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Adaptive Control with Fractional Order Reference Model

Yassine Bensafia and Samir Ladaci

University “20th August 1955” of Skikda,
Department of Electrical Engineering,
BP: 26, Skikda, 21000, Algeria.

{Bensafiay, Samir_ladaci}@yahoo.fr
http://www.univ-skikda.dz

Abstract. *Over the few last years the idea of introducing fractional calculus and systems in adaptive control has found a great interest, for the benefit one can win in the performances given by such systems. In adaptive control, the dynamic behavior of the system is defined by a chosen reference model and an adaptation algorithm modifies the correction to minimize the process output error. In this work, an adaptive control with a fractional order reference model is suggested. The main idea consists of making beforehand an approximation of the fractional reference model using one of the frequency domain approximation methods. After that, we use a classical algorithm of the adaptive control with the resulting reference model. Our objective is to find a control which takes the system to the desired state (the referential signal) with an improved behavior when compared to the integer order model scheme. The results of simulation have confirmed the efficiency proposed fractional order reference model adaptive controller.*

Keywords. *Fractional Adaptive Control, Fractional Control System, MRAC, Approximation Method.*

1. Introduction

Adaptive control with a reference model is part of a set of techniques to automatically adjust the control systems parameters when the characteristics of the process and disturbances are unknown or time varying. The dynamic behavior of the system is defined by the reference model and an adaptation algorithm modifies the correction to minimize the process model output error.

In principle, this type of control is nonlinear as it contains two loops nested against feedback: the correction loop and adaptation loop [1, 2]. There are several types of adaptive systems with reference model; these can be classified according to the structure as follows: MRAC parallel, MRAC series, and MRAC series–parallel.

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The parallel structure (see Figure 1) is the most famous structure, called the method of the output error in the case of the identification [3-6].

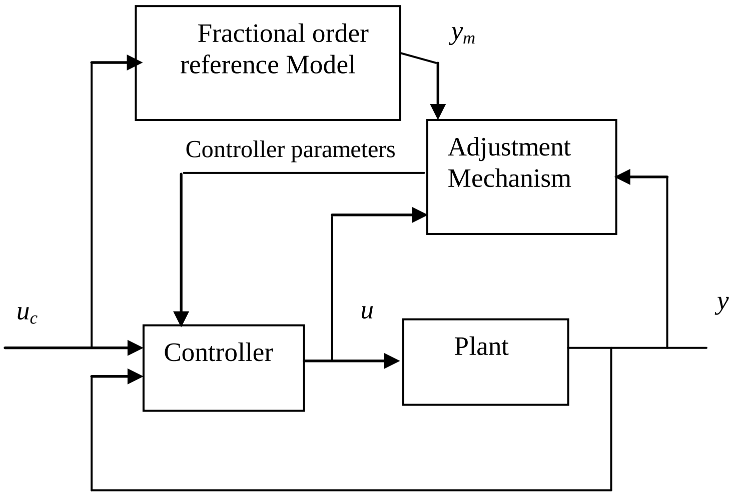

Fig .1. Parallel structure of adaptive control with fractional order reference model.

Many scientific works have shown the importance of Fractional systems and their utility in mathematics, system modeling and control engineering [7-10]. Applications concern various aspects of physical sciences fields, as mechanics, electricity, chemistry, biology, economics, modeling, time and frequency domain system identification and notably control theory, mechatronics and robotics [11]. The interest for the introduction of these systems in adaptive control [3-5] has been first motivated by the very good proven performances of fractional systems relatively to those of integer order.

In this work, an adaptive control with a fractional reference model is suggested. The main idea consists of making beforehand an approximation of the fractional reference model using one of the famous approximation methods (Oustaloup). After that, we use a classical algorithm of the adaptive control with the approximated referential model.

This paper is structured as follows: Section 2 presents the model reference adaptive control (MRAC) problem. Section 3 is an introduction to fractional order systems and Oustaloup approximation method, and then two simulation examples are given in section 4. Finally, some concluding remarks are presented in Section 5.

2. Description of MRAC

2.1. Follow-up Model

Consider SISO system, which can be a represented by a continuous time or discrete-time model:

$$y(t) = \frac{B}{A} u(t) \tag{1}$$

Where u is the control signal and y is the output signal. The symbols A and B denote polynomials in the differential operator p. It is assumed that the degree level $\deg(A) \geq \deg(B)$, the system is causal [1].

We assume that we are trying to find a regulator such that the relationship between the reference signal u_c and the desired signal output y_m, after having approximated the fractional reference model is given by:

$$y_m(t) = \frac{B_m}{A_m} u_c(t) \tag{2}$$

Where A_m and B_m are polynomials in the differential operator p. The generally linear control law is described as:

$$R\,u = T\,u_c - S\,y \tag{3}$$

With R, S and T are polynomials. This control law is against a negative feedback with the transfer operator $-S/R$ and a direct reaction with the transfer operator $-T/R$.

