International Journal of Sciences and Techniques of Automatic control & computer engineering
IJ-STA, Volume 4, N° 2, December 2010, pp. 1354–1369.

Design of a recursive parametric estimation algorithm for an asynchronous machine

Nawel DAMAK and Samira KAMOUN

Automatic Control Unit
Department of Electrical Engineering
National School of Engineering of Sfax,
University of Sfax, B.P.W, 3038, Sfax, Tunisia.
nawel-damak@yahoo.fr, kamounsamira@yahoo.fr

Abstract. This paper deals with the parametric estimation of the squirrel-cage asynchronous machine. The formulation of this parametric estimation problem is based on a least-square method and on knowledge of several experimental measures. We suppose that this asynchronous machine can be represented by a multivariable linear stochastic discrete-time state-space mathematical model, which contains known state variables and unknown time-varying parameters. A recursive parametric estimation algorithm is developed to estimate these parameters. The performances of this algorithm in the simulation test are quite satisfactory.

Keywords. State space MIMO mathematical model; Parametric estimation; Recursive parametric estimation algorithm; Asynchronous machine.

1 Inroduction

In several industrial sectors, the requirements are increasingly strong in terms of real time management of applications, quality, production cost, reliability of the processes, and of person's security. This requires increasingly powerful command tools and dynamic system's methods. These methods are generally based on determining the system mathematical model and its estimated parameters.

The asynchronous machine is the most reliable electric machine, the most robust of its generation, and the least costly in manufacturing. For this purpose, research in parametric estimation of asynchronous machine drew the attention of several researchers working on automatics. Thus, researchers are interested in these problems and many

This paper was recommended for publication in revised form by the editor Staff.
Edition: CPU of Tunis, Tunisia, ISSN: 1737-7749

studies, related to the modeling and to the asynchronous machine parametric estimation, are published in the literature (Bachir, 2002; Dehay, 1996; Finch, 1998; Moreau, 1999; Koubaa, 2006).

This paper aims at studying the problems related to the parametric estimation of an asynchronous machine. This latter, is described as a MIMO state mathematical model, with unknown time-varying parameters. The formulation of this parametric estimation problem is based on a recursive algorithm of a parametric estimation.

The remainder of this paper is organized as follows: in the second paragraph, a recursive parametric estimation algorithm is developed. The third paragraph is devoted to the asynchronous machine modeling. The parametric estimation of the asynchronous machine is studied in the fourth paragraph, and the fifth paragraph concludes the work.

2 Parametric estimation recursive algorithm

The description of a dynamic MIMO system with time-varying parameters can be explained by the following mathematical model:

$$\begin{aligned} x(k+1) &= A(k+1)x(k) + B(k+1)u(k) + v(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k)$ and $y(k)$ represent the state vector, the input and the output vector at the discrete-time k, respectively, $v(k) \in R^n$ is the random disturbance vector which acts on the system, $e(k)$ indicates the random disturbance which affects the measurement of the output $y(k)$, C represents the measurement matrix. The state matrix $A(k)$ and the command matrix $B(k)$ are represented, respectively, as:

$$A(k) = \begin{pmatrix} a_{11}(k) & \ldots & a_{1n}(k) \\ . & \ldots & . \\ . & \ldots & . \\ . & \ldots & . \\ a_{n1}(k) & \ldots & a_{nn}(k) \end{pmatrix} \tag{2}$$

and

$$B(k) = \begin{pmatrix} b_{11}(k) & \ldots & b_{1m}(k) \\ . & \ldots & . \\ . & \ldots & . \\ . & \ldots & . \\ b_{n1}(k) & \ldots & b_{nm}(k) \end{pmatrix} \quad (3)$$

We assume that the system noise $v(k)$ and the measurement noise $e(k)$ are white Gaussian noise.

The parametric estimation problem of the dynamic multivariate system described as a state mathematical model is composed of an estimation procedural development that allows us to estimate the unknown parameters of the $A(k)$ and $B(k)$ matrices.

