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A Comparative Analysis of Two Formulations for Actuator Faults Detection and Isolation: Application to a Waste Water Treatment Process

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Abstract. *The goal in many fault detection and isolation (FDI) schemes is to increase the isolation and identification speed. This paper compares two methods for FDI. The first method is based on adaptive nonlinear observer. This approach uses the model of the system and a bank of adaptive observers to generate residuals in such way to isolate the faulty actuator after detecting the fault occurrence. The second method based on interval observers. The practical domain of the value of each actuator parameter is divided into a certain number of intervals. After verifying all the intervals whether one of them contains the faulty actuator, the faulty value is identified and the corresponding fault is isolated to achieve faster isolation speed.*
Simulation results show the effectiveness and the difference between the two proposed detection and isolation methods using an example of the waste water treatment process described by a nonlinear system model.

Keywords. *Diagnosis, Fault detection and isolation, Nonlinear system, Actuators, Adaptive observer, interval observers.*

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1. Introduction

When a fault occurs in a system, the main problem to be addressed is to raise an alarm, ideally diagnose what fault has occurred, and then decide how to deal with it. The problem of detecting a fault, finding the source and location and then taking appropriate action is the basis of fault tolerant control.

So the objective is to determine if a fault is present in a system (fault detection), as well as to determine the kind, the location (fault isolation), the size, and the time-varying behavior (fault identification) of the fault.

In order to improve the reliability, the operational stability and the production efficiency we use the fault tolerant control, which relies on early fault detection using FDI procedures [12] of faulty elements which can help preventing larger failures or even the destruction of the monitored plant, by stopping the control of the process or by using an adapted control law. This is the main task of fault detection and isolation in dynamic systems. FDI problem has been extensively studied, and many powerful methods have been developed.

Nowadays, the field of model based FDI is growing very fast. The early studies were focused on the design of FDI algorithms for linear system by using parameters estimation approach [10] and [11], parity space approach [7] and [19] or observers based approach [18] with the latter being the most relevant. But the application of linear algorithms are limited when the system process is nonlinear and its nonlinearity can not be ignored; that's why in the last decade, a considerable amount of researches were initiated to deal with these problems. As a consequence recent works treat theoretical development for FDI methods of nonlinear systems, such as differential geometric approach [3], sliding mode observer [2] and [21], adaptive control technique [8], [22], [13] and [5] and observers intervals [14], [15].

This work focuses on the detection and isolation for single actuator fault using a proposed nonlinear adaptive observer which is a well known method for this kind of problem. We design an adaptive observer for each actuator for the fault detection. Then we build a bank of adaptive observers to isolate the actuator's fault.

The method computes a residual vector that is zero when no fault is present and non zero otherwise, to detect that a fault has occurred. The residual will also be different for different faults, to enable diagnosing which fault has occurred.

The parameter estimate based methods rely on the parameter identification procedures; this dependence causes a disadvantage that the speed of the method is not satisfactory because the parameter identification needs a long time.

To this end, we propose a second method based on parameter intervals to make the isolation quicker. In this method, the practical domain of each actuator parameter is partitioned into a certain number of intervals. Each bound of the intervals is used to build an observer. If an interval contains the actuator faulty parameter value and if this interval is small enough, then the residuals of the two correspondent observers will be smaller than the residuals of the other observers and the signs of these two residuals will be different and we isolate the actuator faulty. We will prove that this method bears some resemblance to the method based on adaptive observers. However it does not use the procedure of parameter identification, and therefore is faster than the method based on adaptive observers.

In this paper, we describe the two methods, and apply them to the waste water treatment process. In section 2, the class of the nonlinear systems that we study and the formulation of the nonlinear adaptive observer are expressed. Section 3 presents the principle of the second method using parameter intervals. A briefly description of the waste water treatment process is investigated in section 4. In section 5, simulation results and a brief comparison are provided to show the effectiveness of these proposed computation methods. Conclusions and perspectives on future works end the paper.

2. Fault detection and isolation using adaptive observers

The task of FDI is a chain of processing blocks as shown in figure 1. It consists of the following steps:

- Generation of residuals;
- Testing of residuals;
- Fault diagnosis.

The proposed approach to FDI relies on residual generation method using adaptive observers.

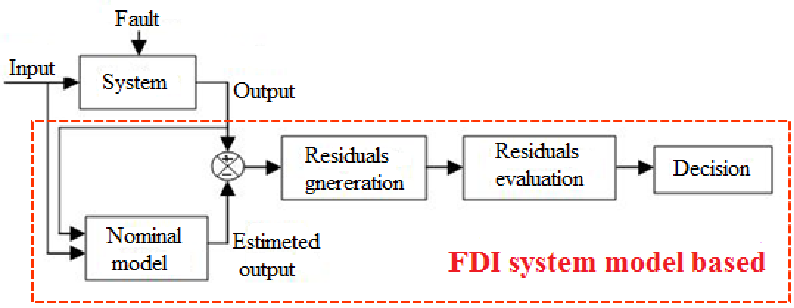

Fig 1. The FDI based model scheme

2.1. Presentation of the system and fault modeling

Consider the nonlinear dynamic system described by:

$$\begin{cases} \dfrac{dx}{dt} = f(x) + \sum_{i=1}^{m} g_i(x) u_j \\ y = Cx \end{cases} \tag{1}$$

where $f(x) \in R^n$ is a nonlinear vector function, $g(x) \in R^{n \times m}$ is a matrix whose elements are nonlinear functions, $u = [u_1, u_2, \ldots, u_m]^T \in R^m$ is the input vector of the system (actuators outputs) , $C \in R^{p \times n}$ and $y \in R^p$ is the output vector.

