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Delay-Dependent H$_\infty$ Control of Uncertain Discrete Delay Systems

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Résumé. In this paper, a delay-dependent solution is given for state-feedback H_∞ control of discrete time systems with norm-bounded uncertainties. Sufficient conditions are obtained for stabilization and for achieving design specifications which are based on Lyapunov-Krasovskii functionals. Numerical examples demonstrate the merit of the present condition in the aspect of conservativeness over other results in the literature.

Key words$-$ *H_∞ control design, Discrete-time systems, Delay-dependent condition, Linear matrix inequality, time-delay.*

1 Introduction

In this paper, a H_∞ control problem for discrete-time systems with time-delay is considered. It is well established in the literature that time-delay is usually the cause of performance degradations for dynamical systems. It can even be, in some circumstances, the cause of instability of the system that we would like to control if such time-delay is not taken into account during the design phase. Time delay may occur either in continuous-time [Fridman and Shaked, 2002,He et al., 2004] or discrete-time systems [Boukas, 2006], [Fridman and Shaked, 2005], [Hmamed and Tissir, 1998,Tissir, 2007]. It is worth noting that most physical systems evolve in continuous-time and it is natural that investigations in stability analysis and controller synthesis are mainly developed for continuous-time systems. However, it is more reasonable that one should use a discrete-time approach for that purpose because the controller is usually implemented digitally (see [Lee and Kwon, 2002] and the references therein). In addition the size of the delay, and especially for unknown delays, makes the transformation into a standard discrete time model hardly realizable.
The delay-independent stabilization provides a controller which stabilizes a system irrespective of the size of the delay. On the other hand, the delay-dependent stabilization is concerned with the size of the delay and usually provides an upper bound of the delay such that the closed-loop system is stable for any delay less than the upper bound. Delay-independent and, delay-dependent conditions for H_∞ control expressed in terms of linear matrix inequalities (*LMIs*) can be easily solved using dedicated solvers [Boyd et al., 1994].

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The goal of this paper consists of considering the class of discrete time linear systems with delay and develop sufficient conditions for H_∞ control design method that depend on the upper bounds of the delays for all admissible uncertainties. The Lyapunov-Krasovskii approach will be used in this paper. Finally, some numerical examples are given to illustrate that the results are less conservative than previous work.

Notation: The following notations will be used throughout the paper. $\Re$ denotes the set of real numbers, $\Re^n$ denotes the n dimensional Euclidean space and $\Re^{m\times n}$ denotes the set of all $m \times n$ real matrices. The notation $X \geq Y$ (respectively $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite). By L_2 we denote the space of sequences $x(k)$, $k = 0, 1, ...$ with the norm $\|x(k)\|_2^2 = \sum_{k=0}^{\infty} x(k)^T x(k) < \infty$.

2 System description and preliminary

Consider the uncertain discrete-time system with delay described by the following equation:

$$\begin{aligned} x(k+1) &= A_0(\Delta)x(k) + A_1(\Delta)x(k-h) + B_0(\Delta)u(k) + B_1 w(k) \\ z(k) &= C_0 x(k) + C_1 u(k) \\ x(k) &= \phi(k), \ \ -h \leq k \leq 0 \end{aligned} \tag{1}$$

with: $A_0(\Delta) = \big(A_0 + \Delta A_0(k)\big)$, $A_1(\Delta) = \big(A_1 + \Delta A_1(k)\big)$ and $B_0(\Delta) = \big(B_0 + \Delta B_0(k)\big)$
Where $x(k) \in \Re^n$is the state vector, $w(k) \in \Re^l$ is the disturbance input which is assumed to be of bounded energy, $u(k) \in \Re^m$ is the control input, $z(k) \in \Re^d$ is the objective vector, h is the discrete delay. A_0, A_1, B_0, B_1 and $C_i, i = 0, 1$ are known real constant matrices, $\Delta A_0, \Delta A_1$ and ΔB_0 are unknown real bounded matrix functions representing time-varying parameter uncertainty. The admissible uncertainties are assumed to be of the form:

$$\big[\Delta A_0(k) \ \ \Delta A_1(k) \ \ \Delta B_0(k)\big] = DF(k)\big[E_0 \ \ E_1 \ \ E_2\big] \tag{2}$$