The formulation of this problem is based on experimental measurements (input and state sequences) originated in the considered system and minimizing a quadratic criterion comprising the prediction error (also called estimation error), which is defined as:

$$\delta(k) = x(k) - x_p(k) \quad (4)$$

with $x_p(k)$ the adjustable model state vector, as follows:

$$x_p(k+1) = \widehat{A}(k+1)x(k) + \widehat{B}(k+1)u(k) \quad (5)$$

where $\widehat{A}(k+1)$ and $\widehat{B}(k+1)$ are the estimated matrices of $A(k+1)$ and $B(k+1)$ at the discrete time $(k+1)$, respectively.

For the estimation of the parameters of the two matrices $A(k+1)$ et $B(k+1)$ of the state mathematical model, we can use a modified version of the recursive parametric estimation algorithm proposed by Kamoun (2007), which has been developed for monovariable systems:

$$\begin{aligned}
\widehat{A}(k+1) &= \widehat{A}(k) + \xi(k)R\delta(k)x^T(k-1) \\
\widehat{B}(k+1) &= \widehat{B}(k) + \xi(k)R\delta(k)u^T(k-1) \\
\delta(k) &= x(k) - \widehat{A}(k)x(k-1) - \widehat{B}(k)u(k-1) \\
\xi(k) &= \frac{l(k)}{\lambda_R[u^T(k-1)u(k-1) + x^T(k-1)x(k-1)]}
\end{aligned} \tag{6}$$

where $l(k)$ is a positive parametric gain and λ_R is the maximum eigenvalue of the matrix R.

Stability condition analysis:

The state error $\delta(k)$ intervening in the parametric estimation algorithm 6 can be described by the following expression:

$$\delta(k) = \tilde{A}(k)x(k-1) + \tilde{B}(k)u(k-1) \tag{7}$$

where $\tilde{A}(k)$ and $\tilde{B}(k)$ are parametric errors represented respectively by:

$$\tilde{A}(k) = A - \hat{A}(k) \tag{8}$$

and

$$\tilde{B}(k) = B - \hat{B}(k) \tag{9}$$

Let us consider a Lyapunov function $X(k)$ on the parametric errors, such us:

$$X(k) = tr[B^T(k)B(k)] + tr[A^T(k)A(k)] \tag{10}$$

we can express the variation of this Lyapunov function, noted by:

$$\Delta X(k+1) = X(k+1) - X(k) \tag{11}$$

The parametric errors $\tilde{A}(k+1)$ and $\tilde{B}(k+1)$ can be described as:

$$\tilde{A}(k+1) = \tilde{A}(k) - \xi(k)R\delta(k)x^T(k-1) \tag{12}$$

and

$$\tilde{B}(k+1) = \tilde{B}(k) - \xi(k)R\delta(k)u^T(k-1) \tag{13}$$

We can easily show that the variation $\Delta X(k+1)$ of the considered Lyapunov function can be expressed as follows:

$$\Delta X(k+1) = -2\xi(k)[u^T(k-1)\tilde{B}^T(k) + x^T(k-1)\tilde{A}^T(k)]R\delta(k) + \xi^2(k)$$
$$\delta^T(k)[u^T(k-1)u(k-1) + x^T(k-1)x(k-1)]R^T R\delta(k) \quad (14)$$

or, in a compact form:

$$\Delta X(k+1) = -\delta^T(k)\phi(k)\delta(k) \quad (15)$$

where $\phi(k)$ is a matrix, which is described as:

$$\phi(k) = 2\xi(k)R - \xi^2(k)\rho^2(k-1)R^T R \quad (16)$$

where the parameter $\rho^2(k-1)$ is represented as follows:

$$\rho^2(k-1) = u^T(k-1)u(k-1) + x^T(k-1)x(k-1) \quad (17)$$

therefore, the stability condition of Lyapunov is given by:

$$\delta^T(k)[2\xi(k)R - \xi^2(k)\rho^2(k-1)R^T R]\delta(k) > 0 \quad (18)$$

from (18), it is easy to obtain the following inequality:

$$\lambda_R\rho^2(k-1)\xi(k) < 2 \quad (19)$$

The parametric gain $\xi(k)$ is positive definite. This makes it possible to confirm that the quantity $\lambda_R\rho^2(k-1)\xi(k)$ is positive. Thus, we can express the inequality, as follows:

$$\xi(k) = \frac{l(k)}{\lambda_R\rho^2(k-1)} \quad (20)$$

where the parameter $l(k)$ must satisfy the condition: $1 < l(k) < 2$.