In this paper, we will discuss the detection and isolation of actuator fault, which affect the input vector of the system so the corresponding faulty model is given below:

$$\begin{cases} \dfrac{dx}{dt} = f(x) + \sum_{j \neq l} g_j(x) u_j + g_l(x) \theta_{u_l} \\ y = Cx \end{cases} \tag{2}$$

Throughout this paper, we assume that only constant actuator fault can occur and $u_j^f = u_j + f_{a_j} = \theta_{u_j}$ is the output of the j^{th} actuator when is faulty (f_{a_j} is a constant) for $t \geq t_f$, $j \in 1,2,\ldots,m$, while u_j is the output when it is healthy.

2.2. The single actuator fault detection and isolation scheme

Considering that the fault is on the i^{th} actuator and $g_i(x) = (g_1(x), g_2(x), \ldots, g_m(x))$, the observer [1], [4] is given by:

$$1 \leq i \leq m \begin{cases} \dfrac{d\hat{x}_i}{dt} = f(x) + \sum\limits_{j \neq i} g_j(x) u_j + g_i(x)\hat{\theta}_{u_i} + H(\hat{x}_i - x) \\ \dot{\hat{\theta}}_{u_i} = -2\gamma(\hat{x}_i - x)^T P g_i(x) \\ \hat{y}_i = C\hat{x}_i \end{cases} \tag{3}$$

where H is a Hurwitz matrix that it can be chosen freely, γ is a scalar and P is a positive definite matrix. The two matrixes P and H are calculated by using the following Lyapunov equation:

$$H^T P + PH = -Q \tag{4}$$

where Q is any positive definite matrix that can be chosen freely.

The basic idea is to estimate the output y_i and compare it to the one generated via the model. The advantage of this proposed method is that it can detect and isolate single fault actuator rapidly by using the FDI scheme. The residual r_i is given by:

$$r_i(t) = \|\hat{y}_i - y\| \tag{5}$$

Then, when we applied a constant fault in one of these actuators, the correspondent residual reach zero after a short time. On the other hand the others residuals take a new constant value and remain at its level.

2.3. The multiple actuator faults detection and isolation scheme

To maintain system reliability, FDI must detect and isolate control system failures quickly and accurately. In this case a new structure of the previous method of single fault detection and isolation is developed. In most of the model based methods the sequence of FDI is: firstly we detect the presence of a fault, secondly we isolate the faulty instrument and finally we identify his value.

- **Fault detection and identification**

In the proposed scheme, we detect the presence of a fault and at the same time we identify its value. We will use this identification for the fault isolation.

For the fault detection and identification we will develop a bank of i adaptive observers, with $i = m$, the number of the system actuators. The adaptive observers form will be as in (3), only that we will consider that all actuators are faulty:

$$1 \leq i \leq m \begin{cases} \dfrac{d\hat{x}_i}{dt} = f(x) + \sum_{j \neq i} g_j(x)\hat{\theta}_{u_{i,j}} + H(\hat{x}_i - x) \\ \dot{\hat{\theta}}_{u_i} = -2\gamma(\hat{x}_i - x)^T P g_i(x) \\ \hat{y}_i = C\hat{x}_i \end{cases} \tag{6}$$

where $\hat{x}_i$ is the state vector of the i^{th} observer; the appropriate gain matrices H and P will have been chosen in order to have a good fault estimation. We will apply to the residual r_i for the fault detection of the form:

$$r_i(t) = \frac{\|\hat{y}_i - y\|}{dt}, \qquad i \in m \tag{7}$$

These residuals allow us to calculate fault estimations by using the following relation:

$$\varphi_i = sgn(r_i)\, max(|r_i|), \qquad i \in m \tag{8}$$

- **Fault isolation**

Once fault is detected, it needs to be isolated. So the fault isolation requires the creation of m banks of m adaptive observers where we will use these m estimations φ_i from estimation vector. The adaptive observer form will be as in (3) where all the banks will be identical except from the system input value:

$$1 \le i \le m \begin{cases} \dfrac{d\hat{x}_i^k}{dt} = f(x) + \sum_{j \ne i} g_j(x) v_j^k + g_i \hat{\theta}_{u_i}^k + H(\hat{x}_i^k - x) \\ \dot{\hat{\theta}}_{u_i}^k = -2\gamma(\hat{x}_i^k - x)^T P g_i(x) \\ \hat{y}_i = C\hat{x}_i^k \end{cases} \tag{9}$$

where the input vector u will be replaced by v^k, $1 \le k \le m$:

$$v^1 = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} \varphi_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; v^2 = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} 0 \\ \varphi_2 \\ \vdots \\ 0 \end{bmatrix}; \cdots; v^m = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \varphi_m \end{bmatrix} \tag{10}$$

Then we will create m banks of adaptive observers, one for each actuator, who will have the same form as in (9). The corresponding residual of the fault isolation is given below:

$$s_{k,i} = \left\| \hat{y}_i^k - y \right\|, \quad k,i \in [1,\cdots,m] \tag{11}$$

where $\hat{y}_i^k$ *i* is the estimated output of the i^{th} observer of the k^{th} bank.

All of the residual of all the bank of adaptive observers rest in this initial value zero the whole period until the time when the fault has been entered, where its leave zero for a short time period. Thus we can detect the fault and the structural residual s_{ki} associated to the isolation bank will indicate us the faulty actuator and consequently the fault estimation value.

3. Fault detection and isolation using parameter interval approach

In the parameter interval based method, each actuator parameter is divided into a certain number of intervals. After occurrence of a fault, the value of the faulty actuator parameter must be in one of these intervals. After checking each interval whether it contains the fault or no, the faulty actuator parameter value is found, the isolation is therefore realized. In this section, we quickly describe the principle of the parameter interval based fault isolation method.

