

Robust Adaptive fuzzy sliding mode PI control of unknown nonlinear systems

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Abstract. *In this paper, we present an adaptive fuzzy control for a class of unknown nonlinear systems. The strategy of control uses the adaptive fuzzy system Takagi-Sugeno (T-S) to approximate the part of the primary control. In order to guarantee the stability and high performance, the auxiliary part is incorporated in the control law. The proposed compensation control is combined a SMC and a proportional integral (PI) removes the fuzzy approximation and attenuates the influence of the external disturbances. The parameters of the control are adjusted on line by the adaptive law with stability and convergence analysis using the Lyapunov approach. The robust and the stability of the control scheme are proved and simulation results are given to verify the effectiveness of the proposed approach.*

Key-words. *Adaptive control, PI control, Sliding mode, Fuzzy logic, Nonlinear system.*

1 Introduction

Due to the complexity of the structure of the controlled system and disturbances, the mathematical model of a practical nonlinear system is in general difficult to derive or too expensive to assess in many practical applications. Several modelling methods based on fuzzy logic have been proposed in recent years [2], [7], [9]. The most important issue for fuzzy logic systems is how to get a system design with the guarantee of stability and control performance. Based on the universal approximation capability, many effective adaptive fuzzy control schemes have been developed to incorporate with human expert knowledge information in systematic way, which can also guarantee stability and performance [4], [5], [10].

It is well known that the sliding mode control method provides a robust controller for nonlinear dynamic systems [1], [3].

However, SMC suffers from a well known problem chattering due to the high gain and high-speed switching control. The undesirable chattering may excite previously unmodelled system dynamics and damage actuators, resulting in unpredictable

stability. One method to alleviate this drawback is to introduce a boundary layer about the sliding plane [1]. This method can lead to stable close loop system without the chattering problem, but there exists a finite steady state error due to the finite steady gain of the control algorithm. Another class of techniques is based on the use of an observer [1]. However, state observer can cause loss robustness. The high-order sliding mode is also used [11]. However, the discontinuity set of controllers is stratified union of manifolds with co dimension varying in the rang form 1 to the relative degree r . unfortunately, the complicated structure of the controller discontinuity set certain causes redundant transient chattering.

In the past several years, active research has been carried out in controller design based on universal approximators, such as fuzzy control [8], [12]. The research of fuzzy model under SMC has attracted many attentions in recent years [4], [10], [13], [14]. The apparent similarities between SMC and fuzzy control motivate considerable research efforts in combining the two approaches for achieving more superior performances such as overcoming some limitations of the traditional SMC.

This paper proposes an adaptive fuzzy control to combine the SMC and the PI controller for nonlinear system with unknown dynamics. The primary control action is approximated by the adaptive fuzzy model type T-S. The fuzzy parameters are estimated on line by the adaptive laws with stability and convergence are analysed in the Lyapunov sense. In order to achieve a good tracking with modelling uncertainties and disturbances, the auxiliary control is incorporated in the control law. The design of this compensation control is based on SMC and PI approach. This compensation controller integrates the PI control with the SMC to eliminate steady state error and the chattering phenomenon.

The paper is organized as follows: the problem formulation is presented in section 2. In section 3, the adaptive fuzzy sliding mode PI control is proposed. Simulation results are presented in section 4.

2. Problem Formulation

Consider a nonlinear system described by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_n = f(\underline{x}, t) + bu + d(t) \\ y = x_1 \end{cases} \quad (1)$$

or equivalent

$$\begin{cases} x^{(n)} = f(\underline{x}, t) + bu + d(t) \\ y = x \end{cases} \quad (2)$$

The $f(\underline{x}, t)$ is unknown and nonlinear function and b is positive constant. $d(t)$ is unknown external disturbances.