The matrix R is a positive symmetric gain where the choice is mainly related to that of the initial condition parametric estimation. Thus, if the initial conditions parametric estimation are close to the real parameters system, we have to use small values in matrix R for the recursive parametric estimation algorithm to be converged. By selecting big values in matrix R, there may be a risk of having fluctuations around the estimated parameters. Therefore, if we select initial conditions far from the real parameters, the recursive estimation parametric algorithm requires correction in the beginning of the iterations. We have, therefore, to opt for a sufficiently big values in matrix R to improve the algorithm convergence speed. These results have been confirmed by numerical simulation.

3 Asynchronous machine modeling

This paragraph is devoted to the modeling of a squirrel-cage asynchronous machine. The passage from continuous model to discrete model is essential for the use of the recursive parametric estimation algorithm (6).

3.1 Continuous mathematical model

The studied asynchronous machine can be described in terms of the following continuous equation:

$$\frac{dx(t)}{dt} = A(t)x(t) + Bu(t) \tag{21}$$

where $x(t)$ and $u(t)$, which represent the state vector and the input vector of the asynchronous machine, respectively, are the following:

$$x^T(t) = [\, i_{ds}(t)\ i_{qs}(t)\ \phi_{dr}(t)\ \phi_{qr}(t)\,] \tag{22}$$

and

$$u^T(t) = [\, u_{ds}(t)\ u_{qs}(t)\,] \tag{23}$$

The variable state of the state vector $x(t)$ is made up of the stator currents $i_{ds}(t)$, $i_{qs}(t)$ and the rotor flows $\phi_{dr}(t)$ et $\phi_{qr}(t)$. These currents and flows sizes are obtained after the transformation of the three fixed phases of the stator and the rotor by an equivalent rolling formed with two windings of quadratic axes d and q.

The matrices $A(t)$ and B are defined by the following expressions, respectively.

$$A(t) = \begin{pmatrix} a_{11} & a_{12} & a_{13}(t) & a_{14}(t) \\ a_{21} & a_{22} & a_{23}(t) & a_{24}(t) \\ a_{31}(t) & a_{32}(t) & a_{33} & a_{34} \\ a_{41}(t) & a_{42}(t) & a_{43} & a_{44} \end{pmatrix} \tag{24}$$

$$= \begin{pmatrix} -\frac{R_s}{L_s - M^2 L_r} & \frac{R_r M}{L_r(L_s - M^2 L_r)} & \frac{M^2 L_r \omega_m(t)}{L_s - M^2 L_r} & \frac{M \omega_m(t)}{L_s - M^2 L_r} \\ \frac{R_s M}{L_r(L_s - M^2 L_r)} & -\frac{R_r L_s}{L_r(L_s - M^2 L_r)} & -\frac{M L_s \omega_m(t)}{L_r(L_s - M^2 L_r)} & -\frac{L_s \omega_m(t)}{L_s - M^2 L_r} \\ -\frac{M^2 L_r \omega_m(t)}{L_s - M^2 L_r} & -\frac{M \omega_m(t)}{L_s - M^2 L_r} & -\frac{R_s}{L_s - M^2 L_r} & \frac{R_r M}{L_r(L_s - M^2 L_r)} \\ \frac{M L_s \omega_m(t)}{L_r(L_s - M^2 L_r)} & \frac{L_s \omega_m(t)}{L_s - M^2 L_r} & \frac{R_s M}{L_r(L_s - M^2 L_r)} & -\frac{R_r L_s}{L_r(L_s - M^2 L_r)} \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_{11} & 0 \\ b_{21} & 0 \\ 0 & b_{33} \\ 0 & b_{43} \end{pmatrix} = \begin{pmatrix} \frac{L_s^2}{L_s - M^2 L_r} & 0 \\ -\frac{M L_s}{L_r(L_s - M^2 L_r)} & 0 \\ 0 & \frac{L_s^2}{L_s - M^2 L_r} \\ 0 & -\frac{M L_s}{L_r(L_s - M^2 L_r)} \end{pmatrix} \tag{25}$$

where L_s, L_r, R_s, R_r and M represent the stator cyclic inductance, the rotor cyclic inductance, the stator resistance, the rotor resistance and the stator-rotor mutual cyclic inductance, respectively.

