above is demonstrated in this section through the application to a real plant. One scenario involving a pump-stuck of tank 2 is considered to illustrate the results given by the fault detection filter and the reconfiguration strategie. The actuator-stuck faults can be modelled by the matrix \(F = B\) and \(q = m\) such that the function \(n_k\) and \((u_k + n_k)\) remains as constant. The pump of tank 2 is stucked to 1.6V and occurred at instant 300s. Then the objective is to keep the level of tank 2 close to the set point value.

![Figure3. Schematic diagram of the IFATIS benchmark](image1)

![Figure3. Schematic diagram of the IFATIS benchmark](image2)

![Figure4. Tank Level without reconfiguration](image3)
In the second case, two scenario involving a reduction of control effectiveness are considered to illustrate the results given by the fault detection filter and the compensation method as well. We consider a loss effectiveness of the first actuator which is represented by a change in matrix $B$ as follow:

$$B_f = B(I - \text{diag}(n_k))$$

To perform a loss of control effectiveness without breaking the system, the $i^{th}$ control input $U_i$ applied to the system is equal to the control input computed by the controller multiplied by a constant coefficient $i$ ($0 < n_i < 1$). In the first scenario, the effectiveness of the first pump is reduced by 10% and appears at instant 400s. According to the actuator fault description given earlier, this fault corresponds to a coefficient $n_1 = +0.1$ and appears abruptly on the system. In the second scenario, the same kind of fault, with a reduction of control effectiveness of 20% is applied to the second pump at instant 700s. The system outputs are displayed on Fig. 7.

Fig.10 clearly demonstrates the FTC method’s ability to compensate for such actuator faults. Indeed, since an actuator fault acts on the system as a perturbation, and due to the presence of the integral error in the controller, the system outputs again reach their nominal values even without fault compensation. It shows that, without FTC in Fig.7, the tanks levels reaches its corresponding...
reference input about 40s after the fault occurrence, whereas it takes only about
10s using the FTC method. These results can be confirmed by examining the
control inputs applied to the system: without the FTC method in Fig. 9, it in-
creases slowly due to the integral error trying to compensate for the fault effect,
whereas the FTC method makes this control input increase quickly (Fig. 11)
and enables the rapid fault compensation.
Figure 9. Zoom on the system inputs $U_1$ and $U_2$ without reconfiguration

Figure 10. System outputs with reconfiguration

Figure 11. Zoom on the system inputs $U_1$ and $U_2$ with reconfiguration
5 Conclusion

The general fault-tolerant control method described in this paper addresses actuator fault but can also be applied to sensor faults. The proposed strategy for FTC relies on FDI supervised control. In addition to providing information to operators concerning the system operating conditions, the fault diagnosis module is especially important in fault-tolerant control systems where one need to know exactly which element is faulty to react safely. The IFATIS Benchmark has been used to demonstrate the benefits of using FTC strategies. The results clearly show the effectiveness of the proposed fault tolerant controller as compared with the classical control architecture. By using the approach, it is shown that the system is able to recover normal performances in a minimum time after fault detection. We have also presented a strategy of control restructuration in case of critical faults which gave good results. When there is complete loss of an actuator; in this case, only a hardware redundancy is effective and could ensure performance reliability.

Références