$\underline{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T = [x_1, x_2, \dots, x_n]^T \in R^n$ is the state vector of the systems which is assumed to be measurable, $u \in R$ and $y \in R$ are respectively the input and the output of the system. The control problem is to obtain the state \underline{x} for tracking a desired state \underline{y}_r . Assume that the given reference y_r is bounded up and have to $(n-1)$ bounded derivatives. The reference vector is denoted as: $\underline{y}_r = [y_r, \dot{y}_r, \dots, y_r^{(n-1)}]^T$.

We define the tracking error:

$$e = y - y_r \tag{3}$$

Then the error vector is given by :

$$\underline{e} = [e, \dot{e}, \dots, e^{(n-1)}]^T \in R^n \tag{4}$$

It is desired that the output error of the system follow:

$$e^{(n)} + k_{n-1}e^{(n-1)} + \dots + k_0e = 0 \tag{5}$$

To ensure a good tracking, the selection of $k_i, i = 1, 2, \dots, n-1$ must satisfy the following Hurwitz polynomial:

$$S = s^{(n)} + k_{n-1}s^{(n-1)} + \dots + k_0s \tag{6}$$

We remark that, when the system (2) is well known, $b \neq 0$ and to guarantee that $\lim_{t \rightarrow \infty} e = 0$. The control law is designed to have the following idealized control [3]:

$$u^* = \frac{1}{b}(-f(x, t) + y_r^{(n)} - \sum_{i=1}^{n-1} k_i e^{(i)}) \tag{7}$$

In fact, most the parameters and structure of the systems are unknown due to environment changes, modelling errors and unmodelled dynamics. Thus, the primary control law (7) can not be implemented. To solve this problem, the direct adaptive fuzzy control scheme is proposed to approximate the idealized control (7).

3. Design of adaptive fuzzy sliding mode proportional integral control

The direct adaptive fuzzy control is designed for nonlinear system with unknown nonlinear dynamics. The strategy of control is based on fuzzy system, SMC and PI approach to ensure stability, tracking and consistent performance.

At first level, the primary control (7) is approximated by the adaptive fuzzy model type T-S. The fuzzy parameters can be tuned on line by adaptive law based on Lyapunov approach.

In the second level, the auxiliary control is incorporated in the control law (7) to attenuate the external disturbances and remove fuzzy approximation error. The design of this control law is derived from PI method and with the SMC to eliminate steady state error and the chattering phenomena.

The proposed controller is given as follows:

$$u = \hat{u}(\underline{x}, \underline{\theta}) + u_s \tag{8}$$

where $\hat{u}(\underline{x}, \underline{\theta})$ and u_s are respectively, the primary control law and the auxiliary control.

Due to the universal approximation property, we use T-S fuzzy system to approximate the primary control u^* . The T-S fuzzy model is described by fuzzy if-then rules, which locally represent linear input-output relations of a system. The fuzzy system is of the following form:

$$R^l : \text{If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \dots, x_n \text{ is } F_n^l \text{ then } u^l \text{ is } \theta_l \tag{9}$$

$$l = 1, 2, \dots, m$$

where R^l represents the l^{th} fuzzy inference rule, $x_i, i = 1, \dots, n$ are measurable variables of system, F_i^l denotes fuzzy set. Each fuzzy set F_i^l is associated with a membership function $\mu_{F_i^l}$ and the center of this membership function is the operating point. By using the weight average fuzzy inference approach [12], we obtain:

$$\begin{aligned} \hat{u}(\underline{x}, \underline{\theta}) &= \frac{\sum_{l=1}^m \prod_{i=1}^n \mu_{F_i^l} \theta_l}{\sum_{l=1}^m \prod_{i=1}^n \mu_{F_i^l}} \\ &= \underline{\theta}^T \underline{\xi}(\underline{x}) \end{aligned} \tag{10}$$

$\underline{\theta} = [\theta_1, \theta_2, \dots, \theta_m]^T$ is parameter vector and $\underline{\xi}(x) = [\xi_1(x), \xi_2(x), \dots, \xi_m(x)]^T$ is a regressive vector with regressor $\xi_l(x) (1 \leq l \leq m)$, (m is the number of rules), which is defined as fuzzy basis function :

$$\xi_l(x) = \frac{\prod_{i=1}^n \mu_{F_i^l}}{\sum_{l=1}^m \prod_{i=1}^n \mu_{F_i^l}} \quad (11)$$

