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Further results on the stabilization of fuzzy constrained control systems

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Abstract This paper deals with the extension of the positive invariance approach to nonlinear systems modeled by Takagi-Sugeno fuzzy systems. The saturations on the control are taken into account during the design phase. Sufficient conditions of asymptotic stability are given ensuring in the same time that the control is always admissible inside the corresponding set. A piecewise Lyapunov function is used.

keywords Nonlinear systems, fuzzy systems, constrained control, positive invariance concept.

1 Notation

– For two vectors $x,\ y\ \in\ \mathbb{R}^n,\ x\ \preceq\ y$ if $x_i\ \preceq\ y_i,\ i\ =\ 1,\ldots,n.$
– A positive definite matrix P is noted $P > 0$.
– we note $P < Q$ if matrix $P - Q$ is negative definite.

2 Introduction

Most plant in the industry have sever nonlinearity associated to saturations on the control. With the development of fuzzy systems, it is now possible to obtain a nonlinear representation by the qualitative knowledge of a system. On the behalf of this idea, some fuzzy models based control system design methods have appeared in the fuzzy control field [15],[16],[19] leading to many applications on nonlinear systems [1], [9]. The nonlinear system is represented by a Takagi-Sugeno (TS) type fuzzy model. However, to the best of our knowledge, the non quadratic saturations on the control are taken into account in the design of the fuzzy control only for a class of fuzzy systems [6].

It is well known that all these plants admit inputs limitation which are modelled by constraints of inequality type. The regulator problem for linear systems with constrained control was widely studied during these two decades. The tool of positive invariance was successfully applied to almost all the systems with constrained control, see for example [2]- [7] and the references therein.

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In this paper, the saturations on the control are taken into account with the fuzzy model. The concept of positive invariance is used to obtain sufficient conditions of local asymptotic stability for the global fuzzy system with constrained control inside a subset of the state space. The main idea of [8] representing the nonlinear system by a set of like uncertain linear subsystems is used in this paper. The problem is then to design a controller which is ”robust” with respect to the upper bound extreme subsystems by taking into account the saturations on the control. A piecewise Lyapunov function used in [8] and [14] is used to analyze and design the controllers which ensure asymptotic stability of the nonlinear system despite the presence of saturations on the control.

In a previous work [11], the same methodology was used with a common Lyapunov function and a piecewise Lyapunov function. The controller gains were designed such that all the level sets associated to the corresponding subsystems contain a same predefined polyhedra to ensure the asymptotic stability inside a common region. Nevertheless, in this work, we show that even a piecewise Lyapunov function is used, no common region is needed at all to guarantee the asymptotic stability of the fuzzy system despite the presence of constraints on the control. Hence, a set of Linear Matrix Inequalities (LMIs) is proposed to built stabilizing controllers.

This paper is organized as follows: Section 3 deals with the problem presentation while Section 4 presents some preliminary results concerning the technique of rewriting equivalently the fuzzy system under the form of r like uncertainty subsystems. The main results of the paper are given by Section 5. An example is studied in Section 6.

3 Problem presentation

Let us consider the following nonlinear system with constrained control that can be described by the T-S fuzzy model:
IF $x_1(t)$ is M_{i1} and $\ldots$ and $x_n(t)$ is M_{in} THEN,

$$\dot{x}(t) = A_i x(t) + B_i u(t), i = 1, \ldots, r \tag{1}$$

where M_{ij} is the fuzzy set, r is the number of IF-THEN rules, $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ is the control which is constrained as follows:

$$u \in \Omega = \{u \in \mathbb{R}^m, -q_2 \preceq u \preceq q_1;\ q_1,\ q_2 \in \mathbb{R}^m\}. \tag{2}$$

Following [16]-[19], the global model is structured in the following form,

$$\dot{x}(t) = A(\alpha)x(t) + B(\alpha)u(t) \tag{3}$$

with,

$$\alpha = \begin{bmatrix}\alpha_1 \ldots \alpha_r\end{bmatrix}^\top$$

$$A(\alpha) = \sum_{i=1}^{r} \alpha_i(x(t))A_i; \tag{4}$$

$$B(\alpha) = \sum_{i=1}^{r} \alpha_i(x(t))B_i; \tag{5}$$

with, $\alpha_i(x(t))$ is the normalized membership function of the inferred fuzzy set $M_i = \prod_{l=1}^{n} M_{il}$ and,

$$\alpha_i(x(t)) \geq 0, i = 1, \ldots, r; \sum_{i=1}^{r} \alpha_i(x(t)) = 1 \tag{6}$$

Let $\mathcal{M}$ be the set of membership functions satisfying (6).