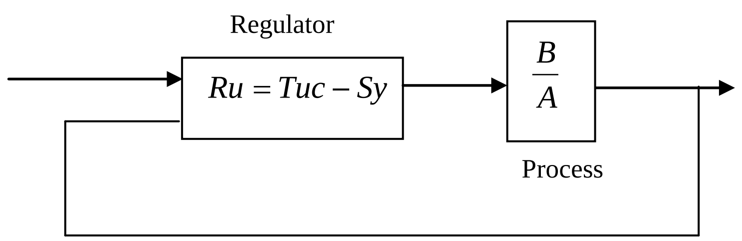

Fig .2. Closed-loop system with a linear regulator.

2.2. MIT rule

The gradient approach to MRAC is based on the assumption that the parameters change more slowly than the other variables in the system. This assumption, which

admits a quasi-stationary treatment, is essential for the computation of the sensitivity derivatives that are needed in the adaptation [12].
Let e denote the error between the system output y, and the reference output y_m. Let θ denote the parameters to be updated. By using the criterion

$$J(\theta) = \frac{1}{2} e^2 \quad (4)$$

the adjustment rule for changing the parameters in the direction of the negative gradient of J J is that

$$\frac{d\theta}{dt} = -\gamma.\frac{\partial J}{\partial \theta} = -\gamma.e\frac{\partial e}{\partial \theta} \quad (5)$$

If it is assumed that the parameters change much more slowly than the other variables in the system, the derivative ∂e/∂θ, i.e., the sensitivity derivative of the system, can be evaluated under the assumption that θ is constant [1].
There are many variants about the MIT rules for the parameter adjustment. For example, the sign-sign algorithm is widely used in [13]; the PI-adjustment rule is used in [14].
The control law is given by the following equation:

$$u = -\theta^T (P_1\varphi) \quad (6)$$

3. Oustaloup Approximation method

The Oustaloup approximation method of a generalized derivator, a differential action which covers the frequency space, is based on a recursive distribution of an infinite number of zeros and negative real poles (to ensure a minimum phase behavior) [11,15-17]. As part of a realist synthesis (practice) based on a finite number of zeros and poles, it should reduce the differential behavior of a generalized bounded frequency range, chosen according to the needs of the application.

The method is based on the function approximation from:

$$H(s) = s^{\alpha}, \qquad \alpha \in R^+$$

By a rational function [3, 5]:

$$G_f(s) = K\prod_{K=1}^{N}\frac{s+\omega_k^{'}}{s+\omega_K} \quad (7)$$

where the poles, zeros, and gain are evaluated from:

$$\omega_K^{'} = \omega_b.\omega_u^{(2K-1-\gamma)/N},\ \omega_K = \omega_b.\omega_u^{(2K-1+\gamma)/N},\ K = \omega_h^{\gamma}$$

ω_u is the unity frequencies gain and the central frequency of a band of frequencies distributed geometrically. Let $\omega_u = \sqrt{\omega_h.\omega_b}$, where ω_h and ω_b are respectively the upper and lower frequencies.

γ is the order of derivative, and N is the order of the approximating function.

4. Examples

4.1. Example 1: (the reference model is of integer order)

The system is described using the following equation:

$$G(s)=\frac{y(s)}{u(s)}=\frac{2s+8}{s^2-3s-2} \tag{8}$$

The referential model is defined by:

$$Gm(s)=\frac{y_m}{u_m}=\frac{1}{0.2s+1} \tag{9}$$

Let us assume that we want to minimize the error:

$$e=y-y_m \tag{10}$$

The recurrence equation of the system described above which is obtained after the discretization $(T=0.04)$ is given by :

$$\mathrm{y(k+2)=2.131.y(k+1)-1.127.y(k)+0.09171u(k+1)-0.07811.u(k)}$$

The recurrence equation of the original model described above which is obtained after the discretization $(T=0.04)$ is given by:

$$\mathrm{y_m(k+1)=0.8187.y_m(k)+0.1813.u_m(k)}$$

Let $\mathrm{k, l}$ and m be respectively the degree of the polynomials R, S and T, such that:

$$\mathrm{k=degR=degAm+degB-degA}\ =1+1-2=0$$

$$l=\deg S=\deg R=0$$

$$m=\deg T=\deg B_m=0$$

Therefore, the vector of regulation parameters is:

$$\theta=(s_0,t_0)$$

Let us define the regression vector (or measurement vector) φ us follows:

$$\varphi^T=-\left[\frac{\delta e}{\delta r_1}\ \ldots\frac{\delta e}{\delta r_k}\ \frac{\delta e}{\delta s_0}\ \ldots\frac{\delta e}{\delta s_l}\ \frac{\delta e}{\delta t_0}\ \ldots\frac{\delta e}{\delta t_m}\right]$$

$$=\frac{b_0}{A_0A_m}\left[s^{k-1}u\ldots u\ \ s^l y\ldots y\ \ -s^m u_c\ldots-u_c\right] \tag{11}$$

$$=\frac{b_0}{A_0A_m}\left[y\quad -u_c\right]$$, such that $b_0=1$ and $A_0=s+1$.