3.1. Nonlinear dynamic system and fault modeling

Considering the nonlinear dynamic system (2), it can be described as:

$$\begin{cases} \dot{x} = f(x, \theta_{u_i}, u) \\ y = Cx \end{cases} \tag{12}$$

where: θ_{u_i} is the constant parameter vector of the actuator such as $\theta_{u_i}^{0}$ is its nominal value. For actuator fault isolation each component of the parameter vector θ_{u_i} is divided into certain number of intervals.

This method is based on the monotonous characteristic of w function. In [15] the studies only focus on the faults of the dynamic part of the system. In this paper, this method is extended to actuator fault isolation problem for nonlinear dynamic systems. f is a monotonous function of the parameter θ_{u_i} at any considered point x in the state space. There is a fault in this dynamic system (12), if the dynamic difference:

$$\Delta f(x, \theta_{u_i}, \theta_{u_i}^{0}, u) = f(x, \theta_{u_i}, u) - f(x, \theta_{u_i}^{0}, u) \tag{13}$$

between the system and its nominal model caused by the difference of actuators parameter vectors $\Delta\theta_{u_i} = \theta_{u_i} - \theta_{u_i}^{0}$ is significant.

3.2. Fault detection and isolation

After the fault occurrence, the fault isolation task is followed by the fault detection procedure. For fault detection, an existing method [14] is used. The time of the fault occurrence and the time when it is detected are considered as the same which is noted as t_f .

We assume that the considered faults are caused by the change of single actuator parameter. For m actuators parameters θ_{u_1} , θ_{u_2} , …, θ_{u_m} we partition the possible domain of each parameter into a certain number of intervals. For example, the parameter θ_{u_j} is partitioned as p intervals, the bounds of i^{th} interval are $\theta_{u_j}^{a(ij)}$ and $\theta_{u_j}^{b(ij)}$. After fault occurrence, the faulty actuator parameter value must be in one of the parameter intervals. To verify if an interval contains the faulty value, an actuator parameter filter is built for this interval. A parameter filter consists of two isolation observers which correspond to two bounds of the interval.

3.3. The actuator fault detection and isolation scheme

For the model (12), the parameter filter with respect to actuator fault can be described with the isolation observers given below:

$$\begin{cases} \dot{\hat{x}}^{a(ij)} = f(\hat{x}^{a(ij)}, \theta_{u_j}^{oba(i)}, u) + k(y - \hat{y}^{a(ij)}) \\ \dot{\hat{y}}^{a(ij)} = C\hat{x}^{a(ij)} \\ \varepsilon^{a(ij)} = y_h - \hat{y}_h^{a(ij)} \end{cases} \tag{14}$$

$$\begin{cases} \dot{\hat{x}}^{b(ij)} = f(\hat{x}^{b(ij)}, \theta_{u_j}^{obb(i)}, u) + k(y - \hat{y}^{b(ij)}) \\ \dot{\hat{y}}^{b(ij)} = C\hat{x}^{b(ij)} \\ \varepsilon^{b(ij)} = y_h - \hat{y}_h^{b(ij)} \end{cases} \tag{15}$$

where:

- $\theta_{u_j}^{oba(i)} \in R^m$, $\theta_{u_j}^{obb(i)} \in R^m$ are the parameter vectors of the observers corresponding to actuator parameter vector;
- $\varepsilon^{a(ij)} \in R$, $\varepsilon^{b(ij)} \in R$ are the estimation errors ;
- y_h is the h^{th} component of y ;
- $\hat{y}_h^{a(ij)}$ and $\hat{y}_h^{b(ij)}$ are the h^{th} component respectively of $\hat{y}^{a(ij)}$ and $\hat{y}^{a(ij)}$.

We assume that before the fault occurrence, the observer's states $\hat{x}^{a(ij)}$ and $\hat{x}^{b(ij)}$ have converged to the system state x, so: $\varepsilon^{a(ij)}(t < t_f) = \varepsilon^{b(ij)}(t < t_f) = 0$ since $\theta_{u_j}^{oba(i)}(t < t_f) = \theta_{u_j}^{obb(i)}(t < t_f) = \theta_{u_j}^{0}$.

But at the time t_f, when the fault is occurred the s^{th} actuator parameter changes:

$$\forall t \geq t_f \quad \begin{cases} \theta_{u_s}^f = \theta_{u_s}^0 + \Delta_u^f \\ \theta_{u_l}^f = \theta_{u_l}^0 \end{cases} \tag{16}$$

and the j^{th} parameter of the observers change in order to isolate the fault:

$$\theta_{u_j}^{oba(i)}(t) = \begin{cases} \theta_{u_j}^0, t < t_f \\ \theta_{u_j}^{a(i)}, t \geq t_f \end{cases} \quad ; \quad \theta_{u_l}^{oba(i)}(t) = \theta_{u_l}^0, \forall t, l \neq j \tag{17}$$

$$\theta_{u_j}^{obb(i)}(t) = \begin{cases} \theta_{u_j}^0, t < t_f \\ \theta_{u_j}^{b(i)}, t \geq t_f \end{cases} \quad ; \quad \theta_{u_l}^{obb(i)}(t) = \theta_{u_l}^0, \forall t, l \neq j \tag{18}$$

Where: $\theta_{u_j}^{a(i)}$ et $\theta_{u_j}^{b(i)}$ are the bounds of the i^{th} interval of j^{th} actuator parameter.