A_i and B_i are constant Matrices of appropriate size and each pair (A_i, B_i) is assumed to be stabilizable.

In general, the control is given by,

$$u(t) = F(\alpha)x(t) \tag{7}$$

This control leads to the following closed-loop system,

$$\dot{x}(t) = [A(\alpha) + B(\alpha)F(\alpha)]\, x(t) \tag{8}$$

The main objective of this paper is to design the controller $F(\alpha)$ such that the global system is asymptotically stable at the origin despite the presence of constraints on the control. To achieve this objective, two techniques will be used: The first consists in rewriting equivalently the initial system (1) by using a state space repartition allowing to introduce r like uncertain subsystems as used before by many authors. The second concerns the use of the so-called positive invariance approach which will enable one to construct regions where the control never saturates.

4 Preliminary results

In this section, we recall the technique of rewriting equivalently the fuzzy system (3) under the form of r like uncertainty subsystems as proposed in [8].

Let us consider the nonlinear system with constrained control that can be described by the T-S fuzzy model (1). Following the idea of [8], one can divide the input space into fuzzy subspaces and build a linear model, called the local model, in each subspace. Then, the membership function is used to connect smoothly the local models together to form a global fuzzy model of the nonlinear system. Define in the state space the set of subspaces $\{S_j \quad j = 1, \cdots, r\}$ as follows :

$$S_j = \{x \in \mathbb{R}^n / \alpha_j(x) \geq \alpha_i(x), i = 1, \cdots, r, i \neq j\}, \tag{9}$$

The characteristic function of the set S_j is defined by :

$$\eta_j = \begin{cases} 1, & x \in S_j \\ 0, & x \notin S_j \end{cases}; \qquad \sum_{j=1}^{r} \eta_j = 1 \tag{10}$$

On every subspace S_j the fuzzy system can be denoted by:

$$\dot{x}(t) = (A_j + \Delta A_j(t))x(t) + (B_j + \Delta B_j(t))u(t) \tag{11}$$

with,

$$\Delta A_j(t) = \sum_{i=1, i\neq j}^{r} \alpha_i(t)(A_i - A_j);$$
$$\Delta B_j(t) = \sum_{i=1, i\neq j}^{r} \alpha_i(t)(B_i - B_j)$$

The idea of this approach is to choose on every subspace $S_j, j \in 1, \ldots, r$, the fuzzy subsystem (11) and consider that the interaction of the corresponding system with all the remainder $r-1$ subsystems is taken into account by the like uncertainty terms $\Delta A_j(t)$ and $\Delta B_j(t)$. It also is assumed that if the jth subsystem is in the jth subspace, it will stay in this subspace for a time $t_j > \tau, \tau > 0$ is a fixed constant. The number of traversing time instants among the regions is also assumed to be finite.

Remark 1. It is useful to note that $\Delta A_j(t)$ and $\Delta B_j(t)$ are known at any time and the studied system is not an uncertain system. However, in order to obtain simpler stability conditions, this technique assumes that the terms $\Delta A_j(t)$ and $\Delta B_j(t)$ are like uncertain terms and are bounded.

Following the idea of [8], we assume that an upper bound of each like uncertainty term is known and is given by,

$$[\Delta A_j(t)]^T[\Delta A_j(t)] \leq E_{1j}^T E_{1j}, \forall t \succeq 0; j = 1, \ldots, r \tag{12}$$

$$[\Delta B_j(t)]^T[\Delta B_j(t)] \leq E_{2j}^T E_{2j}, \forall t \succeq 0; j = 1, \ldots, r \tag{13}$$

Note that the details about the estimation of the upper bounds according to (12)-(13) are widely developed in [8]. Using these upper bounds one can define the following extreme subsystems:

$$\dot{x}(t) = (A_j + E_{j1})x(t) + (B_j + E_{j2})F_j x(t),$$
$$x(t) \in S_j, j = 1, \ldots, r$$

Hence, we obtain r distinct linear time-varying subsystems. The stabilization problem of the unconstrained fuzzy system (1) has been studied in [8] by using these extreme subsystems.