The adjustable fuzzy parameters of $\hat{u}(x, \underline{\theta})$ are tuned on line using the Lyapunov approach. In order to guarantee that the parameters are bounded, we introduce the projection algorithm [12] to restrict them in the closed set Ω .

$$\dot{\underline{\theta}} = \begin{cases} -\gamma_1 \sigma b \underline{\xi}(x) & \text{if } (|\underline{\theta}| < M \text{ or } (|\underline{\theta}| = M \text{ and } \gamma_1 \sigma b \underline{\theta}^T \underline{\xi}(x) > 0)) \\ \text{Proj}[-\gamma_1 \sigma b \underline{\xi}(x)] & \text{otherwise} \end{cases} \quad (12)$$

where γ_1 is fixed adaptive gain. Proj[.] represents the projection operator which is defined by :

$$\text{Proj}[-\gamma_1 \sigma b \underline{\xi}(x)] = -\gamma_1 \sigma b \underline{\xi}(x) + \gamma_1 \sigma b \frac{\underline{\theta} \underline{\theta}^T \underline{\xi}(x)}{|\underline{\theta}|^2} \quad (13)$$

The auxiliary control integrates the PI control with the SMC to eliminate steady state error and the chattering phenomena. The input and output of the continuous time PI controller is given by:

$$u_s = k_p \sigma(x, t) + k_i \int \sigma(x, t) dt \quad (14)$$

k_p and k_i are control gain to be designed. $\sigma(x, t) = 0$ represents a time varying sliding surface which is defined in the state space R^n by the scalar equation:

$$\sigma(x, t) = e_0^{(n-1)} + k_{n-1} e_0^{(n-2)} + \dots + k_1 e_0 \quad (15)$$

Equation (12) can be rewritten as:

$$u_s = -\underline{\theta}_{pi}^T \phi(\sigma) \quad (16)$$

where $\underline{\theta}_{pi} = [k_p, k_i]^T$ is an adjustable parameters and

$\phi(\sigma) = [\sigma(x, t), \int \sigma(x, t) dt]^T$ is a regressive vector.

The adjustable fuzzy parameters of $\underline{\theta}_{pi}$ are tuned on line using the Lyapunov approach. In order to guarantee that the parameters are bounded, we introduce the projection algorithm [12] to restrict them in the closed set Ω .

$$\dot{\underline{\theta}}_{pi} = \begin{cases} -\gamma_2 \sigma b \phi(\sigma) & \text{if } (|\underline{\theta}| < N \text{ or } (|\underline{\theta}| = N \text{ and } \gamma_2 \sigma b \underline{\theta}^T \phi(\sigma) > 0)) \\ \text{Proj}[-\gamma_2 \sigma b \phi(\sigma)] & \text{otherwise} \end{cases} \quad (17)$$

where γ_2 is fixed adaptive gain. $\text{Proj}[\cdot]$ represents the projection operator which is defined by:

$$\text{Proj}[-\gamma_2 \sigma b \phi(\sigma)] = -\gamma_2 \sigma b \phi(\sigma) + \gamma_2 \sigma b \phi(\sigma) \frac{\underline{\theta}_{pi} \underline{\theta}_{pi}^T \rho}{|\underline{\theta}_{pi}|^2} \quad (18)$$

Hence, the resulting control law is as follows:

$$u = \underline{\theta}^T \xi(x) - \underline{\theta}_{pi}^T \phi(\sigma) \quad (19)$$

Theorem. Consider the nonlinear system (2). If the control (19) is applied with the primary control and the auxiliary control are given respectively by (10) and (16), the parameters vector $\underline{\theta}$ and $\underline{\theta}_{pi}$ are adjusted by the adaptive law (12)-(19). The closed-loop system is stable in sense that all the signals are bounded and the tracking error will converge to zero asymptotically.