Our index of isolation is: $v^{ij}(t) = sgn(\varepsilon^{a(ij)}(t))sgn(\varepsilon^{b(ij)}(t))$, there are two cases [16]:

- For the case where the interval contains the faulty parameter value it will be:

$$sgn(\varepsilon^{a(ij)}(t)) = -sgn(\varepsilon^{b(ij)}(t)) \tag{19}$$

- For the case where the interval does not contain the faulty value, it exists $t_e \geq t_f$ that:

$$sgn(\varepsilon^{a(ij)}(t_e)) = sgn(\varepsilon^{b(ij)}(t_e)) \tag{20}$$

4. Description of the wastewater treatment process model

The increasing pace of industrialization, urbanization and population growth that our planet has faced over the last century has considerably increased environmental pollution and habitat destruction, and it negatively affected water, air and soil qualities. In this context, wastewater treatment has become one of the most important environmental issues, as it reduces or prevents pollution of natural water resources promotes sustainable water re-use, protects the aquatic environment and improves the status of aquatic ecosystems.

During the operation of a biological wastewater treatment process, many disturbances and faults can occur. The nature of these changes can be either sudden or slow and they can be related to normal or faulty process operation, provoking real or apparent deviations from the normal operation. This biochemical process is highly complex system, with a great number of components interacting to achieve the system’s purpose. In this system, all components are related in a complex manner, which means that a fault in one component can often cause the failure of the entire system. To prevent this event, it is essential to detect faults immediately in order to enable the controlling system to take actions, so that the system can still fulfill its purpose. In the last decades, the biological treatment processes has proven to be an effective way to deal with polluted wastewater. The activated sludge process (Fig.2) is the most generally applied biological wastewater treatment method [9].

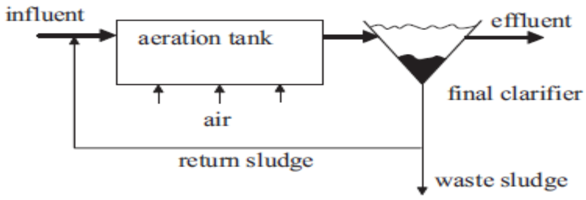

Fig. 2. The conventional activated sludge scheme

In the activated sludge process, a bacterial biomass suspension is responsible for the removal of pollutants. The fundamental phase of the mathematical modeling for the processes of water treatment by activated sludge consists in determining the reaction rates of the macroscopic variables of the system to know the rate of: biomass growth, substrate degradation and dissolved oxygen uptake. These variables, as well as inputs and outputs, are collected in mathematical expressions constituting the model of the process. The mathematical model [17] of the activated sludge process is based on the equations, resulting from mass balance considerations, carried out on each of the reactant of the process:

$$\text{Variation} = \pm\,\text{Conversion} + \text{Feeding} - \text{Drawing off}$$

All the details about the system can be found in [6]. The FDI scheme will monitor the four actuators Q_{in}, Q_L, Q_r and Q_w.

5. Application

In this section, the FDI is applied to a wastewater treatment process model using these two methods.

5.1. Synthesis of the observer using the first method

The process model is a nonlinear system with the same form as in (1):

$$\begin{cases}\dot{S}_I = \dfrac{Q_{in}}{V_r}(S_{I,in} - S_I) \\ \dot{S}_S = \dfrac{Q_{in}}{V_r}(S_{S,in} - S_S) - \dfrac{1}{Y_H}\rho_1 + \rho_3 \\ \dot{X}_I = \dfrac{Q_{in}}{V_r}(X_{I,in} - X_I) - \dfrac{Q_r}{V_r}(X_{I,rec} - X_I) + f_{X_1}\rho_2 \\ \dot{X}_S = \dfrac{Q_{in}}{V_r}(X_{S,in} - X_S) - \dfrac{Q_r}{V_r}(X_{S,rec} - X_S) + (1 - f_{X_1})\rho_2 - \rho_3 \\ \dot{X}_H = \dfrac{Q_{in}}{V_r}(X_{H,in} - X_H) - \dfrac{Q_r}{V_r}(X_{H,rec} - X_H) + \rho_1 - \rho_2 \\ \dot{S}_O = \dfrac{Q_{in}}{V_r}(S_{O,in} - S_O) - Q_L\dfrac{\beta}{C_S}(C_S - S_O) - \dfrac{1-Y_H}{Y_H}\rho_1 \\ \dot{X}_{H,rec} = \dfrac{Q_{in}+Q_r}{V_{dec}}X_H - \dfrac{Q_r+Q_w}{V_{dec}}X_{H,rec} \\ \dot{X}_{I,rec} = \dfrac{Q_{in}+Q_r}{V_{dec}}X_I - \dfrac{Q_r+Q_w}{V_{dec}}X_{I,rec} \\ \dot{X}_{S,rec} = \dfrac{Q_{in}+Q_r}{V_{dec}}X_S - \dfrac{Q_r+Q_w}{V_{dec}}X_{S,rec} \end{cases} \tag{21}$$

where:

$$x^T = \begin{bmatrix} S_I & S_S & X_I & X_S & X_H & S_O & X_{H,rec} & X_{S,rec} \end{bmatrix} \tag{22}$$

$$u^T = \begin{bmatrix} Q_{in} & Q_L & Q_r & Q_w \end{bmatrix} \tag{23}$$

$$y^T = \begin{bmatrix} S_I & S_S & X_I & X_S & X_H & S_O \end{bmatrix} \tag{24}$$

As we will indicate later on, the algorithm for this model is constituted by a bank of four adaptive observers for monitoring these four actuators for the case of a simple fault [20]. The faulty model for the first actuator (Q_{in}) is:

$$\dot{x} = f(x) + g_2(x)Q_L + g_3(x)Q_r + g_4(x)Q_w + g_1(x)\theta_{u_1} \tag{25}$$