5 Main results

In this section, we propose sufficient conditions of local asymptotic stability for the system with constrained control, by using a piecewise Lyapunov function. This result is based on the technique of rewriting equivalently the fuzzy system (3) under the form of r like uncertainty subsystems as proposed in [8]. We consider that the control is constrained as follows:

$$u \in \Omega = \{u \in \mathbb{R}^m, -q_2 \preceq u \preceq q_1;\ q_1,\ q_2 \in \mathbb{R}^m\}. \tag{14}$$

The objective is then to design for such a subsystem a feedback control given by:

$$u(t) = F_j x(t), x(t) \in S_j \tag{15}$$

which guarantees the asymptotic stability of the like uncertain subsystem (11) despite the presence of the saturations (14). The subsystem in closed-loop is given by:

$$\dot{x}(t) = [(A_j + B_j F_j) + (\Delta A_j(t) + \Delta B_j(t) F_j)]\, x(t) \tag{16}$$

Note that the control in system (3) can be considered in this approach as a switching control formed by all the subsystems controls and given by,

$$u(t) = \sum_{j=1}^{r} \eta_j F_j x(t) \tag{17}$$

In the constrained case, recall that model (16) remains valid every time only if the state is constrained to evolve in a specified region defined by

$$\mathcal{D}_j = \{x \in \mathbb{R}^n / -q_2 \preceq F_j x \preceq q_1;\ \ q_1, q_2 \in \mathbb{R}^m\}; \tag{18}$$

Note that these domains are convex and unbounded for $m < n$.

In this work, we follow the approach proposed in [12], [4], [5]. This approach uses the following piecewise smooth quadratic Lyapunov function candidate:

$$V(x) = x^T P x(t) \tag{19}$$

where $P = \sum_{j=1}^{r} \eta_j P_j$. Let us define the level set of this function by :

$$\Psi(P, \rho) = \{x \in\ \mathbb{R}^n | V(x) \preceq \rho;\ \rho \succ 0\}$$

In a previous work [11], the same methodology was used with a common Lyapunov function for all the r upper bound extreme subsystems and a piecewise Lyapunov function. The controller gains F_j were designed such that all the level sets associated to matrices $P_j, j = 1, \ldots, r$ contain a same predefined polyhedra Γ to ensure the asymptotic stability inside a common region. Nevertheless, in this work, we show that even a piecewise Lyapunov function is used, no common region is needed at all to guarantee the asymptotic stability of the fuzzy system despite the presence of constraints on the control.The aim of this approach consists in giving conditions allowing the choice of a stabilizing controller (15) in such a way that :

– $V(x)$ is Lyaponuv function of the fuzzy system.
– There exist a positif scalar ρ such that $\Psi(P, \rho) \subseteq \bigcap \mathcal{D}_j$.

Hence, for all $x \in \Psi(P, \rho)$ the system trajectory converge to the origin and the control never saturates.

for this, we recall bellow the result of stabilizability of the unconstrained fuzzy system, using the idea of [8] based on the upper extreme subsystems. The conditions of asymptotic stability for the fuzzy system (3) are given according to the following definition.

Definition 1. *The system (3) is said to be quadratically stabilizable if there exists a control law (7), a positive symmetric matrix P and a scalar $\gamma > 0$ such that the following condition is satisfied:*

$$\dot{V}(x(t)) = x(t)^T \left\{ [A(\alpha) + B(\alpha)F(\alpha)]^T P + P[A(\alpha) + B(\alpha)F(\alpha)] \right\} x(t) \leq -\gamma \|x\|^2 \tag{20}$$

$\forall x(t) \in \mathbb{R}^n$, $\forall \alpha \in \mathcal{M}$, $\forall t > 0$ *where* $V(x) = x^T P x$ *is a Lyapunov function.*

It is worth noting that if the system (3) is quadratically stabilizable, then function $V(x)$ is a Lyapunov function for the closed-loop system (8). Then, the equilibrium point $x = 0$ will be uniformilly asymptotically stable in the large.