The varying matrix parameters $A(t)$ are the parameters that depend on the mechanic speed $\omega_m(t)$.

The asynchronous machine presents a symmetry. We have:
$a_{11} = a_{33}$, $a_{12} = a_{34}$, $a_{21} = a_{43}$, $a_{22} = a_{44}$, $a_{13}(t) = -a_{31}(t)$, $a_{14}(t) = -a_{32}(t)$, $a_{23}(t) = -a_{41}(t)$, $a_{24}(t) = -a_{42}(t)$, $b_{11} = b_{33}$ et $b_{21} = b_{43}$.

3.2 Discrete mathematical model

The passage from the continuous mathematical model to the discrete mathematical model is made possible by the first order development of matrices $A(t)$ and B. The discrete model can have the following formulation:

$$x(k+1) = A_d x(k) + B_d u(k) \tag{26}$$

with

$$A_d = I + AT_e$$
$$B_d = T_e B$$

where I is the identity matrix. As far as the simulation step T_e is concerned, we have to use a maximum calculation step $T_e max$. By evaluating the matrix trace, we can, therefore, determine the maximum calculation step by:

$$trace(A(t)) = \frac{1}{T_e} \tag{27}$$

which imposes for the simulation step:

$$\Delta T_e = \frac{1}{|trace(A(t))|} \tag{28}$$

In this application, the maximum simulation step to be used is 1,099ms.

The passage from the continuous mathematical model to the discrete mathematical model provides the following constant machine parameters:

$a_{11} = 0.22$; $a_{12} = 0.98$; $a_{21} = 0.73$; $a_{22} = -0.02$, $b_{11} = 0.7$; $b_{21} = -0.66$.

There are two types of variation for time-varying parameters. The first corresponds to the parameters $a_{13}(k)$, $a_{14}(k)$, $a_{41}(k)$ and $a_{42}(k)$. The second corresponds to parameters $a_{23}(k)$, $a_{24}(k)$, $a_{31}(k)$ and $a_{32}(k)$. The curves of these two types variation are given in details in Figure 1.

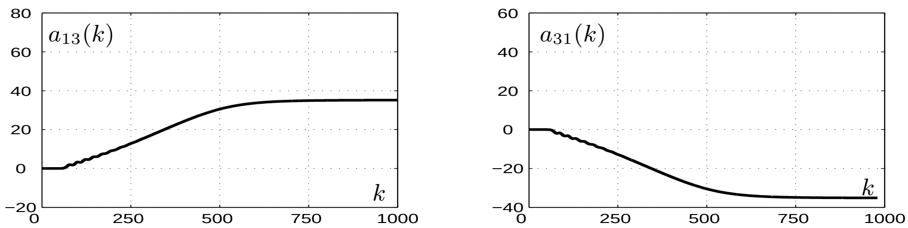

Fig. 1. Evaluation curves of the parameters $a_{13}(k)$ and $a_{31}(k)$.

4 Asynchronous machine parametric estimation

The estimation of the asynchronous machine parameters with the recursive parametric estimation algorithm, we must have the input $u(k)$ and state vector variable $x(k)$ in each discrete instant k. The curves of the stator voltages and of the stator currents, as well as the rotor flux are shown in Figures 2, 3 and 4.