Where E_0, E_1, E_2 and D are constant matrices of appropriate dimension and $F(k)$ is unknown real time varying matrix satisfying

$$F^T(k)F(k) \leq I \tag{3}$$

Introduce the new terms:
$\widetilde{A}_0 = A_0 + DF(k)E_0$, $\widetilde{A}_1 = A_1 + DF(k)E_1$ and $\widetilde{B}_0 = B_0 + DF(k)E_2$.
Then, the motion of the system (1) can be described as follows:

$$\begin{aligned} x(k+1) &= \widetilde{A}_0 x(k) + \widetilde{A}_1 x(k-h) + \widetilde{B}_0 u(k) + B_1 w(k), \\ z(k) &= C_0 x(k) + C_1 u(k). \end{aligned} \tag{4}$$

It is assumed, in the analysis part, that
A0. *The eigenvalues of* $A_0 + A_1$ *are all of absolute value less than* 1.
We address first the following two analysis problems.

Problem 21 *For $u(k) = 0$ and for a given scalar γ find whether the system is asymptotically stable and the following holds:*

$$J = \|z(k)\|_2^2 - \gamma^2 \|w(k)\|_2^2 < 0, \;\; \forall 0 \neq \{w(k)\} \in L_2$$
$$\textit{for } \; \phi(k) = 0, \;\; -h \leq k \leq 0 \tag{5}$$

Problem 22 *Find a control law $u(k)$ that stabilizes the system (1) and (5) is satisfied for a given scalar γ.*

Once solutions are obtained to the above problems, the problem of finding a state-feedback control law which stabilizes the system and achieves (5) for a prescribed γ will be considered.

3 Stability

In this section, we present delay-dependent condition that can be used to check if the discrete time system we are considering is stable for the case where $u(k) = 0, k \geq 0$.

Theorem 31 *The discrete-time system (1) is robustly stable for a prescribed scalar $\gamma > 0$, if there exist positive definite symmetric matrices $P_1 = P_1^T \in \Re^{n\times n}, Q = Q^T \in \Re^{n\times n}$ and $R = R^T \in \Re^{n\times n}$, matrices $P_i \in \Re^{n\times n}, i = 2, ..., 6$ and a positive scalar ε such that the following LMI is verified*[1]*:*

$$\Gamma = \begin{pmatrix} \Gamma_{11} & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & * & * & * \\ -P_4 & -P_5 & -P_6 & -\frac{1}{h}R & * & * \\ B_1^T P_2 & B_1^T P_3 & 0 & 0 & -\gamma^2 I & * \\ D^T P_2 & D^T P_3 & 0 & 0 & 0 & -\varepsilon I \end{pmatrix} < 0 \tag{6}$$

where

$$\begin{aligned}
\Gamma_{11} &= P_2^T(A_0 - I) + (A_0 - I)^T P_2 + P_4 + P_4^T + Q + C_0^T C_0 + \varepsilon E_0^T E_0 \\
\Gamma_{21} &= P_1 + P_3^T(A_0 - I) + P_5^T - P_2 \\
\Gamma_{22} &= hR - P_3 - P_3^T \\
\Gamma_{31} &= A_1^T P_2 - P_4 + P_6^T + \varepsilon E_1^T E_0 \\
\Gamma_{32} &= A_1^T P_3 - P_5 \\
\Gamma_{33} &= -P_6 - P_6^T - Q + \varepsilon E_1^T E_1
\end{aligned} \tag{7}$$

Proof :
To prove our Theorem, assume A0. and consider the following change of variables:

$$y(k) = x(k+1) - x(k),$$
$$0 = -y(k) + (\widetilde{A}_0 - I)x(k) + \widetilde{A}_1 x(k-h) + B_1 w(k) \tag{8}$$

[1] The symbol * stands for symmetric block in matrix inequalities

and taking into account that

$$x(k-h) = x(k) - \sum_{j=k-h}^{k-1} y(j) \tag{9}$$

We consider the following Lyapunov-Krasovskii candidate functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{10}$$

with

$$V_1(k) = x^T(k)P_1x(k),\ V_2(x(k)) = \sum_{j=k-h}^{k-1} x^T(j)Qx(j),\ V_3(k) = \sum_{m=-h}^{-1}\sum_{j=k+m}^{k-1} y^T(j)Ry(j). \tag{11}$$

where $P_1 > 0, Q > 0$ and $R > 0$. We apply the Lyapunov-Krasovskii method and require that $\Delta V(k)$ is strictly negative to guarantee the asymptotic stability of the system and that $\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k)$ is strictly negative in order to satisfy (5).
we now compute $\Delta V_1(x(k))$:

$$\begin{aligned} \Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= x^T(k+1)P_1x(k+1) - x^T(k)P_1x(k) \end{aligned} \tag{12}$$