$g_1(x)$, $g_2(x)$, $g_3(x)$ et $g_4(x)$ are the four columns of the matrix $g(x)$, the corresponding observer is given by:

$$\begin{cases} \dot{\hat{x}}_1 = f(x) + g_2(x)Q_L + g_3(x)Q_r + g_4(x)Q_w + g_1(x)\hat{\theta}_{u1} + H(\hat{x}_1 - x) \\ \dot{\hat{\theta}}_{u1} = -2\gamma(\hat{x}_1 - x)Pg_1(x) \\ \hat{y}_1 = C(\hat{x}_1) \end{cases} \tag{26}$$

Where $\hat{x}_1$ is the estimation of the state vector and $\hat{\theta}_{u1}$ is the fault estimation for the first observer. The residual r_1 is given by:

$$r_1(t) = \|\hat{y}_1 - y\| \tag{27}$$

The three other observers (θ_2, θ_3 and θ_4) have the same form.

In the case of multiple faults, firstly we should create a bank of four adaptive observers for the fault detection and identification. Secondly, to isolate the fault, we crate four banks of four adaptive observers where we use these four estimation φ_i from estimation vector.

5.2. Synthesis of the observer using the second method

We will treat the fault that can occur at the one of the four actuators of the system (Q_{in} Q_L Q_r and Q_w). Each of these actuators is divided into 5 parameter intervals, for each of them, a parameter filter is built. The values of the parameter filters for Q_{in} , Q_L , Q_r and Q_w are shown in the following tables:

Table.1. The values of the parameter filter of Q_{in} (Q_{in}^0 =2500 l/h)

No	1	2	3	4	nominal
Q_{in}^a	2410	2430	2450	2470	2490
Q_{in}^b	2430	2450	2470	2490	2510

Table.2. The values of the parameter filter of Q_L (Q_L^0 =43 l/h)

No	1	2	3	4	nominal
Q_L^a	32	34	36	38	42
Q_L^b	34	36	38	42	44

Table.3. The values of the parameter filter of Q_r (Q_r^0 =1800 l/h)

No	1	2	3	4	nominal
Q_r^a	500	800	1100	1400	1700
Q_r^b	800	1100	1400	1700	2000

Table.4. The values of the parameter filter of Q_w (Q_w^0 =600 l/h)

No	1	2	3	4	Nominal
Q_w^a	100	200	300	400	500
Q_w^b	200	300	400	500	700

Let $j = 1$ corresponds to parameter Q_{in}. The isolation observer for the i^{th} interval $[\theta_{u_j}^{a(i)} \quad \theta_{u_j}^{b(i)}]$ of the 1^{st} actuator parameter is given by:

$$\begin{cases}
\dot{\hat{S}}_I^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(S_{I,in} - \hat{S}_I^{\alpha(ij)}) + k_1(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{S}}_S^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(\hat{S}_{S,in} - \hat{S}_S^{\alpha(ij)}) - \dfrac{1}{Y_H}\rho_1 + \rho_3 + k_2(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_I^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(X_{I,in} - \hat{X}_I^{\alpha(ij)}) - \dfrac{Q_r}{V_r}(X_{I,rec} - \hat{X}_I^{\alpha(ij)}) + f_{X_1}\rho_2 + k_3(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_S^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(X_{S,in} - \hat{X}_S^{\alpha(ij)}) - \dfrac{Q_r}{V_r}(X_{S,rec} - \hat{X}_S^{\alpha(ij)}) + (1 - f_{X_1})\rho_2 - \rho_3 + k_4(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_H^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(X_{H,in} - \hat{X}_H^{\alpha(ij)}) - \dfrac{Q_r}{V_r}(X_{H,rec} - \hat{X}_H^{\alpha(ij)}) + \rho_1 - \rho_2 + k_5(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{S}}_O^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)}}{V_r}(\hat{S}_{O,in} - \hat{S}_O^{\alpha(ij)}) - Q_L\dfrac{\beta}{C_S}(C_S - \hat{S}_O^{\alpha(ij)}) - \dfrac{1 - Y_H}{Y_H}\rho_1 + k_6(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_{H,rec}^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)} + Q_r}{V_{dec}}\hat{X}_H^{\alpha(ij)} - \dfrac{Q_r + Q_w}{V_{dec}}\hat{X}_{H,rec}^{\alpha(ij)} + k_7(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_{I,rec}^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)} + Q_r}{V_{dec}}\hat{X}_I^{a(ij)} - \dfrac{Q_r + Q_w}{V_{dec}}\hat{X}_{I,rec}^{\alpha(ij)} + k_8(S_S - \hat{S}_S^{\alpha(ij)}) \\
\dot{\hat{X}}_{S,rec}^{\alpha(ij)} = \dfrac{\theta_{u_1}^{ob\alpha(i)} + Q_r}{V_{dec}}\hat{X}_S^{\alpha(ij)} - \dfrac{Q_r + Q_w}{V_{dec}}\hat{X}_{S,rec}^{\alpha(ij)} + k_9(S_S - \hat{S}_S^{\alpha(ij)})
\end{cases} \tag{28}$$

where: $\alpha = \begin{cases} a \\ b \end{cases}$

a, b correspond respectively to the actuator interval bound parameter $\theta_{u_j}^{a(ij)}$ and $\theta_{u_j}^{b(ij)}$

$$\begin{cases} \theta_{u1}^{oba(i)} = \theta_{u1}^{oba(i)} = Q_{in}^{0} & ,t < t_f \\ \theta_{u1}^{oba(i)} = \theta_{u1}^{a(i)} = Q_{in}^{a(i)} \ , \ \ \theta_{u1}^{obb(i)} = \theta_{u1}^{b(i)} = Q_{in}^{b(i)} & ,t \geq t_f \end{cases} \tag{29}$$

5.3. Simulation and comparison results

In this section, we will give the results from the two developed methods for fault actuator and visualize the process outputs, the residuals and the fault estimation.
Initially, we will give the results without fault, and then we will observe the case of a simple and multiple actuator faults.