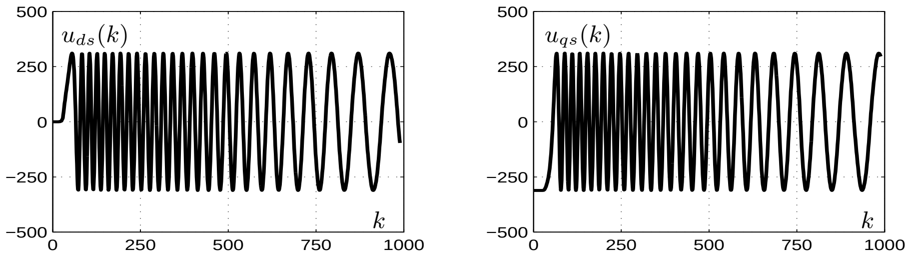

Fig. 2. Evaluation curves of the stator voltage $u_{ds}(k)$ and $u_{qs}(k)$.

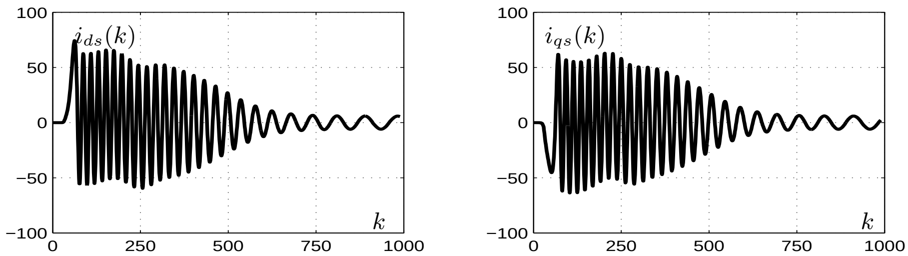

Fig. 3. Evaluation curves of the stator currents $i_{ds}(k)$ and $i_{qs}(k)$.

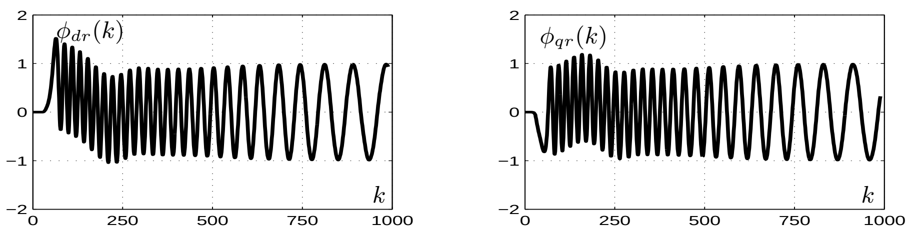

Fig. 4. Evaluation curves of the rotor flux $\phi_{dr}(k)$ and $\phi_{qr}(k)$.

We use the fixed parameter gain $l = 1.6$ and the following R matrix gain:

$$R = \begin{pmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{pmatrix} \tag{29}$$

The evaluation curves of the estimated parameters $\hat{a}_{11}(k)$, $\hat{a}_{12}(k)$, $\hat{a}_{21}(k)$ and $\hat{a}_{22}(k)$ are shown in Figures 5 and 6.

The evaluation curves $\hat{a}_{13}(k)$ and $\hat{a}_{31}(k)$, which represent the two variation types are given in Figure 7.

We show in Figure 8 the evaluation curves of the estimated parameters $\hat{b}_{11}(k)$ and $\hat{b}_{21}(k)$.

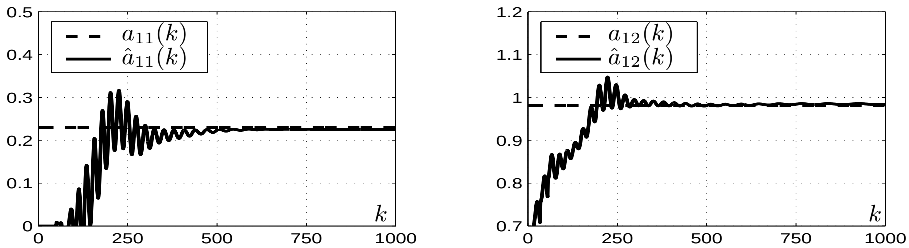

Fig. 5. Evaluation curves of the estimated parameters $\hat{a}_{11}(k)$ and $\hat{a}_{12}(k)$.