Lemma 1. *[8]: The fuzzy system (3) is quadratically stabilizable if and only if there exists a set of feedback gains $(F_1, F_2, \ldots, F_r)$ such that the following closed loop subsystems with the accurate upper bounds are quadratically stable:*

$$\dot{x}(t) = (A_j + E_{j1})x(t) + (B_j + E_{j2})F_j x(t), \quad x(t) \in S_j, j = 1, \ldots, r \tag{21}$$

Recall that the stability result obtained by [8] is based on the use of Lemma 6 and a piecewise Lyapunov function candidate (19), as used by [14].

The use of the lemma 1 and the result of [12] enable us to state the main result of this paper concerning the asymptotic stability of the fuzzy system (3) with the saturations (2).

Theorem 1. *: If there exist a set of symmetric positives definite matrix $P_i \in \mathbb{R}^{n \times n}$ and a positive scalar ρ such that:*

$$(A_j + B_j F_j)^T P_j + P_j(A_j + B_j F_j) + (E_{j1} + E_{j2}F_j)^T P_j + P_j(E_{j1} + E_{j2}F_j) < 0; \quad j = 1, \ldots, r; \tag{22}$$

$$\Psi(P, \rho) \subset \mathcal{D}_j, j = 1, \ldots, r \tag{23}$$

Then, the fuzzy system (3) with the feedback control (17) is asymptotically stable $\forall x_0 \in \Psi(P, \rho)$.

Proof. Conditions (22) imply that the function $V(x) = x^T P x$ is a Lyapunov functions of all the upper bound extreme subsystems (21). Recall that the level set $\Psi(P, \rho)$ of the Lyapunov function is positively invariant w.r.t the upper bound extreme subsystems. According to Lemma 6 and Definition 5, this set is also a level set (region of stability) for the like uncertain subsystems (16), that is, the set $\Psi(P, \rho)$ is also positively invariant w.r.t the like uncertain subsystems (16). Thus, the control is always admissible i.e., $-q_2 \preceq F_j x(t) \preceq q_1, \forall t \succeq 0$ by virtue of conditions (23). Consequently, each control $u(t) = F_j x(t)$ is admissible $\forall x_0 \in \Psi(P, \rho)$ and the linear subsystem (16) is always valid inside this region of linear behavior. Hence, it is obvious that by applying the switching control (17) to the like uncertain fuzzy system (11), the control remains admissible by virtue of the following,

$$-q_2 \preceq F_j x(t) \preceq q_1, \forall t \succeq 0 \text{ implies}$$
$$-q_2 \preceq \sum_{j=1}^{r} \eta_j F_j x(t) \preceq q_1, \forall t \succeq 0; j = 1, \ldots, r$$

where η is defined by (10). In order to guarantee that this implication remains satisfied even if the state switches from a subspace S_j to a different subspace $S_i, i \neq j$, it is necessary to take the initial state inside the common domain $\Psi(P, \rho)$. The positive invariance property of the set $\Psi(P, \rho)$, implies that all the uncertain subsystems (16) remain linear despite the presence of the saturations. This fact, allows the application of the Lemma 6 and Definition 5 to these like uncertain subsystems to obtain r upper bound extreme subsystems by using the assumptions (12). If in addition the feedback controllers F_j satisfy conditions (22), then the global fuzzy system (3) with the feedback control (17) is asymptotically stable at the origin $\forall x_0 \in \Psi(P, \rho)$ despite the presence of saturations. ∇

It is worth noting that to include a symmetric ellipsoid inside a non symmetrical polyhedra, it is sufficient to realize this only inside the symmetrical part of the polyhedra. This means in our case, to realize (23) only with $q = min(q_1, q_2)$. It is well known that to obtain condition (23), one has only to satisfy the following inequalities [13],

$$\rho f_j^i P^{-1} (f_j^i)^T \preceq q_i^2, j = 1, \ldots, r; i = 1, \ldots, m, \tag{24}$$

where f_j^i is the ith row of matrix F_j, $q = min(q_1, q_2)$. These inequalities can be transformed by the use of Schur complement to the following LMI,

$$\begin{bmatrix} \mu_i & y_j^i \\ * & X \end{bmatrix} \geq 0, i = 1, \ldots, m \tag{25}$$

where y_j^i is the ith row of matrix $Y_j = F_j X$, $X = P^{-1}$ and $\mu_i = q_i^2/\rho$.