5.3.1. The first method: Adaptive observer

- **Case1: No fault**

Figure (3) shows the result of the six process outputs and the four residuals. It is mentioned that these initial residual values are not equal to zero and they need a certain time to converge to zero. This necessary time depends on the two matrix H and P, the time to converge to 0 depends on the P and the oscillation of the residue is conditioned by the H; finally the value of the residual, if there is a fault, depends on the constant γ.

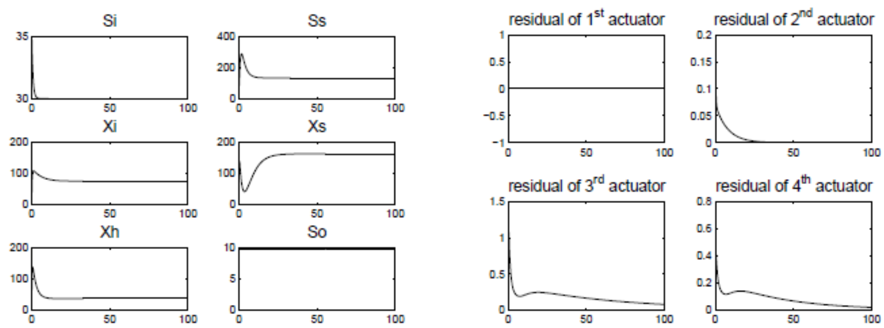

Fig. 3. Outputs process and residuals r_i (No fault)

- **Case2: Single fault**

To show in detail the fault isolation algorithm we have chosen the example where the faulty actuator parameter is $Q_{in}^{f} = 2420 l/h$. Q_L , Q_r and Q_w are maintained at their nominal value.

We have applied a fault at time $t_f = 50\ days$ in the first actuator Q_{in} . In figure (4) we presented the default effect on the six process outputs and the four residuals r_i associated to the four observers. At the beginning the four residuals needs a short time period to converge. From the figure, we see that all residuals leave zero at $t = 50\ days$ but after a very short period, $r_1(t)$ that corresponds to the input Q_{in} return to its initial value.

While we observe two possible situations for the three others residuals: that is to stabilize on new values, like the r_2 the residual of the second input Q_L, or they converge to a new value, as the r_3 and r_4 corresponding to Q_r and Q_w inputs. Consequently, we have isolated the fault actuator correctly and rather quickly. In this case, the isolation time is $t_{iso} = 5.5\ days$, because the fault appears at $t_f = 50\ days$ and it has been isolated at $t_I = 55.5\ days$.

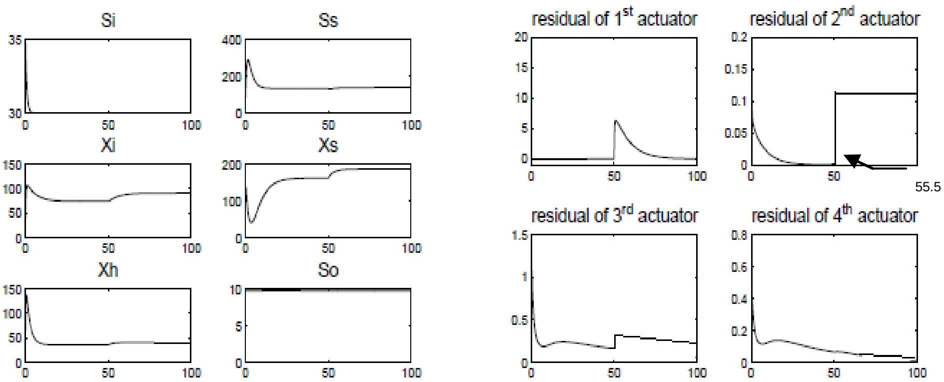

Fig. 4. Outputs process and residuals r_i (single fault)

- **Case 3: Single fault with output noise**

We will present the case where each output corrupted by a Gaussian distributed white noise vector with zero mean and a variance equal to 0.3. At time $t_f = 50\ days$ a single fault occurs in the first input Q_{in} . As we see in figure (5) we can easily conclude that results are similar with the case without noise, so we can say that the fault's effect on outputs is independent of the noise vector.

The noise that occurred on the system have a influence on residuals, but the effect can not be inhibited us to detect the fault. Therefore, we conclude that we have isolated a fault in the first actuator by using the same method that is developed later.