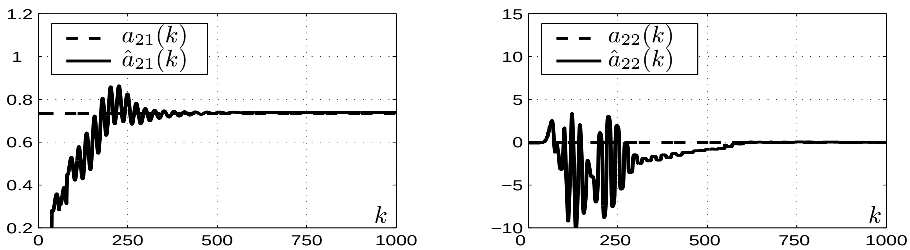

Fig. 6. Evaluation curves of the estimated parameters $\hat{a}_{21}(k)$ and $\hat{a}_{22}(k)$.

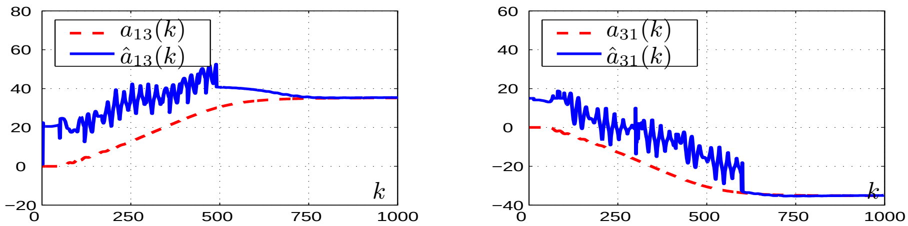

Fig. 7. Evaluation curves of the estimated parameters $\hat{a}_{13}(k)$ and $\hat{a}_{31}(k)$.

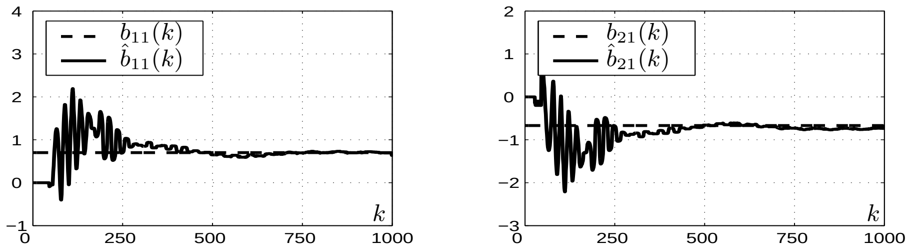

Fig. 8. Evaluation curves of the estimated parameters $\hat{b}_{11}(k)$ and $\hat{b}_{21}(k)$.

The estimated parameters related to constant parameters have oscillation in the beginning of the estimation. Starting from the iteration $k = 500$, the recursive algorithm of the parametric estimation (6) converges towards the real values.

To evaluate the estimation quality of the recursive parametric estimation algorithm (6), we consider an error $\delta_{a_{ij}}$, which can be expressed by:

$$\delta_{a_{ij}} = \frac{1}{200} \sum_{k=1}^{200} (a_{ij} - \hat{a}_{ij})^2 \tag{30}$$

with $i, j = 1, ..., 4$.

Table 1 shows that the errors in the real and the estimated parameters are small. We deduce, that the estimated values are close to the real ones.

Table 1. Average quadratic error between the real and the estimated parameters

Parameter	$\hat{a}_{11}(k)$	$\hat{a}_{12}(k)$	$\hat{a}_{21}(k)$	$\hat{a}_{22}(k)$	$\hat{b}_{11}(k)$	$\hat{b}_{21}(k)$
$\delta_{a_{ij}}$	10^{-5}	10^{-5}	10^{-5}	10^{-3}	10^{-4}	0.005

Concerning the varying-time parameters, we deduce that the estimated parameters follow the real ones but there is a gap in the variation slope. From the discrete instant $k = 750$, the estimated parameter converges towards the real parameter without a gap.