Proof:

Consider the following Lyapunov function as:

$$V = \frac{1}{2} \sigma^2 + \frac{1}{2\gamma_1} \Phi \Phi^T + \frac{1}{2\gamma_2} \Psi \Psi^T \quad (20)$$

where

$$\Phi = \underline{\theta} - \underline{\theta}^* \quad \text{and} \quad \Psi = \underline{\theta}_{pi} - \underline{\theta}_{pi}^*$$

The optimal parameters are given by:

$$\underline{\theta}^* = \arg \min_{\underline{\theta} \in \Omega_u} \left(\sup_{\underline{x} \in R^n} |u^* - \hat{u}(\underline{x}, \underline{\theta})| \right) \quad (21)$$

$$\underline{\theta}_{pi}^* = \arg \min_{\underline{\theta}_{pi} \in \Omega_s} \left(\sup_{\underline{x} \in R^n} |u_s - \underline{\theta}_{pi}^T \phi(\sigma)| \right) \quad (22)$$

where $\Omega_u = \{\underline{\theta} / |\underline{\theta}| \leq M\}$ and $\Omega_s = \{\underline{\theta}_{pi} / |\underline{\theta}_{pi}| \leq N\}$ are the convex compact sets which contain feasible parameter sets for $\underline{\theta}$ and $\underline{\theta}_{pi}$. M and N are pre-specified constants.

The time derivative of V (19) is:

$$\dot{V} = \sigma \dot{\sigma} + \frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \tag{23}$$

$$\dot{\sigma} = x^{(n)} - y_r^{(n)} + \sum_{i=1}^{n-1} k_i e^{(i)} = b(\hat{u}(\underline{x}, \underline{\theta}) - u^*) + bu_s + d(t) \tag{24}$$

$$\dot{V} = \sigma b(\hat{u}(\underline{x}, \underline{\theta}) - u^*) + \sigma bu_s + \sigma d(t) + \frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \tag{25}$$

$$\dot{V} = \sigma b(\hat{u}(\underline{x}, \underline{\theta}) - \hat{u}(\underline{x}, \underline{\theta}^*) + \hat{u}(\underline{x}, \underline{\theta}^*) - u^*) + bu_s + d(t) + \frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \tag{26}$$

Substituting (16) into (26):

$$\begin{aligned} \dot{V} &= \sigma b \Phi^T \xi(x) + \sigma b(\hat{u}(\underline{x}, \underline{\theta}^*) - u^*) + \sigma b \underline{\theta}_{pi}^T \phi(\sigma) + \sigma d(t) + \\ &\frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \\ \dot{V} &= \sigma b \Phi^T \xi(x) + \sigma b(\hat{u}(\underline{x}, \underline{\theta}^*) - u^*) + \sigma b(\underline{\theta}_{pi}^{*T} \phi(\sigma) - \underline{\theta}_{pi}^T \phi(\sigma)) - \\ &\sigma b \underline{\theta}_{pi}^{*T} \phi(\sigma) + \sigma d(t) + \frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \end{aligned}$$

Define the minimum fuzzy approximation error:

$$w = b(\hat{u}(\underline{x}, \underline{\theta}^*) - u^*)$$

The \dot{V} can be written as:

$$\begin{aligned} \dot{V} &= \sigma b \Phi^T \xi(x) + \sigma w + \sigma b \underline{\theta}_{pi}^{*T} \phi(\sigma) + \sigma d(t) + \frac{1}{\gamma_1} \dot{\Phi} \Phi^T + \frac{1}{\gamma_2} \dot{\Psi} \Psi^T \\ \dot{V} &= \sigma w + \sigma d(t) - \sigma b \underline{\theta}_{pi}^{*T} \phi(\sigma) + \frac{1}{\gamma_1} (\dot{\Phi} + \gamma_1 \sigma b \xi(x) + \Phi^T) + \frac{1}{\gamma_2} \Psi^T (\dot{\Psi} + \gamma_2 \sigma b \underline{\theta}_{pi} \phi(\sigma)) \end{aligned}$$

By consideration of the update laws (12)-(19), the \dot{V} can be rewritten as:

$$\dot{V} \leq |\sigma| (|w| + |d(t)|) - \sigma b \underline{\theta}_{pi}^{*T} \phi(\sigma)$$

hence $\dot{V} \leq 0$

4. Simulation results

In this section, we test the algorithm on two examples. The first is to let the servomechanism to track a desired trajectory. The second is to let the output of a first order nonlinear system to track a constant trajectory.