Let $P = \begin{pmatrix} P_1 & 0 & 0 \\ P_2 & P_3 & 0 \\ P_4 & P_5 & P_6 \end{pmatrix}$, using (8) and (9), we can write the term of (12) as follows:

$$\Delta V_1(k) = y^T(k)P_1y(k) + 2\widetilde{x}^T(k)P^T \begin{pmatrix} y(k) \\ 0 \\ 0 \end{pmatrix} = y^T(k)P_1y(k)$$

$$+2\widetilde{x}^T(k)P^T \begin{pmatrix} y(k) \\ -y(k) + (\widetilde{A}_0 - I)x(k) + \widetilde{A}_1x(k-h) + B_1w(k) \\ x(k) - x(k-h) - \sum_{j=k-h}^{k-1} y(j) \end{pmatrix} \tag{13}$$

where $\widetilde{x}(k) = \begin{pmatrix} x^T(k) & y^T(k) & x^T(k-h) \end{pmatrix}^T$
For $V_2(k)$, by standard manipulation, we have

$$\begin{aligned} \Delta V_2(k) &= V_2(x(k+1)) - V_2(x(k)) \\ &= \sum_{j=k+1-h}^{k} x^T(j)Qx(j) - \sum_{j=k-h}^{k-1} x^T(j)Qx(j) \\ &= x^T(k)Qx(k) - x^T(k-h)Qx(k-h) \end{aligned} \tag{14}$$

For $V_3(k)$, one has

$$\begin{aligned} \Delta V_3(k) &= \sum_{m=-h}^{-1}\sum_{j=k+1+m}^{k} y^T(j)Ry(j) - \sum_{m=-h}^{-1}\sum_{j=k+m}^{k-1} y^T(j)Ry(j) \\ &= hy^T(k)Ry(k) - \sum_{j=k-h}^{k-1} y^T(j)Ry(j) \end{aligned} \tag{15}$$

By Cauchy-Schwartz inequality

$$h\sum_{j=k-h}^{k-1} y^T(j)Ry(j) \geq \Big(\sum_{j=k-h}^{k-1} y^T(j)\Big)R\Big(\sum_{j=k-h}^{k-1} y(j)\Big) \tag{16}$$

Hence

$$\Delta V_3(k) = hy^T(k)Ry(k) - \frac{1}{h}\Big(\sum_{j=k-h}^{k-1} y^T(j)\Big)R\Big(\sum_{j=k-h}^{k-1} y(j)\Big) \tag{17}$$

It follows from (13), (14) and (17) that

$$\Delta V(k) \leq \widetilde{x}^T(k)\Xi\widetilde{x}(k) + y^T(k)P_1y(k) + 2\widetilde{x}^T(k)P^T\begin{pmatrix}0\\0\\-I\end{pmatrix}\sum_{j=k-h}^{k-1} y(j)$$

$$+2\widetilde{x}^T(k)P^T\begin{pmatrix}0\\B_1\\0\end{pmatrix}w(k) + x^T(k)Qx(k)$$

$$-x^T(k-h)Qx(k-h) + hy^T(k)Ry(k) - \frac{1}{h}\Big(\sum_{j=k-h}^{k-1} y^T(j)\Big)R\Big(\sum_{j=k-h}^{k-1} y(j)\Big) \tag{18}$$

where $\Xi = P^T\begin{pmatrix}0 & I & 0\\ \widetilde{A}_0 - I & -I & \widetilde{A}_1\\ I & 0 & -I\end{pmatrix} + \begin{pmatrix}0 & (\widetilde{A}_0 - I)^T & I\\ I & -I & 0\\ 0 & \widetilde{A}_1^T & -I\end{pmatrix}P$