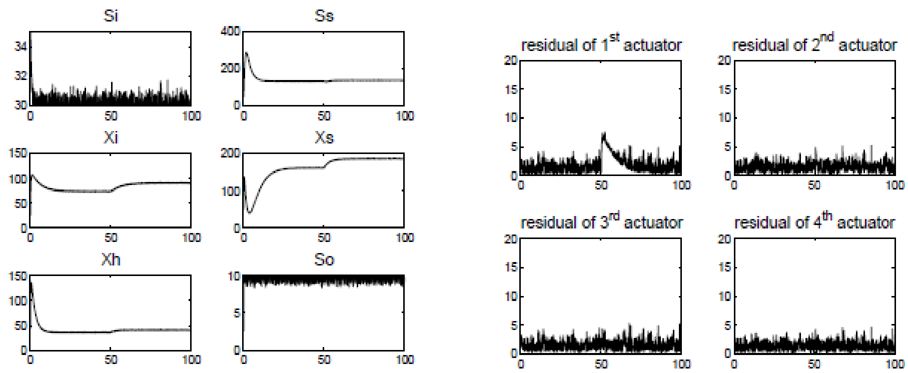

Fig. 5. Outputs process and residuals r_i with output noise (single fault)

- **Case 4: Multiple faults**

To illustrate the case where multiple faults occur on the system, we have applied a constant fault with magnitude $f_{a3} = 60l/h$ at time $t_{f_1} = 50\ days$ in the third actuator Q_r and another one $f_{a_4} = 50l/h$ in the fourth actuator Q_w at time $t_{f_2} = 65\ days$.

The fault of the third actuator is still occurred when the fault at fourth actuator has been introduced. Figure (6) shows, the fours residual to the observer, where at time $t_{f_1} = 50\ days$ all of them leave zero so the first fault is detected from the detection and identification bank and the candidate values are $\varphi_1 = 22$, $\varphi_2 = 30.5$, $\varphi_3 = 32$ and $\varphi_4 = 25$. Then at time $t = 60\ days$, all the residuals r_i have returned to zero and at time $t_{f_2} = 65\ days$, the second fault have been detected with $\varphi_1 = 50$, $\varphi_2 = 52$, $\varphi_2 = 53$ and $\varphi_4 = 51$.

In figure (7), we can see the eight residuals, $s_{3,i}$ and $s_{4,i}$ of the third and fourth isolation bank. The dashed line separates the first from the second fault. In the third bank $s_{3,i}$, before the dashed line and at time $t_{f1} = 50\ days$, only the residual $s_{3,3}$, associated to the third actuator, leaves zero but the other residuals corresponding to the other three actuators stays at zero. In the contrary all the residuals of the fourth bank leave zero for a short time period. Therefore we conclude that we have isolated the actuator fault.

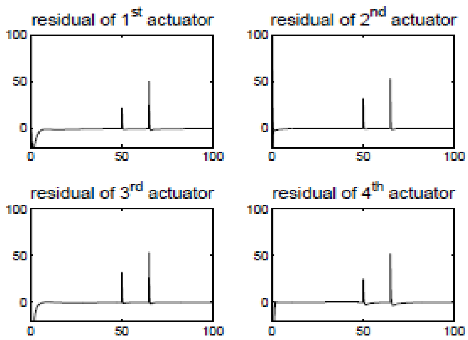

Fig. 6. Residuals r_i for to the detection and identification bank

After the dashed line and at time $t_{f_2} = 65\ days$, in the fourth bank $s_{4,i}$ only the residual $s_{4,4}$ associated to the fourth actuator leaves zero, the others stay at zero. All of the residuals $s_{3,i}$ of the third bank leave zero as envisaged. At time $t_R = 70\ days$ all of the residual return to zero, so new faulty actuators can be treated.

5.3.2. The second method: Parameter interval

- **Case1: No fault**

Figure (8) shows the result of the six process outputs if we use this second method, we can see that it is the same as the first one.

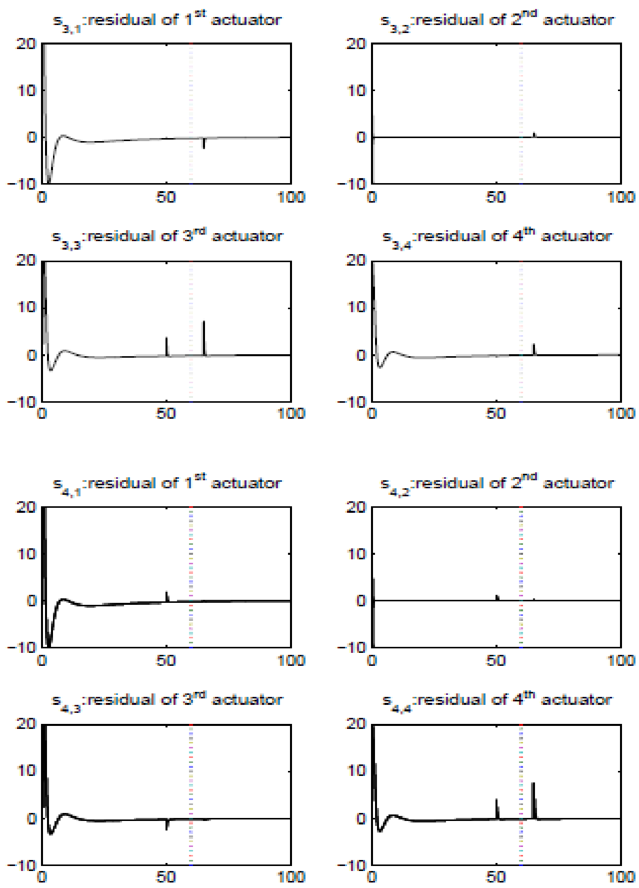

Fig. 7. Residuals $s_{k,i}$ for to the 3^{rd} and 4^{th} isolation banks

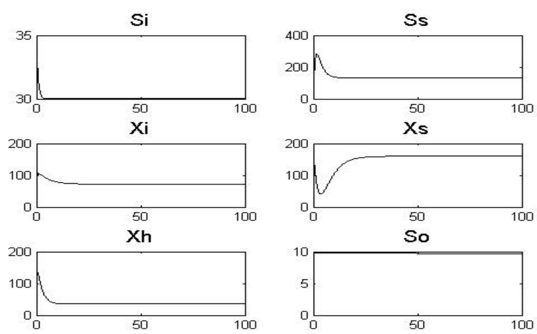

Fig. 8. Outputs process (No fault)

- **Case1: Single fault**

We have applied a fault at time $t_f = 50\ days$ in the first actuator Q_{in}. Figure 9 shows the default effect on the six process outputs.