Example 1. Taken the servomechanism as given in [7]. It is modelled by the following second order differential equation:

$$m\ddot{q} + l\dot{q} + \Delta f(q) = \tau + d \quad (27)$$

where

\dot{q} : Velocity

q : Position

$\Delta f(q)$: Nonlinear term depending on q .

m, ℓ : Mass and damping

τ : Torque and d : Disturbance.

We suppose that the position $x_1 = q$ and the velocity $x_2 = \dot{q}$ are available from measurements.

The dynamic equations of the servomechanism can be described in space state as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(\underline{x}, t) + u + d(t) \\ y = x_1 \end{cases} \quad (28)$$

$f(\underline{x}, t) = -lx_2 - \Delta f(x_1)$, $u = \tau$ with $\Delta f(x_1) = 0.4 \sin(x_1)$.

In the first step, we need to define some fuzzy sets to cover the state space. The choice of the number of fuzzy set and the constant M is related to knowledge of expert on the system. We consider $M = 16$ and three fuzzy membership functions are chosen as in figure 1, which are defined by:

$$\mu_{F_1^1}(x_1) = \exp(-0.5(x_1 + \pi/3)); \quad \mu_{F_2^1}(x_2) = \exp(-0.5(x_2 + \pi/3));$$

$$\mu_{F_1^2}(x_1) = \exp(-0.5(x_1)); \quad \mu_{F_2^2}(x_2) = \exp(-0.5(x_2));$$

$$\mu_{F_1^3}(x_1) = \exp(-0.5(x_1 - \pi/3)); \quad \mu_{F_2^3}(x_2) = \exp(-0.5(x_2 - \pi/3));$$

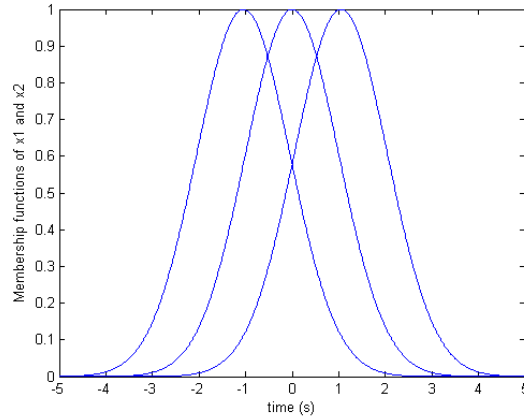


Fig. 1. Membership functions

Then there are 9 rules to approximate the primary control law $\hat{u}(x, \theta)$. The fuzzy rules are defined by the following linguistics description:

$$R^l : \text{If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ then } \hat{u}^l \text{ is } \theta^l \quad (29)$$

Where $l = 1, 2, 3$ and $m = 9$ is the number of the fuzzy rules.

The choice of the number of fuzzy set and the constant M is related to knowledge of expert on the system. For the first simulation, the control objective is to maintain the system to track the desired angle trajectory:

$$y_r = \frac{\pi}{10} * (\sin(0.01t) + 0.3 * \cos(0.03t)).$$

The sliding surface is defined as:

$\sigma(x, t) = k_1 e_0(t) + k_2 \dot{e}_0(t)$ with $k_1 = 5$ and $k_2 = 1$. The initial consequent parameters $\theta(0)$ of fuzzy control are chosen randomly in the interval $[-1, 1]$ and $k_p(0) = 14$ with $k_i(0) = 24$. Let the learning rate $\gamma_1 = 0.5, \gamma_2 = 2$. From figure 2, it can be seen that, the tracking performance is obtained with unknown nonlinear dynamics and in presence of disturbances. The corresponding fuzzy control signal is given in figure 3, which demonstrates that the chattering phenomenon is disappeared. The figures 5 and 6, illustrates the behaviour and the convergence of the parameters of PI control.