Now, we consider the asymptotical stability with $w(k) = 0$, then (18)becomes

$$V(x(k)) \leq \Omega^T(k,j)\Upsilon\Omega(k,j) \tag{19}$$

where

$$\Upsilon = \begin{pmatrix}\Upsilon_{11} & \Upsilon_{21}^T & \Upsilon_{31}^T & -P_4^T\\ \Upsilon_{21} & \Upsilon_{22} & \Upsilon_{32}^T & -P_5^T\\ \Upsilon_{31} & \Upsilon_{32} & \Upsilon_{33} & -P_6^T\\ -P_4 & -P_5 & -P_6 & -\frac{1}{h}R\end{pmatrix} \tag{20}$$

$$\begin{aligned}
\Upsilon_{11} &= P_2^T\big(\widetilde{A}_0 - I\big) + \big(\widetilde{A}_0 - I\big)^T P_2 + P_4 + P_4^T + Q\\
\Upsilon_{21} &= P_1 + P_3^T\big(\widetilde{A}_0 - I\big) + P_5^T - P_2\\
\Upsilon_{22} &= hR - P_3 - P_3^T\\
\Upsilon_{31} &= \widetilde{A}_1^T P_2 - P_4 + P_6^T\\
\Upsilon_{32} &= \widetilde{A}_1^T P_3 - P_5\\
\Upsilon_{33} &= -P_6 - P_6^T - Q
\end{aligned}$$

and

$$\Omega(k,j) = \left(\widetilde{x}^T(k)\ \sum_{j=k-h}^{k-1} y^T(j))\right)^T$$

It is clear that Υ is negative definite since Γ is negative definite. The performance requirement of (5), defining $\widetilde{\Omega}(k,j) = \left(\widetilde{x}^T(k)\ \sum_{j=k-h}^{k-1} y^T(j)\ w^T(k))\right)^T$ we require that

$$\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \leq \widetilde{\Omega}^T(k)\Theta\widetilde{\Omega}(k) + \overline{D}F(k)\overline{E} + \overline{E}^T F^T(k)\overline{D}^T < 0 \tag{21}$$

where $\overline{D} = \left(D^T P_2\ D^T P_3\ 0\ 0\ 0\right)^T,\ \ \overline{E} = \left(E_0\ 0\ E_1\ 0\ 0\right)^T$,

$$\Theta = \begin{pmatrix} \Theta_{11} & \Theta_{21}^T & \Theta_{31}^T & -P_4^T & P_2^T B_1 \\ \Theta_{21} & \Theta_{22} & \Theta_{32}^T & -P_5^T & P_3^T B_1 \\ \Theta_{31} & \Theta_{32} & \Theta_{33} & -P_6^T & 0 \\ -P_4 & -P_5 & -P_6 & -\frac{1}{h}R & 0 \\ B_1^T P_2 & B_1^T P_3 & 0 & 0 & -\gamma^2 I \end{pmatrix} \tag{22}$$

and

$$\begin{aligned}
\Theta_{11} &= P_2^T(A_0 - I) + (A_0 - I)^T P_2 + P_4 + P_4^T + Q + C_0^T C_0 \\
\Theta_{21} &= P_1 + P_3^T(A_0 - I) + P_5^T - P_2 \\
\Theta_{22} &= hR - P_3 - P_3^T \\
\Theta_{31} &= A_1^T P_2 - P_4 + P_6^T \\
\Theta_{32} &= A_1^T P_3 - P_5 \\
\Theta_{33} &= -P_6 - P_6^T - Q
\end{aligned}$$

Bounding the norm-bounded uncertainties as [Xie, 1996]:

$$\overline{D}F(k)\overline{E} + \overline{E}^T F^T(k)\overline{D}^T \leq \varepsilon\overline{E}^T\overline{E} + \varepsilon^{-1}\overline{D}\overline{D}^T \tag{23}$$

where ε is a positive number.
Since summation in the inequality (21) from $k = 0$ till $k = \infty$ implies (5) and we obtain by Schur complements that LMI (6) holds.