To compensate the increase of the estimation error $\delta(k)$, we rely on the parametric gain $l(k)$ by varying it exponentially or by using Fuzzy Logic techniques. These techniques allow us to have a Fuzzy supervisor which has as an input the estimation error and as an output the supervised parametric gain. The obtained supervised gain creates an adaptation for the parametric gain $l(k)$ of the parametric variation. If the estimation error is big, the gain has a small value, and if the error is small, the gain has a big value. In the two cases, the gap between the estimated and the real parameters decreases. We obtain the best estimation by integrating the supervised gain shown in Figure 9. The estimated parametric curves are shown in Figure 10.

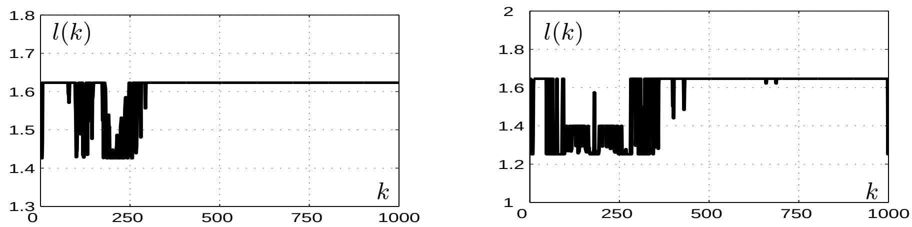

Fig. 9. Evaluation curves of the supervised gain for the estimation of $a_{13}(k)$ and $a_{31}(k)$.

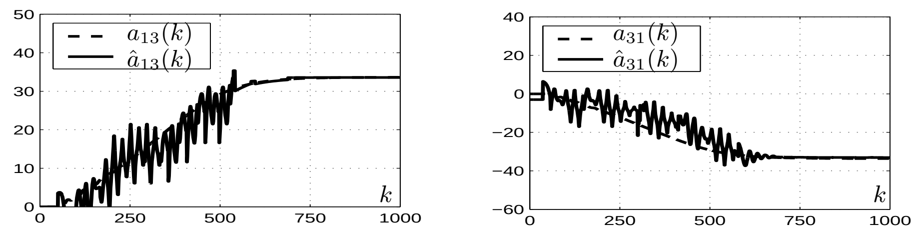

Fig. 10. Evaluation curves of the estimated parameters $\hat{a}_{13}(k)$ and $\hat{a}_{31}(k)$.

Figure 10 shows that the parametric estimation is improved by using a supervised gain in the recursive parametric estimation algorithm (6). Indeed, during the variation instants, i.e, the first five hundred iterations, the increase of the estimation error brings about a gap between the estimated parameter and the real one. This increase is compensated by the Fuzzy supervisor integrated in the parametric estimation algorithm. We obtain, therefore, in each discrete instant k, the appropriate parametric gain value according to the estimation error criterion. This makes the parametric estimation algorithm more robust towards asynchronous machine estimation variations.

Effect of the state noise on the convergence of the Algorithm

The variance of the state noise must to be lower than 0.03, so that the estimated parameters converges towards the real values. If the variance of the state noise is superior than 0.03 the estimated parameters $\hat{a}_{33}(k)$, $\hat{a}_{34}(k)$, $\hat{a}_{43}(k)$ and, $\hat{a}_{44}(k)$ don't converge towards the real values. Table 3 chow the error $\delta_{a_{ij}}$ (30) of these parameters in the case of the variance of the state noise is lower than 0.03 (var(v(k))<0.03) and, in the case of the variance of state noise is superior than 0.03 (var(v(k))>0.03). In the case of the variance of the state noise is lower than 0.03 (var(v(k))<0.03) is small, but in the case of the variance of state noise is superior than 0.03 (var(v(k))>0.03) this error is important. We deduce that the choice of a small variance of state noise is necessary for obtaining the convergence of all parameters.