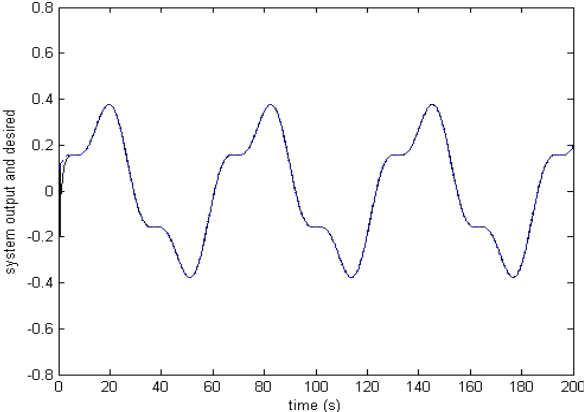


Fig. 2. : Reponses of $y(t)$ and $y_r(t)$

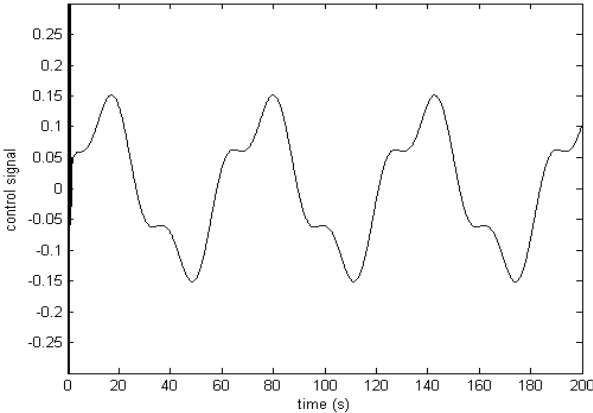


Fig. 3. The control signal

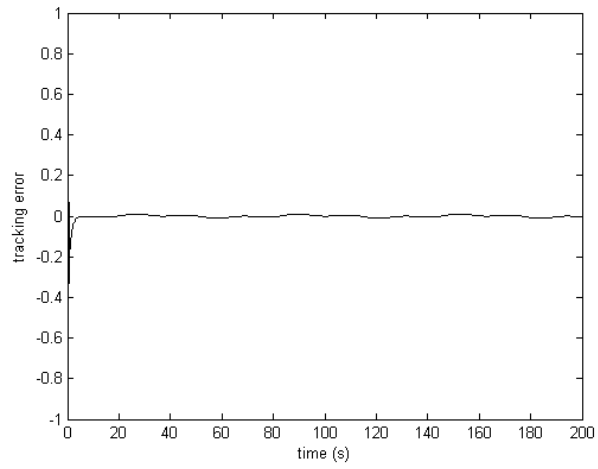


Fig. 4. The tracking error

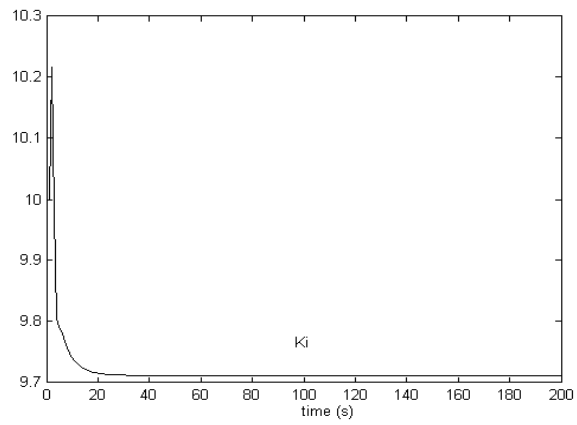


Fig. 5. Evolution of k_i

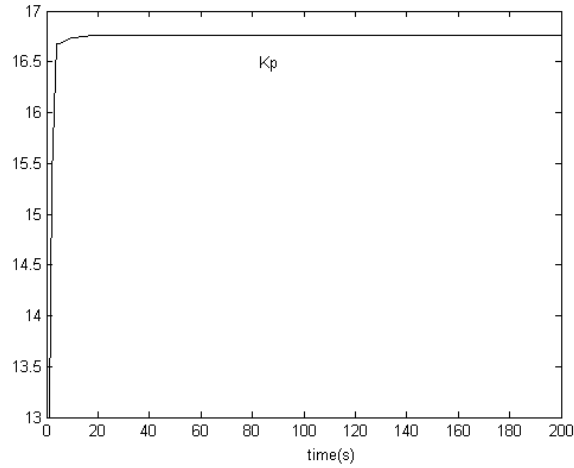


Fig. 6. Evolution of k_p

Example 2. In this example, we test the proposed adaptive fuzzy control on the regulation control of a first order nonlinear system. The dynamic equation of system is given by [12]:

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t) + d(t) \quad (30)$$

The control objective is to maintain the system to track a constant trajectory: $y_r = 0$. The external disturbances $d(t) = 2 * rand(1,40)$ was injected at the output of the system.

To build the proposed approximators, we define six fuzzy sets over the intervals $[-3,3]$ for the state x , the Gaussian membership functions are:

$$\mu_{F^1}(x) = 1/(1 + \exp(x + 2))$$

$$\mu_{F^2}(x) = \exp(-(x + 1.5)^2)$$

$$\mu_{F^3}(x) = \exp(-(x + 0.5)^2)$$

$$\mu_{F^4}(x) = 1/(1 + \exp(x - 0.5)^2)$$

$$\mu_{F^5}(x) = \exp(-(x - 1.5)^2)$$

$$\mu_{F^6}(x) = 1/(1 + \exp(5(x - 20)))$$

We consider $N = 6$ and the initial consequent parameters $\underline{\theta}(0)$ of fuzzy control are chosen randomly in the interval $[0,2]$. Choose the sliding surface as $\sigma = e(t)$. The initial values: $x(0) = 3$, $k_p(0) = 14$ and $k_i(0) = 24$. Let the learning rate $\gamma_1 = 0.5, \gamma_2 = 2$.

It can be seen from figure 7 and figure 9 that the good tracking is obtained with the unknown nonlinear dynamic and in the presence of model uncertainties and external disturbance.

Figure 8 shows the proposed control which demonstrates the chattering of traditional SMC is eliminated. From figure 10 and 11 we remark that a good convergence of the parameters of the PI control is obtained.