The result of Theorem 7 is now used for the control synthesis.

Theorem 2. *: For given positive scalars ρ, if there exist symmetric definite positive matrices $X_1, \ldots, X_r$ and matrices $Y_1, \ldots, Y_r$, solutions of the following LMIs:*

$$X_j(A_j + E_{1j})^T + Y_j^T (B_j + E_{2j})^T + (A_j + E_{1j})X_j + (B_j + E_{2j})Y_j < 0, \tag{26}$$

$$\begin{bmatrix} \mu_i & y_j^i \\ * & X_k \end{bmatrix} \geq 0, \tag{27}$$

$$X_k > 0,$$

$$j = 1, \ldots, r; i = 1, \ldots, r; k = 1, \ldots, r$$

where $\mu_i = q_i^2/\rho$, y_j^i is the ith row of matrix Y_j;
Then, the fuzzy system (3) with the feedback control (17) with,

$$F_j = Y_j X^{-1} \tag{28}$$
$$P_i = X_i^{-1} \tag{29}$$

is asymptotically stable at the origin $\forall x_0 \in \Psi(P,\rho)$.

Proof. Follows readily from Theorem 6. ∇

This result is easily applied to design controllers: solving the LMI's (26)-(27) by any common available software (in our case we used the matlab LMI control toolbox), matrices P_i and the controllers gains F_i can be computed easily according to the equalities (28) and (29).

6 Example

Let us Consider the following constrained nonlinear system,

$$\begin{aligned}\dot{x}_1(t) &= -2.1x_1 + 1.5x_2(t) + 2.5u_1(t) + 0.5u_2(t)\\ \dot{x}_2(t) &= 3.5x_1(t) - 0.5[0.5 + ln(x_1^2+1)]x_2(t) + u_1(t) - 1.5u_2(t)\end{aligned}$$

where the control is constrained as follows:

$$-q_2 \preceq u \preceq q_1; q_1 = \begin{bmatrix} 35\\ 45\end{bmatrix}; q_2 = \begin{bmatrix} 40\\ 45\end{bmatrix}$$

Now we give the exact approximation of the nonlinear system by a TS model. For this, assume that $x_1(t) \in [-\gamma, \gamma]$, then one can write,

$$ln(x_1^2+1) = M_1^1(x_1(t)).0 + M_1^2(x_1(t)).ln(\gamma^2+1) \tag{30}$$

with,

$$\begin{aligned} M_1^1(x_1(t)) &= \frac{ln(\gamma^2+1) - ln(x_1^2+1)}{ln(\gamma^2+1)} = \alpha_1(t)\\ M_1^2(x_1(t)) &= 1 - M_1^1(x_1(t)) = \frac{ln(x_1^2+1)}{ln(\gamma^2+1)} = \alpha_2(t)\end{aligned}$$

The fuzzy model which represents exactly the nonlinear system is given by,

$$\text{If } x_1(t) \text{ is } M_1^1 \text{Then } \dot{x}(t) = A_1x(t) + B_1u(t); -q_2 \preceq u \preceq q_1$$

$$\text{If } x_1(t) \text{ is } M_1^2 \text{Then } \dot{x}(t) = A_2x(t) + B_2u(t); -q_2 \preceq u \preceq q_1$$

where matrices A_1, A_2, B_1 and B_2 are given by,

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.1 & 1.5\\ 3.5 & -0.25\end{bmatrix}; B_1 = \begin{bmatrix} 2.5 & 0.5\\ 1 & -1.5\end{bmatrix}\\ A_2 &= \begin{bmatrix} -2.1 & 1.5\\ 3.5 & -0.5(0.5 + ln(\gamma^2+1))\end{bmatrix}; B_2 = B_1.\end{aligned}$$