Remark 31 *In deriving Theorem 31 we have taken the matrix P that contains free matrices P_2, P_3, P_4, P_5 and P_6. In contrast to the descriptor method used in [Fridman and Shaked, 2005] where the matrix P has the form $P = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix}$. Thus our result is more general since it posses more degrees of freedom. In a similar way, [He et al., 2004] has added in the derivative of the Lyapunov-Krasovskii functional some null terms containing free matrices. It has been shown, that in some cases the introduction of free matrices leads to less restrictive results.*

4 Stabilizability

The aim of this section is to solve problem 22 that is to design a state feedback controller which stabilizes the system and achieves a prescribed bound on the H_∞-norm of the closed-loop with uncertainties. Using the system dynamics (4) and the controller expression:

$$u(k) = Kx(k), \tag{24}$$

we get the following system:

$$\begin{cases} x(k+1) = \widetilde{A}_{0c}x(k) + \widetilde{A}_1 x(k-h) + B_1 w(k) \\ z(k) = C_{0c}x(k) \end{cases} \tag{25}$$

If we let $\widetilde{A}_{0c} = \widetilde{A}_0 + \widetilde{B}_0 K,\ \ C_{0c} = C_0 + C_1 K$, based on the results on stability, Replacing A_0 and C_0 in Theorem 31 with A_{0c} and C_{0c}, respectively, we obtain the following results in term of inequalities.

Theorem 41 *For some given scalars $\gamma > 0$, system (4) is robustly stabilizable via feedback control law (24) and satisfies (5) for all non-zeros $w \in L_2[0,\infty)$, if there exist positive definite symmetric matrices $P_1 = P_1^T \in \Re^{2n\times n}, Q = Q^T \in \Re^{n\times n}$ and $R = R^T \in \Re^{n\times n}$, matrices $P_i \in \Re^{n\times n}, i = 2, ..., 6$, $K \in \Re^{m\times n}$ and a positive scalar ε such that the following condition is verified:*

$$\Pi = \begin{pmatrix} \Pi_{11} & \Pi_{21}^T & \Pi_{31}^T & -P_4^T & P_2^T B_1 & P_2^T D & \varepsilon\big(E_0+E_2K\big)^T & \big(C_0+C_1K\big)^T \\ \Pi_{21} & \Pi_{22} & \Pi_{32}^T & -P_5^T & P_3^T B_1 & P_3^T D & 0 & 0 \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & -P_6^T & 0 & 0 & 0 & 0 \\ -P_4 & -P_5 & -P_6 & -\frac{1}{h}R & 0 & 0 & 0 & 0 \\ B_1^T P_2 & B_1^T P_3 & 0 & 0 & -\gamma^2 I & 0 & 0 & 0 \\ D^T P_2 & D^T P_3 & 0 & 0 & 0 & -\varepsilon I & 0 & 0 \\ \varepsilon\big(E_0+E_2K\big) & 0 & 0 & 0 & 0 & 0 & -\varepsilon I & 0 \\ C_0+C_1K & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{pmatrix} < 0 \tag{26}$$

where

$$\begin{aligned} \Pi_{11} &= P_2^T\big(A_0+B_0K-I\big) + \big(A_0+B_0K-I\big)^T P_2 + P_4 + P_4^T + Q \\ \Pi_{21} &= P_1 + P_3^T\big(A_0+B_0K-I\big) + P_5^T - P_2 \\ \Pi_{22} &= hR - P_3 - P_3^T \\ \Pi_{31} &= A_1^T P_2 - P_4 + P_6^T + \varepsilon E_1^T\big(E_0+E_2K\big) \\ \Pi_{32} &= A_1^T P_3 - P_5 \\ \Pi_{33} &= -P_6 - P_6^T - Q + \varepsilon E_1^T E_1 \end{aligned} \tag{27}$$

Remark 41 *In Theorem 41 there are some nonlinearities due to the product between the variables P_2, P_3, ε and K. This can be overcomed by using some relaxation techniques [Boyd et al., 1994,Tarbouriech and Garcia, 1999] or by iterating with respect to K. Inequalitie (26) then reduce to linear matrix inequalitie LMI in all the remaining decision variables. Thus latter decision variables are solved using the techniques of [Gahinet et al., 1995]. This procedure can be derived as follows:*

Algorithm:

Step 1: Fix K, γ sufficiently large and h sufficiently small to have a feasible solution, solve the LMI (26) for $P_i, i = 1, \ldots, 6$, Q, R and ε.

Step 2: Fix P_2, P_3 and ε obtained in the previous step, and let $\gamma = \gamma - \mu$ with μ sufficiently small positive scalar, solve the LMI (26) for P_1, P_4, P_5, P_6, Q, R and K.

Step 3: If γ less than a prescribed performance index γ. Go to step 4 else go to step 2.
Step 4: Let $h = h + h_{step}$, solve the LMI (26).
Step 5: If the LMI have a solution go to step 4 else $h = h - h_{step}$, stop.