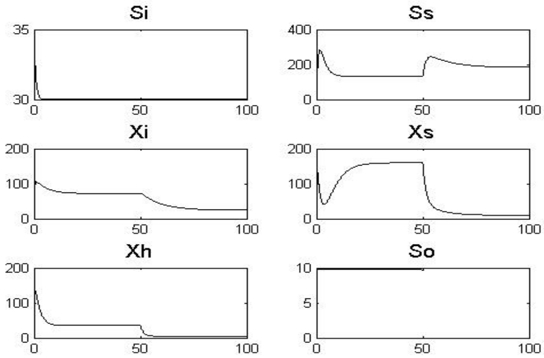

Fig. 9. Outputs process (single fault)

Figure 10 show that the filter of the 1st interval does not send the non containing signal. This is the case where $s = j$ and the interval contains the faulty parameter value. Therefore the fault is on Q_{in} and in the first interval.

It shows also that after t_f, the signals of two observers estimation errors are always different, so this interval cannot be excluded from"containing faulty parameter value", and we can assume that the parameter Q_{in} is the faulty actuator parameter.

Figure 11 presents the results of the 2^{nd} parameter filter of Q_L. Since the fault is not on this parameter, so the sign of the prediction errors $\varepsilon^a(t)$ and $\varepsilon^b(t)$ become the same after a period of the fault occurrence time.

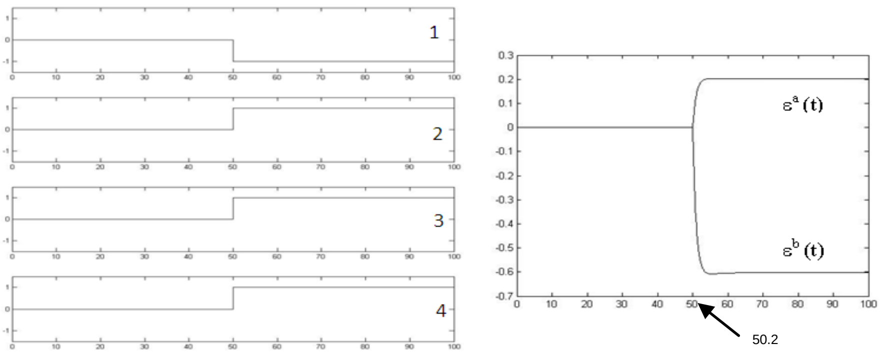

Fig. 10. The filter and the observer's estimation of the 1st interval.

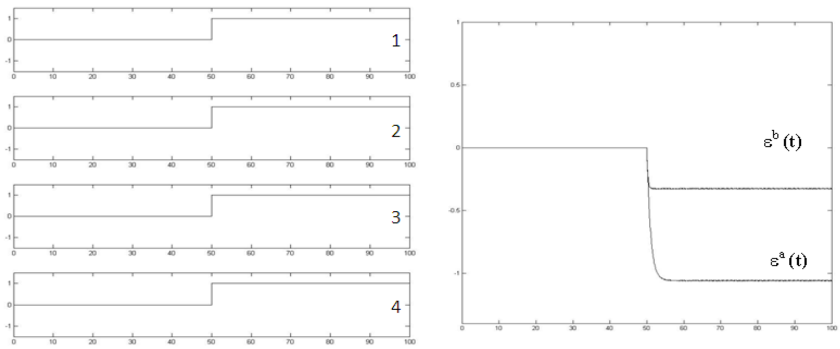

Fig. 11. The filter and the observer's estimation of the 2nd interval

5.3.3. Comparison of the two methods

Simulation runs have been used to compare these two methods. For various values of the faulty actuator parameter Q_{in} , the isolation times are presented in Table5.

Table.5. The values of the isolation time.

Faulty actuator parameter	2410	2420	2435	2460	2480
Isolation time (days) (1st meth)	7	5.5	3	4	5.1
Isolation time (days) (2nd meth)	1.3	0.20	0.20	0.1	0.1

So we can also do a comparison by using Q_L , Q_r or Q_w and we can conclude that these experimental results based on parameter intervals are faster than those based on adaptive observers. Though it is not so accurate as the detection and isolation results based on the *2nd* method, but it requires less computation and it is effective for non-linear systems diagnosis.

The use of an interval notion contributes to the fault detection speed in a positive way and it is also fits large kind of nonlinear dynamics systems. The only required conditions for the type of the nonlinear system is that the dynamic of the system is a monotonous function with respect to the considered parameter. This method does not need any parameter identification procedure. It is proven that if the parameter intervals are small enough the isolation speed will be fast enough. But it can not solve the problem of multiple faults. This problem consist the interest of our future works

6. Conclusion

Fault detection and isolation for nonlinear dynamics systems is the subject of this paper. The objective is to compare two methods based on the model. Experimental results show that the two detection and isolation methods are both effective and more accurate than others methods.

The first method using adaptive observers and the isolation can be carried out for the single and the multiple actuator faults, but the isolation speed is not ideal. However the second one which is based on parameter intervals can solve this problem but only for the single actuator fault. Some simulation results illustrate these advantages.

In our work we only focus on the faults of the actuator parameter, that is why one interesting future research direction is to extend this 2nd method firstly for multiple actuator faults and secondly to sensor fault isolation problem for nonlinear dynamic systems.

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