Table 3. The error $\delta_{a_{ij}}$ of the parameters $\hat{a}_{33}(k)$, $\hat{a}_{34}(k)$, $\hat{a}_{43}(k)$ and $\hat{a}_{44}(k)$

parameter	$\hat{a}_{33}(k)$	$\hat{a}_{34}(k)$	$\hat{a}_{43}(k)$	$\hat{a}_{44}(k)$
$\delta_{a_{ij}}$(var(v(k)) $<$ 0.03)	10^{-5}	10^{-5}	10^{-5}	10^{-3}
$\delta_{a_{ij}}$(var(v(k)) $>$ 0.03)	0.1	1.9	1.6	0.16

5 Conclusion

In this paper, we dealt with parametric estimation of an asynchronous machine parametric estimation. This machine has been described as a state mathematical model, which is continuous, multivariate, linear, stochastic, with known state variables but unknown varying-time parameters and is able to vary according to time. The parametric estimation, with the use of a recursive parametric estimation algorithm, is studied after the passage from the continuous mathematical model to discrete mathematical model by the exponential matrix method. A recursive parametric estimation algorithm is developed. The stability analysis of the estimation scheme is studied. However, the estimation of the varying time is imprecise. To improve the convergence, we have used a varying parametric gain. This gain varies, exponentially, to compensate the error estimation increase, and it takes distinct values according to error estimation values. We obtain, therefore, a supervised gain. This supervised gain has been obtained by Fuzzy Logic techniques. The variance of the state noise has to be small, so that the estimated parameters converges towards the real values. Otherwise there will be some parameters which diverge. By applying it on the asynchronous machine parametric estimation, the presented recursive parametric estimation algorithm has brought about reliable performance.

Références

Bachir, I. (2002)
Contribution au diagnostic de la machine asynchrone par estimation paramétrique.
Thèse de Doctorat, Université de Poitiers.

Damak, N. et Kamoun, S. (2007)
Estimation paramétrique de systèmes décrits par des modèles mathématiques d'état.
Actes des septièmes journées scientifiques des jeunes chercheurs en Génie Electriques et Informatique, "Nouvelles Tendances Technologiques en Génie Electrique et Informatique", pp. 365-375, Mars 2007, Mounastir.

Damak, N. (2007)
Sur l'estimation des systèmes dynamiques décrits par des modèles mathématiques d'état.
Mastère en Automatique et Informatique Industrielle, Ecole Nationale d'Ingénireurs de Sfax.

Dehay, G. (1996)
Application des moindres carrés récursifs à l'estimation temps réel des paramètres de la machine asynchrone.
Thèse de doctorat, Université de Technologie de Compiègne.

Finch, J.W., D.J. Atkinson and P.P. Acarnley (1998)
Full-order estimator for induction motor states and parameters.
IEEE Proc. Electr. Power Applications.

Guesbaoui, Touhami, H. O. et Iung, C. (1994)
Identification of asynchrone machine parameters by a récursive least-squares method.
the 3^{rd} *I.E.E.E. Conference, Glasgow.*

Jemni, A. (1997)
Estimation paramétrique des systèmes à représentation continue-Application au génie Electrique.
Thèse de doctorat, Université de Poitiers.

Kamoun, M.(1994)
Modélisation, identification et commande adaptative décentralisée de processus discrets de grande dimension.
Thèse de Doctorat d'Etat en Génie Electrique (Automatique), Ecole Nationale d'Ingénieurs de Tunis, Université de Tunis 2, Tunis.

Kamoun, S. (2003)
Contribution à l'identification et à la commande adaptative de systèmes compplexes.
Thèse de Doctorat en Automatique et Informatique Industrielle, Ecole Nationale d'Ingénieurs de Sfax, Université de Sfax, 246 pages.

S. KAMOUN, (2007)
Design of parametric and state estimation algorithms for stochastic systems.
IJ-STA, Volume 1, December 2007, pp. 120-135,

Koubaa, Y. (2006)
Asynchronous machine parameters estimation using recursive method.
Science Direct. Simulation Modelling Practise and Theory, 14,pp. 1010-1021.

Moazzam, M.H.; T. Hesketh, and D.J. Clements (1997)
Recursive identification of certain structured time-varying state-space models.
IEE Proc. Control Theory Appl, vol. 144, No5.

Moreau, S. (1999)
Contribution à la modélisation et à l'estimation paramétrique des machines électriques à courant alternatif: Application au diagnostic.
Thèse de doctorat, Paris, 225 pages.

Zarchan P., Musoff H., (2000)
Fundamentals off Kalman Filtering:a Practical Approch, New York, AIAA.