5 Numerical examples

In this section, numerical computations is performed to illustrate the advantages of the results compared with existing ones.

Example 51 *Consider the uncertain discrete time delay system described by (4) with the following matrices:*

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1.01 \end{pmatrix}, \;\; A_1 = \begin{pmatrix} -0.02 & -0.005 \\ 0 & -0.01 \end{pmatrix},$$

$$B_0 = \begin{pmatrix} 0 \\ 0.01 \end{pmatrix}, \;\; B_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \;\; C_0 = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

$$C_1 = 0.1, \;\; D = 0.2I, \;\; E_0 = E_1 = 0.01I, \;\; E_2 = 0. \tag{28}$$

In [Lee and Kwon, 2002], considering the stabilization problem only with norm-bounded uncertainties, a maximum value of $h = 41$ was obtained for the case of constant delay. In [Fridman and Shaked, 2005], for $\gamma = 180.07$ the system with the above norm-bounded uncertainties is stabilizable for all constant $h \leq 67$.
Applying theorem 41, for $K = \begin{pmatrix} -111.9899 & -81.9951 \end{pmatrix}$ We can show that the system (4) is stabilizable for all constant $h \leq 71$ and the minimum bound $\gamma = 164.141$. Clearly, our method produces much less conservative results, thus demonstrating its validity.

Example 52 *Let us consider a system described by (4) and suppose that the system data are as follows:*

$$A_0 = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}, \;\; A_1 = \begin{pmatrix} 0.01 & 0.1 \\ 0 & 0.1 \end{pmatrix},$$

$$B_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \;\; B_1 = C_0 = C_1 = 0, \;\; h = 10. \tag{29}$$

Applying theorem 41, for $K = \begin{pmatrix} 2.0005 & 2.9051 \end{pmatrix}$ we obtain by using the software Matlab:

$$P_1 = \begin{pmatrix} 6.6058 & -0.1463 \\ -0.1463 & 17.9371 \end{pmatrix},$$

$$Q = \begin{pmatrix} 1.9343 & -0.1174 \\ -0.1174 & 2.9663 \end{pmatrix},$$

$$R = \begin{pmatrix} 0.2873 & 0.2867 \\ 0.2867 & 0.7328 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 1.9683 & -0.8618 \\ 1.8801 & 1.8911 \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 5.8425 & -1.7195 \\ 5.3242 & 14.0693 \end{pmatrix},$$

$$P_4 = \begin{pmatrix} -0.0107 & -0.0030 \\ -0.0101 & -0.0172 \end{pmatrix},$$

$$P_5 = \begin{pmatrix} 0.0085 & 0.0165 \\ 0.0217 & 0.0428 \end{pmatrix},$$

$$P_6 = \begin{pmatrix} 0.0076 & -0.0049 \\ 0.0124 & 0.0075 \end{pmatrix}. \tag{30}$$

By numerical simulation, we show in figure 1 and figure 2 the trajectories and the feedback controller of the discrete-time system with time delay (29). These figures show that the closed-loop system is stable under the feedback controller.

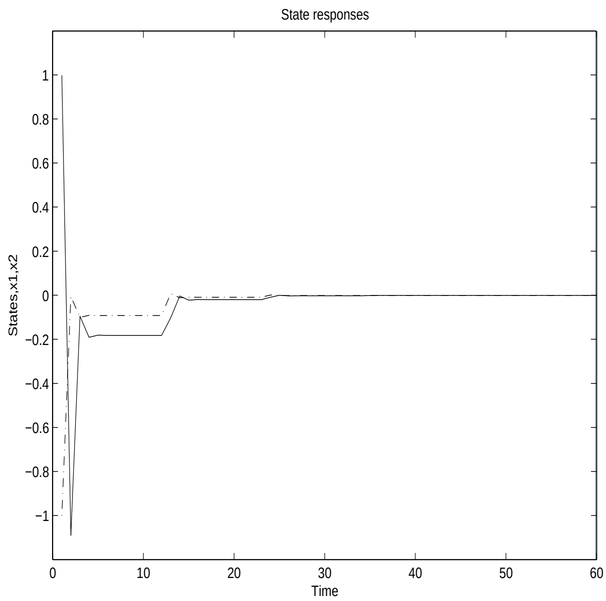

Fig. 1. The behaviors of the states x_1 and x_2.