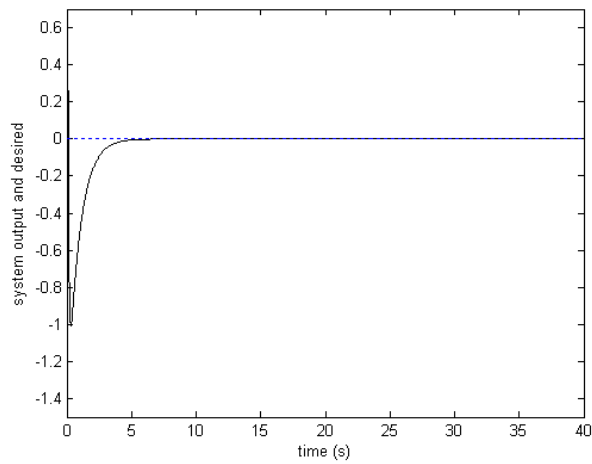


Fig. 7. Responses of $y(t)$ and $y_r(t)$

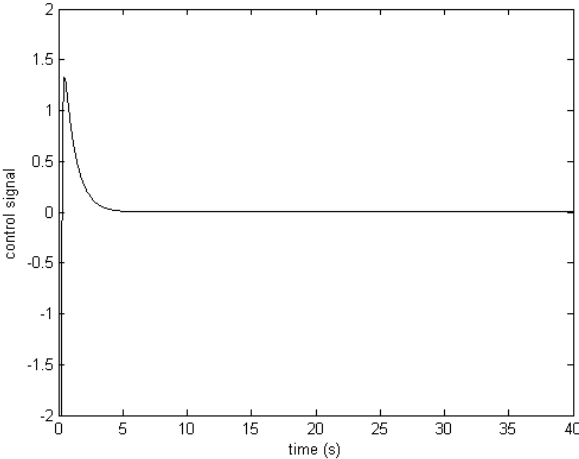


Fig. 8. The control signal

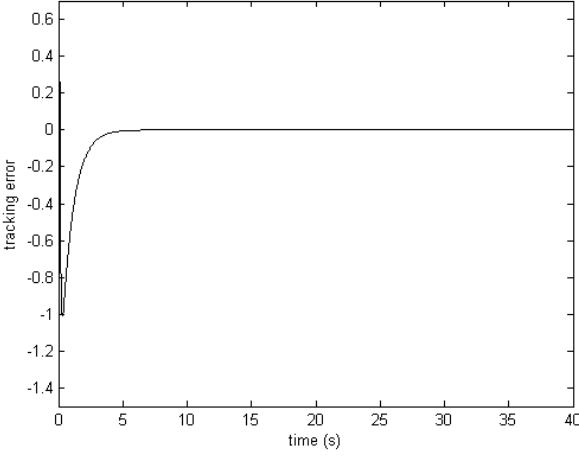


Fig. 9. The tracking error

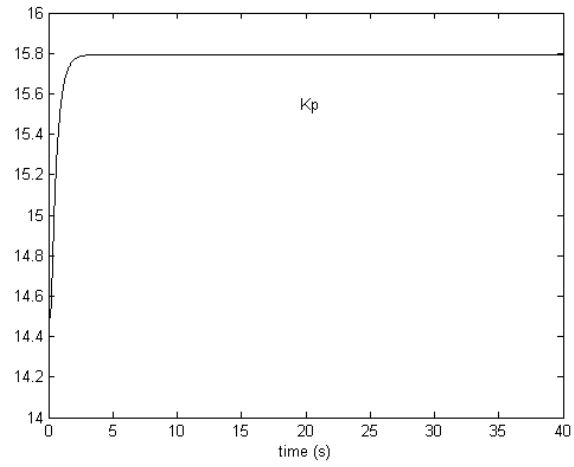


Fig. 10. Evolution of k_p

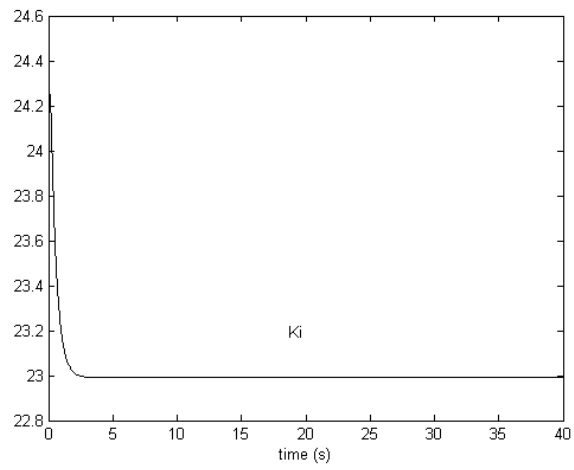


Fig. 11. Evolution of k_i

5. Conclusion

In this paper, an adaptive fuzzy control has been proposed for a class of unknown nonlinear systems. We introduced the fuzzy system T-S to approximate the part of the primary control.

The auxiliary part of control is incorporated to ensure the stability and the tracking performance. The synthesis of this second part of control is consisting to combine a SMC and a PI controller. The drawback of chattering in SMC is avoided and the robustness is guaranteed. The closed loop system is stable in the sense of Lyapunov approach. The simulation results illustrate the effectiveness and robustness of the proposed fuzzy control method.

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