For this fuzzy system composed of two subsystems . The following upper bounds can be taken :

$$E_{11} = 0.25|A_2 - A_1|; E_{21} = 0; E_{12} = 0.25|A_1 - A_2|; E_{22} = 0.$$

Solving the LMI (26)-(27) for $\gamma = 15$ we find:

$$P_1 = \begin{bmatrix} 0.1044 & 0.0050 \\ 0.0050 & 0.0356 \end{bmatrix}; P_2 = \begin{bmatrix} 0.0796 & -0.0395 \\ -0.0395 & 0.0511 \end{bmatrix}$$

The obtained gain controllers are given by,

$$F_1 = \begin{bmatrix} -0.3501 & -0.7210 \\ 1.0798 & 0.1654 \end{bmatrix}; F_2 = \begin{bmatrix} 0.1946 & -0.4226 \\ 0.7014 & -0.4580 \end{bmatrix}$$

The set of positive invariance $\Psi(P, \rho)$ is depicted in the Figure 1 together with the sets $\mathcal{D}_j$.

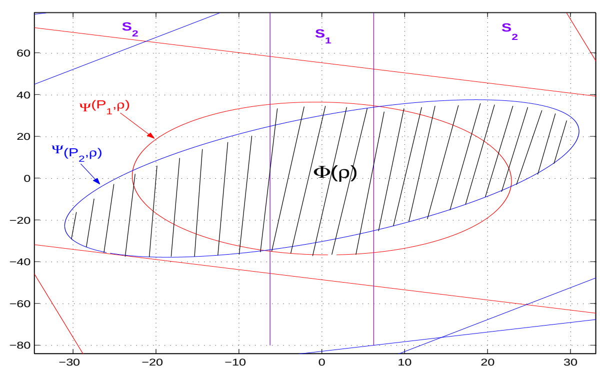

Figure1. The set $\Phi(\rho)$ representation.

7 Conclusion

In this paper, the problem of constrained nonlinear systems represented by fuzzy systems is studied. The positive invariance tool is used. Sufficient conditions of asymptotic stability are obtained despite the presence of saturations on the control by using a piecewise Lyapunov function. The used approach is the one followed in [8] with like uncertain subsystems and upper bound subsystems. The obtained results are successfully applied to a nonlinear systems with constrained control, represented by Takagi-Sugeno (TS) type fuzzy model.

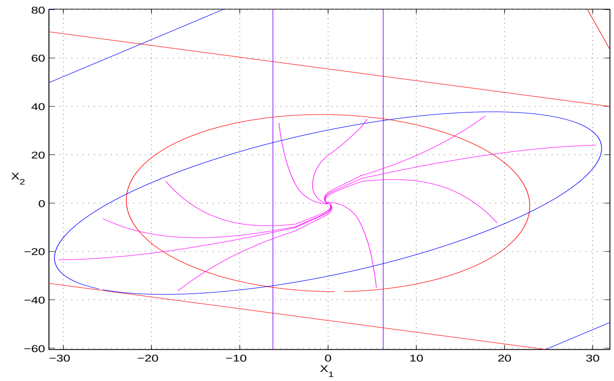

Figure2. This figure presents the evolution of the state of the system in closed-loop inside the common set of positive invariance $\Phi(\rho)$ for different initial states.

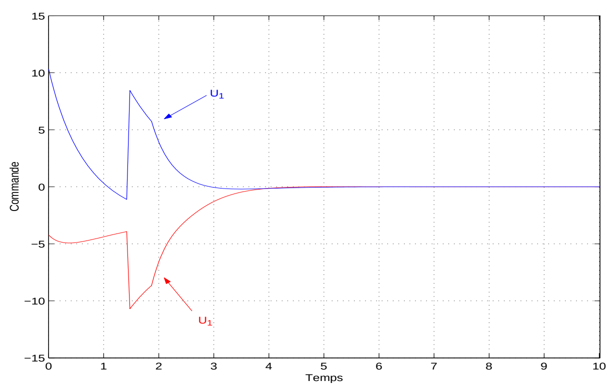

Figure3. The control evolution for an initial state inside the common set of positive invariance $\Phi(\rho)$.

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