$$\varphi^T=\frac{1}{(s+1).(0.2s+1)}\left[y\quad -u_c\right]$$

The recurrence equation of φ^T after the discretization $(T = 0.04)$ is given by

$$\varphi^T(k+2) = 1.78\,\varphi^T(k+1) - 0.7866\,\varphi^T(k) + [0.003696\,y(k+1) + 0.003412\,y(k) - 0.003696\,u_c(k+1) + 0.003412\,u_c(k)]$$

The command law is given using the following equation:

$$u = -\theta^T(P_1\varphi) \qquad (12)$$

The real output of the system is shown in Fig. 3.

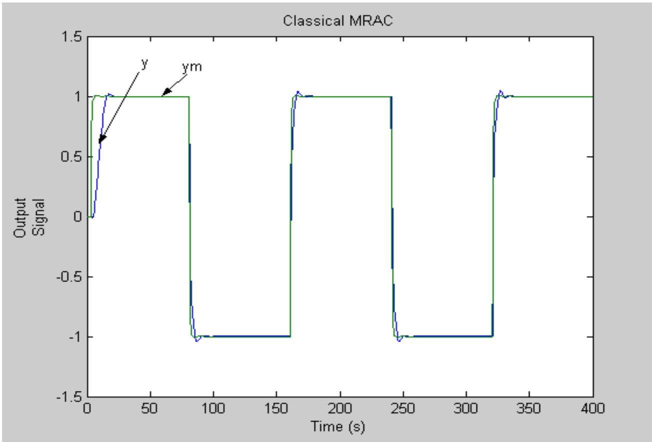

Fig .3. The real output of the system y and the reference output y_m

The following figure (Fig. 4) shows us the error between the real output of the system and the real output of the original model.

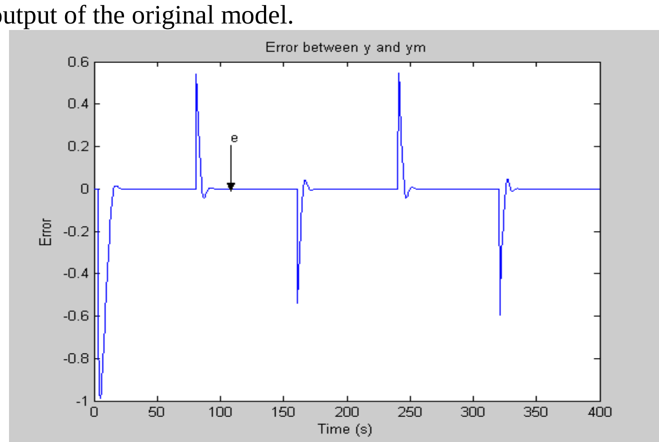

Fig .4. The error signal between y and y_m

From Figures Fig .3 and Fig .4, we remark that the output of the system follows the referential one and the small oscillations are caused by the sudden change of the referential signal.

4.1. **Example 2:** (the reference model is of fractional order)

The system is described using the following equation:

$$G(s)=\frac{y(s)}{u(s)}=\frac{2s+8}{s^2-3s-2} \tag{13}$$

The fractional original model is given by:

$$G_m(s)=\frac{y_m}{u_m}=\frac{1}{0.2s^{0.25}+1} \tag{14}$$

The fractional original model approximated using the method of Oustaloup is:

$$G_m(s)=\frac{s+13.34}{2.1246s+14.1834} \tag{15}$$

The recurrence equation of the previous system obtained after the discretization $(T=0.04)$ is given by:

$$\mathrm{y(k+2)=2.131.y(k+1)-1.127.y(k)+0.09171u(k+1)-0.07811.u(k)}$$

The recurrence equation of the original model described above after the discretization (T=0.04) is given by:

$$\mathrm{y_m(k+1)=0.7656.y_m(k)+0.4707.u_m(k+1)-0.2503u_m(k)}$$

Let k, l and m be respectively the degree of the polynomials R, S and T, such that:

$$\mathrm{k=degR=degA_m+degB-degA=1+1-2=0}$$

$$\mathrm{l=degS=degR=0}$$

$$\mathrm{m=degT=degB_m=1}$$

Therefore the regulation parameters vector is:

$$\theta=(s_0,t_0,t_1)$$

Let us define the regression vector φ :

$$\varphi^T=-\left[\frac{\delta e}{\delta r_1}\ \frac{\delta e}{\delta r_2}\cdots\frac{\delta e}{\delta r_k}\ \frac{\delta e}{\delta s_0}\ \frac{\delta e}{\delta s_1}\cdots\frac{\delta e}{\delta s_l}\ \frac{\delta e}{\delta t_0}\ \frac{\delta e}{\delta t_1}\cdots\frac{\delta e}{\delta t_m}\right]$$

$$=\frac{b_0}{A_0A_m}\left[s^{k-1}u\ldots u\ \ s^l y\ldots y\ \ -s^m u_c\ldots-u_c\right] \tag{16}$$

$$=\frac{b_0}{A_0A_m}\begin{bmatrix} y & -s\,u_c & -u_c\end{bmatrix}$$, such that $b_0=1$ and $A_0=s+1$.

$$\varphi^T=\frac{1}{(s+1).(2.1246s+14.1834)}\begin{bmatrix} y & -s\,u_c & -u_c\end{bmatrix}$$

The recurrence equation of φ^T after the discretization (T=0.04) is given by:

$$\varphi^{\mathrm{T}}\mathrm{(k+2)=1.726}\ \varphi^{\mathrm{T}}\mathrm{(k+1)-0.7356}\,\varphi^{\mathrm{T}}\mathrm{(k)+[0.0003405\,y(k+1)-0.0003073\,y(k)}$$
$$\mathrm{-0.0003405\,u_c(k+1)-0.0003073\,u_c(k)]}$$

The command law is given using the following equation:

$$u=-\theta^T(P_1\varphi)$$

The real output of the system is shown in figure Fig. 5.

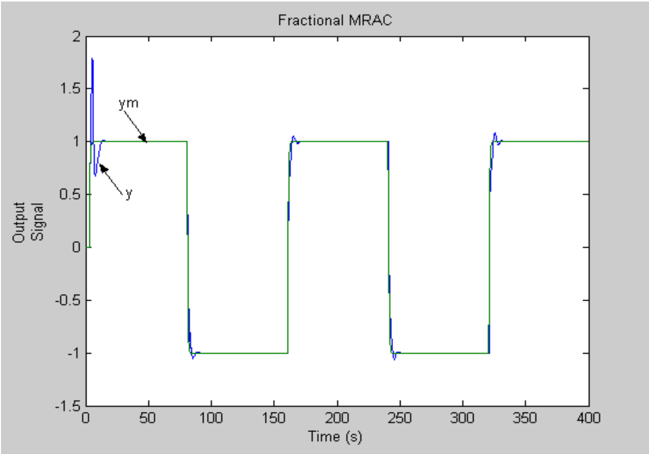

Fig .5. The real system output y and the fractional order model output y_m

The following figure shows us the form of the error e:

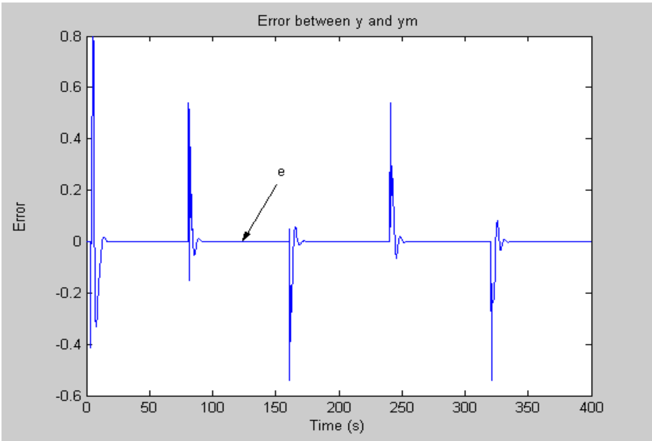

Fig .6. The error signal between y and y_m

From figures Fig. 5 and Fig. 6, we can remark that the output of the system follows the original (reference) model and the small oscillations are caused by the sudden change of the original signal. Hence, the system becomes more precise using the fractional original model with a better behaviour regarding the output performance.

4. Conclusion

In this work, an adaptive control with a fractional reference model is suggested. The main idea consists of making beforehand an approximation of the fractional reference model using one of the famous approximation methods (Oustaloup). After that, we use a classical algorithm of the adaptive control with the approximated referential model.

Our objective is to find a control which takes the system to the desired state (the referential signal) with improved performance behavior. The results of simulation have confirmed the efficiency of the adaptive control with a fractional reference model. Further work will concern the introduction of such fractional order filters in the adaptation algorithm and control law.

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