6 Conclusions

Sufficient conditions have been derived to guarantee robust stability and robust stabilization dependent of delay for uncertain linear discrete systems with the H_∞ controller. The uncertainties considered are time-varying and norm-bounded, we have proposed an iterative

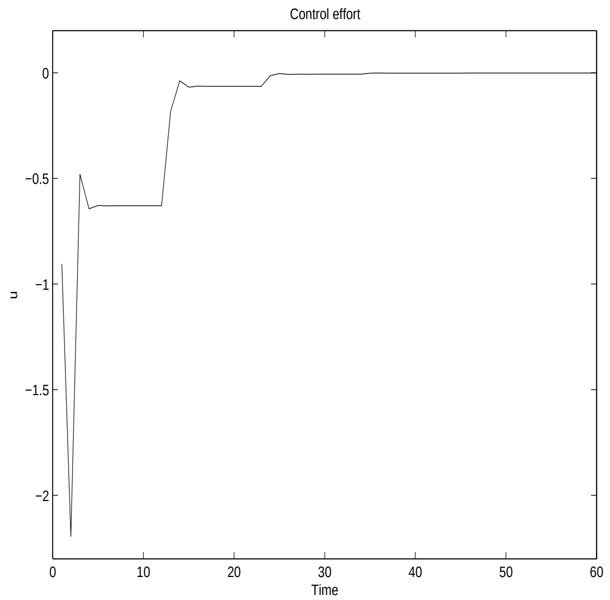

Fig. 2. The behaviors of the control law u.

procedure based on numerical optimization. The algorithm is simply implemented on Matlab software. Two numerical examples and simulations are given to demonstrate the validity of our main results.

Références

[Boukas, 2006] E.-K Boukas. 2006 "Discrete-Time Systems with Time-Varying Time Delay: Stability and Stabilizability", Mathematical Problems in Engineering, Vol. 2006, Article ID 42489, Pages 110.

[Boyd et al., 1994] S. Boyd, L. EL Ghaoui, E. Feron, V. Balakrishnan, 1994 "Linear matrix inequalities in systems and control theory", studies in Applied Mathematics, 15, SIAM, USA.

[Fridman and Shaked, 2005] E. Fridman and U. Shaked, 2005 "Delay-Dependent H_∞ Control of Uncertain Discrete Delay Systems" European Journal of Control, 11, 29-37.

[Fridman and Shaked, 2002] E. Fridman and U. Shaked, 2002 "A Descriptor system approch to H_∞ control of linear time-Delay systems" IEEE Trans. Autom. Control, 47, 253-270.

[Gahinet et al., 1995] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, 1995 "LMI Control Toolbox for Use with Matlab" Natick, MA: Math Works.

[He et al., 2004] Y. He, M. Wu, J. H. She, and G. P. Liu, 2004 "Parameter-Dependent Lyapunov Functional for Stability of Time-Delay Systems with Polytopic-Type Uncertainties" IEEE Trans. Automat. Contr., 49, 828-832.

[Hmamed and Tissir, 1998] A. Hmamed and E. Tissir, 1998 "Further results on the stability of discrete time matrix polynomials," Int. J. Systems Sci., 29, 819-821.

[Lee and Kwon, 2002] Y. S. Lee and W. H. Kwon , 2002 "Delay-dependent robust stabilization of uncertain discrete-time state-delayed systems" 15th Triennial IFAC world congress, Barcelona, Spain.

[Nesterov and Nemirovsky, 1994] Y. Nesterov and A. Nemirovsky, 1994 ”Interior Point Polynomial Methods convex Programming”. studies in Applied Mathematics, vol. 13, SIAM, Philadelphia.

[Tarbouriech and Garcia, 1999] S. Tarbouriech and G. Garcia 1999 ”Stabilization of Neutral Linear Time-Delay Systems with Saturating Actuators” Proceedings of the 38^{th} Conference on Decision and Control., 2011-2016.

[Tissir, 2007] E. Tissir, 2007 ”Robust Control Design for Retarded Discrete-Time Systems” Journal of Dynamic Systems, measurement and control, 199, Issue 1, 72-76.

[Xie, 1996] L. Xie , 1996 ”Output feedback H_∞ control of systems with parameter uncertainty”, International Journal of Control, 63, 741-750.